A
COMPENDIOUS SYSTEM
OF
Natural Philosophy:

With NOTES,

Containing the

MATHEMATICAL DEMONSTRATIONS,

AND

Some Occasional REMARKS.

In FOUR PARTS.

VOL. I.

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THE PREFACE.

IT being an usual Complaint with those who are unacquainted with Geometry, that they are discouraged by the Mathematical Demonstrations, from perusing Books of Natural Philosophy; *I apprehended that some Papers I had drawn up for the Use of my Pupils in the University, would not be altogether unacceptable, if published in such Form, that the Propositions, or Substance of the Book, might be read without Interruption from the Demonstrations. I have therefore in the following Treatise laid down the said Propositions, and endeavoured to prove them in a familiar and easy Manner, without Geometry, by way of* Text: *And for the sake of those*

who are ſkilled in that Science, have added the Demonſtrations, with ſome occaſional Remarks, by way of Notes. *And whereas the Writers on this Subject have appropriated to themſelves a Stile too* technical *for Beginners, I have, in* hopes *of being more eaſily underſtood, ſometimes choſe a plainer and more popular Way of Expreſſion, though perhaps not always ſo accurate.*

In the Introduction to the firſt Part, Notice is taken of the Method of Philoſophiſing *made Uſe of by* Des Cartes, *and others before him, ſo far as the Deſign of this* Compendium *required. I ſhall add here a few Conſiderations relating to the Method which prevails at this Time.*

In the preſent Method of Philoſophiſing, all Matter is conſidered (with reſpect to its Subſtance) as homogeneous, *or of the ſame Kind; and no other Cauſe or Principle of Action in Matter*

is allowed of, but what is well established by Facts.

Some Philosophers admit Elementary Fire, *as they call it, among their Principles; or, which comes to the same thing, they consider Fire as endowed with active Powers distinct from those of other Matter.* Keill, *in his Letter to Dr.* Cockburn, De Legibus Attractionis, aliisque Physices Principiis, *makes use of three Principles,* viz. 1. Empty Space. 2. The infinite Divisibility of Quantity. 3. The Attraction of Matter. *And affirms, that all* Physics *depend thereon.*

The first of his Principles the Reader may perhaps think ridiculous; but he may consider, that at that Time of Day, the Notion of a Plenum *was not wholly exploded: The laying down* Empty Space *as a first Principle was only calling out for* Elbow-Room *and a* clear Stage *-------But not to trouble the Reader with what others have done, I have*

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chosen and every where stuck to three; *and as oft as a* Phænomenon *occurred, which I could not account for by them, I have given it up as a Difficulty; not despairing, but that when all the Circumstances of the Phænomenon shall be thoroughly known, they alone may be found sufficient. It seems not consistent with the Regard a Philosopher should have to the Uniformity of Nature, every where observable, to call in a new Principle at every knotty Point. Those which I make use are,*

First, Attraction of Gravitation. *That is, a Disposition in Bodies to move towards each other, even when at great Distances asunder.*

Secondly, Attraction of Cohesion. *That is, a like Disposition in Bodies to move towards each other, but distinct from the former, in as much as it is observed to take Place only when the Bodies are very* near *together.*

Thirdly, Repulsion, *or a Disposition in Bodies, whereby in some Cases they*

endeavour

endeavour to avoid, or fly from each other.

The first of these is Matter of daily Observation. Thus a Ball let go from the Hand falls to the Ground.

The second may be seen in the following Instance. A small Portion of a Fluid forms itself into a Sphere or Drop: Which can only happen from a Disposition in the Particles of which it consists, to come as near as possible to each other.

An Instance of the third is this. If Air inclosed in a Bladder, be squeezed into a less Compass, the Air within, when the Pressure is taken off, restores the Bladder to its former Size: A plain Indication that the Particles of which the Air consists, endeavour to avoid or fly from each other (a).

These Dispositions in Bodies are not the Result of any Mechanical Cause

(a) *See another Instance of this Disposition, in Part* III. *Page* 161. *Note* m *of this* Compendium.

whatever; that is, such as may arise from the Effluvia *of Bodies, or the Action of any other* material Substance (b): *They are therefore the Act of an* immaterial Cause, *in Virtue of which* inactive Matter *performs the Offices for which it was designed.*

From

(b) Demonstration. *In the first Place it is well known, that if* Gravity *acts upon Bodies with the same Degree of Intenseness, whether they be in Motion or at Rest, it may be demonstrated, that Bodies, when projected, will describe* Parabola's; *and that when vibrating in* Cycloids, *their Vibrations will be* isocronous, *&c. In the next Place it is as well known, that Bodies when projected do describe* Parabola's, *and that when vibrating in* Cycloids, *their Vibrations are* isocronous, *&c. From which two Propositions it demonstratively follows, that if Gravity be the cause of the abovementioned Effects, it must act upon Bodies with the same Force, whether they be in Motion or at Rest.*

Again, it is well known, that, if Attraction of Cohesion *acts upon Rays of Light with the same Degree of Intenseness, whatever be the Velocity they move with, it may be demonstrated, that the Ratio of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction, will be given. But in Refraction of Light, the Ratio of those Sines is given in Fact; if therefore* Attraction of Cohesion *be the Cause of the Refraction of Light, it must act upon Rays of Light with the same Intenseness, whatever Velocity they move with.*

But no Effluvia *of Bodies, no* material Substance, *and in short no* material Cause *whatever, can act with the same Intenseness, or have the same Effect upon a Body in Motion, as upon the same Body at Rest; because as it is very*

From the first and third Principle (c), *together with the Properties of Matter enumerated in the first Chapter of this* Compendium, *which Properties must be always understood, the Elasticity or Spring of the Air, and from thence the Nature and Propagation of Sound are accounted for. And from the Spring of the Air considered as being augmented by Heat, and diminished by Cold, as it is observed to be, and the Air's being*

very well known to Mathematicians, to whom I address this Note, Body *can only act upon* Body, *according to the Sum or Difference of their Motions. It remains therefore, that the two Dispositions herein mentioned, are not the Result of any* material Cause *whatever: Which is one Part of the Proposition to be demonstrated.*

As to the other Disposition in Bodies, their Repulsion, *since Rays of Light are also affected by it, as it appears they are by an Experiment of Sir* Isaac Newton's, *referred to in the foregoing Note, it may very reasonably be supposed, though we don't at present know the exact Law of its Action, to affect Bodies in Motion after the same Manner that it would do the same at Rest, and that it therefore is also the Result of no* material Cause *whatever.*

(c) *The Law or Manner wherein these Principles are observed to act in different Circumstances, are determined from Facts, in Part* I. *Chap.* 3. *The Law of the third, so far as it relates to the Air, will be found in Part* II. *Chap.* 3. *of this* Compendium.

at

at the ſame time affected by the firſt Principle, the Phænomena *of the Winds are explained. By the ſecond Principle, the Coheſion of Matter, the various Degrees of Hardneſs obſervable in it, the Diſſolution of Bodies by Fluids, with other* chemical *Operations; and in particular the* Phænomena *of Fermentation, and conſequently the Cauſes of Thunder and Lightning,* &c. *By this Principle alſo the riſing of Fluids in ſmall Tubes, and from thence the aſcent of Sap in Vegetables are accounted for; all which Particulars, except the two firſt, are treated of in the ſecond Part of this* Compendium: *as alſo the Refraction of Light, and conſequently all that Train of* Phænomena *depending thereon, which is the Subject of the third Part. By the firſt Principle, the ſeveral Circumſtances relating to falling Bodies, and to the Motion of Projectiles, together with the Doctrine of* Pendulums, *(treated of in the firſt) and*

and likewise all those which relate to the Pressure of Fluids (treated of in the second Part) are determined. And above all is deduced that most curious Doctrine of central Forces, *just touched upon in the first, but largely and fully explained in the fourth Part, by which (assuming that the heavenly Bodies were at first put into Motion by their Creator) we are enabled to assign the Cause of the Continuation thereof, with all its Modifications and Irregularities; to determine the necessary Shape of those Bodies; and to account for the ebbing and flowing of the Sea,* &c. *Difficulties, too great seemingly for human Reason to surmount!*

Some of the Phænomena *which I have not been able to give a satisfactory Account of, from the abovementioned Principles, are the Reflection of Light, its Emission from luminous Bodies, and the Formation and Ascent of Vapour. This may be only owing to the want of better Acquaintance with the Circum-*

stances

ſtances of thoſe Phænomena; *that is, more ſufficient* Data, *or Facts to found their Solution upon: So that we are not to conclude immediately, that the Principles are inſufficient; but rather to wait with Patience: The Diligence of others may render that eaſy, which our utmoſt Efforts at preſent are not able to ſurmount.*

However, as a Reader unacquainted with Studies of this Kind, may wonder that ſo many of the Phænomena *or Appearances of Nature, ſhould be accounted for by ſo few Principles; and becauſe it may be a Means of giving him ſome Inſight into the Subject of theſe* Sheets, *I will here lay down the following Propoſitions, which are immediately deducible from the Principles, and alſo nearly connected with the* Phænomena *to be accounted for by them; by means of which, he will more readily perceive the Connection or Relation between the one and the other.*

PROPOSITION

PROPOSITION I.

Matter is an *unactive* Substance; that is, it is quite indifferent either to Motion or Rest; insomuch that the least Force imaginable acting upon it will move it, if at Rest; and alter its Course when moving: And further no Body has a Power either of putting it self into Motion, or of stopping or altering its Course when it does move. Notwithstanding which, every Body acts upon all others (provided their Distances be not too great) perpetually endeavouring to put them into Motion.

Hence we have the true Idea of the Gravity or Weight of Bodies belonging to the Earth, and their Disposition to descend towards it on all Occasions. Those Bodies are equally indifferent to Motion or Rest; but by Virtue of the first Principle are attracted, that is, drawn towards the Sun, the Moon, the

rest

rest of the Planets, and the Earth; but towards this last more strongly than towards any of the rest; and so they tend, gravitate, *or are* heavy *towards that. The Reason that they are attracted more forcibly towards the Earth, than towards those other Bodies, is, that although it be one of the Laws of the first Principle, that it operates according to the Quantity of Matter in Bodies, and therefore the Attraction of the Sun should be the most prevalent, in as much as that Body contains the most Matter; yet it is another Law of that Principle or Disposition, that it acts more strongly according to the nearness of Bodies to each other: This latter Consideration in the present Case, overbalances the former; and so the Bodies about us tend towards the Earth.*

PROPOSITION II.

If a large round Body be covered every where with smaller ones to an equal Height or Distance from its Surface;

face; and if thoſe ſmaller ones tend towards the large Body, by virtue of the firſt Principle, and are, at the ſame time, diſpoſed to fly from each other by virtue of the third; and ſuppoſing farther, that when they touch or are very near each other, their Diſpoſition to avoid each other exceeds their Tendency to the large Body, and when they are at a certain greater Diſtance from each other, that Diſpoſition is leſs than their Tendency to the large Body: Then will thoſe ſmaller Bodies keep at certain Diſtances from each other, and conſtitute an *elaſtic*, *compreſſible* Subſtance ſurrounding that large Body, and *gravitating* towards it on all Sides.

Hence an Idea of the Nature and Condition of the Atmoſphere *ſurrounding the Earth, with all its Properties.*

N. B. *When I ſay a Body* tends *to another, I don't mean that it* moves *towards it, but only that it would move towards*

towards it, if nothing prevented. Thus, a Bird while mounting aloft into the Air, tends *towards the Earth, as much as one that is falling down; for the one would fall as well as the other, if nothing prevented.*

PROPOSITION III.

If, while the abovementioned ſmaller Bodies are in the State ſuppoſed in the foregoing Propoſition, any one, or more of them, be made to move (ſuppoſe for Inſtance half way) towards the next, it will by virtue of the third Diſpoſition, drive or impel thoſe it comes nearer to, cloſer together; which Bodies, when that other moves back again (as it will immediately do, being repelled by them) will recede from each other again: That is, a kind of tremulous Motion will be communicated to them by that other, and for the like Reaſon, by them to the next; and ſo on through the whole, or at leaſt to a great Diſtance from where it began.

Hence

Hence we may form an Idea how Sound *is excited by the Tremors of a Body during its Vibration, and propagated through the Air.*

PROPOSITION IV.

One of the Laws by which the ſecond Principle is obſerved to act, is, that Bodies act upon one another, not in Proportion to the Quantity of Matter they contain, as by the firſt Principle; but only according to the Largeneſs of their Surfaces, and the nearneſs of the Surface of one Body to that of another.

From hence we may underſtand, that ſuch Particles of Bodies as are flat or ſquare, and ſo ſituated among each other as to touch or be very near one another in many Points, will conſtitute what we call an hard Body, *and thoſe Particles*

which are more round, or ſo ſituated that leſs Portions of their Surfaces are near together, will attract one another with a leſs Force, and ſo form a ſofter Body; *thoſe which are round, or nearly ſo, will attract one another ſtill leſs, and alſo ſlide more eaſily over one another, and ſo form what we call a* fluid Body.

Proposition V.

When two Bodies meet together, if the Particles which conſtitute the one, be diſpoſed, by Virtue of the ſecond Principle, to move towards thoſe of the other with a greater Degree of Force, than the Particles of either Body are diſpoſed to move towards themſelves; thoſe of the firſt will leave it, and run in among thoſe of the ſecond: And for the ſame Reaſon, thoſe of the ſecond will ſever from that, leave it and enter in between thoſe of the firſt. And if the

the Motion with which this is done, be very violent, and the Bodies be of the inflammable Kind, their Particles by thus rubbing and clashing one against another will be sufficiently heated to take Fire, and will burst out into Flame.

Hence Dissolutions, Fermentations, Explosions, Eruptions of Vulcano's, Thunder, Lightning, Aurora Borealis. *With all other* Phænomena *of that Tribe.*

PROPOSITION VI.

If a Pipe, open at both Ends, and of a very small Bore, have one End dipped into Water, the Water will run up into the Pipe above the Surface of the Water on the outside (being drawn up by the Tendency it has by the second Principle to that Part of the inner Surface of the Pipe, which is just above it, as it rises) till that inner

Surface which is ſtill juſt above it, be loaded with as great a Weight of Water as that Tendency can ſupport.

Hence a right Notion of the Aſcent of Sap *in Vegetables*; *the* Suction of Fluids by Spunges: *With all other* Phænomena *reducible to that Head.*

Proposition VII.

If a Body moving right forwards, but obliquely with reſpect to the Surface of another Body, at length comes ſo near that Body as to be diſpoſed by the ſecond Principle to tend towards it; inſtead of continuing to go right on, it will turn out of its Way towards that Body before it comes at it; and conſequently will ſtrike or enter it in a nearer Place, and in a leſs oblique Direction, than it would have done, in caſe it had gone right on. If it enters the Body, it ſtill keeps turning out

out of its Courſe the ſame Way as before, till it has got ſo far within it, that there ſhall be as many Particles of the Body behind it to attract it backwards, as there are before it near enough to attract it forwards: After which it goes right on in its laſt acquired Direction, till it comes near the other Side; for while it is ſurrounded with as many Particles to attract it one Way as another, it is the ſame thing as if it were not attracted at all. When it has got ſo near the other Side, that there are fewer Particles before it to attract it forwards, than there are behind it, near enough to attract it backwards, it then begins to turn out of its Courſe towards the inſide of the Body; that is, from that Side of the Body towards which it is going; and continues to bend its Courſe the ſame Way, till it has got ſo far out of the Body, that there are no Particles of the Body behind it, near enough to it to attract it any more.

After which it pursues an undisturbed Course in the Direction it acquired last of all.

Hence we have a just Idea of the Refraction of Light *with all the* Phænomena *arising therefrom; which are no other than so many Cases of this Proposition.*

Proposition VIII.

If several Bodies be moving right forwards, and at length be attracted by another Body, as supposed in the foregoing Proposition, but some with greater Degrees of Force or Intenseness, than others; those which are attracted with the greatest Force, will turn the farthest out of their Way towards that Body; and consequently if all of them, before this happened, were moving in one Direction, they will be made to part from each other, and move different Ways.

Hence

Hence an Idea of the different Refrangibility of the Rays of Light.

PROPOSITION IX.

If a Body be made to move from another Body, towards which by the first Principle it tends, its Motion will be retarded continually; that is, it will move ſlower and ſlower: If it moves towards that Body, its Motion will be continually increaſed; and unleſs it be made to move directly to or from it, its Courſe will always keep bending towards it, ſo that it ſhall deſcribe a Curve, concave, or hollow, on the Side next the Body.

Hence all the Phænomena *of* falling Bodies, *and of* Projectiles.

PROPOSITION X.

If a Body, that by the firſt Principle tends towards another Body, moves

towards it on the Surface of an inclined Plane, its Motion (as in the foregoing Propoſition) will be continually increaſed; and if it moves from it on the ſame Plane, its Motion will be retarded continually, but leſs in Proportion to the Obliquity of the Plane: (that is, leſs in Proportion as the Plane deviates from the Perpendicular) the Interpoſition of the Plane preventing in ſome Meaſure the Effect its Tendency to the other Body, would otherwiſe produce. And the Velocity it acquires by rolling down one Plane, will by virtue of its *Inactivity*, or that Diſpoſition Bodies have to continue their State of Motion or Reſt, inable it to roll up another fitly diſpoſed.

Hence the Solution of the Phænomena *of* Bodies deſcending on inclined Planes, *and the* Vibration of Pendulums.

PROPOSITION

PROPOSITION XI.

If a Body, acted upon by the firſt Principle, be cauſed to move to or from another, in any other Direction than ſuch as paſſes through the Center of that Body, the firſt Body when left to itſelf, will begin to bend its Courſe towards that other Body; and if the Direction it moves in and the Velocity it moves with, be properly adjuſted to each other, it will move quite round that Body without touching it or coming to it: and if that Adjuſtment be ſuch, that the Body ſhall return to the Place it ſet out from with the ſame Velocity and Direction it had when it was there before, it will revolve round that other Body over and over again in the ſame Path.

From

From hence we have the Solution of the Motion of the primary Planets round the Sun, *and of the* secondary ones round the Primary.

Proposition XII.

If a Body be revolving about another as in the last Proposition, and a third Body approaches them, towards which they both shall also tend, the Motion of the revolving Body will be disturbed: That is, its Path will be altered, and Irregularities in its Course will ensue; because its Tendency to that third Body in some Parts of its Course will conspire with, and in others perhaps be opposite to its own Motion. And not only so, but the Tendency it has to the Body about which it revolves, will in some Situations be increased, and in others be diminished by the Action of the third; which will also conduce towards altering its Course.

Hence

Hence the Lunar Irregularities, *and all other* Diſturbances in the Motion of the Heavenly Bodies *on their too near Approach towards each other.*

PROPOSITION XIII.

Imagine a large Body covered all over with ſmaller ones tending to its Center: Suppoſe alſo a diſtant Body, towards which they all tend, but the little ones with leſs Degrees of Force than they do towards the Body they cover. Then will ſuch of thoſe ſmaller Bodies, as are neareſt the diſtant one, loſe Part of their Tendency to the Body they cover; and ſo will thoſe ſmaller ones which are fartheſt off, or placed on the oppoſite Side the large Body. But, as to thoſe ſmaller Bodies, which are at the ſame Diſtance from the diſtant Body with the Center of the large Body itſelf, their Tendency

Tendency to the Body they lie upon, will be increaſed. The reſt will have their Tendency increaſed or diminiſhed more or leſs, according to their Nearneſs to thoſe whoſe Tendency is increaſed or diminiſhed the moſt *(d)*.

Hence ariſes the Difference in the Weights Bodies *have upon the Earth's Surface, at the Approach and Departure of the Heavenly Bodies, (but chiefly of the Moon,) to or from that Side of the Earth where the Bodies are; and conſequently the* ebbing and flowing of the Sea, *the Water riſing where its Weight or Tendency to the Earth is diminiſhed, and ſinking at the ſame Time in thoſe Places where its Weight is augmented. That the Approach and Departure of the Moon ſhould cauſe a greater Difference in the Weight of Bodies on the Earth, than the Ap-*

(d) What is affirmed in this and the foregoing Propoſition, depends on a Train of Reaſoning too long to be inſerted here. To underſtand it throughly, read Chapters the 18th and 19th of Part the Fourth.

proach and Departure of the other Heavenly Bodies, is owing to the nearneſs of the Moon to the Earth; which Conſideration in this Caſe overbalances the Conſideration of her Smallneſs, the above-mentioned Effects depending in a great Meaſure, on the Proportion the Diameter of the Earth bears to the Diſtance of the Heavenly Bodies.

PROPOSITION XIV.

If a Body, whoſe Parts tend to the Center thereof, conſiſts wholly of a Fluid, or be partly ſolid and partly fluid, provided ſome of the Fluid be at the Surface, and very diſtant Parts thereof communicate with each other; and the Body have no Motion about its Axis, it will ſettle into a ſpherical Form, the mutual Tendency of its Parts towards each other, contracting it into the leaſt poſſible Shape. But if it revolves about its Axis, all its

Parts

Parts will endeavour to fly off from that Axis; but ſuch as are fartheſt from the Axis, more than the reſt: Conſequently thoſe Parts in its Surface, which are the fartheſt from the Extremities of that Axis, being alſo fartheſt from the Axis itſelf, will have a greater Endeavour to fly off, than ſuch as are nearer thoſe Extremities; beſides, as is evident, the former will endeavour to fly off directly from the Center, but the latter not ſo. The above-mentioned Endeavour therefore in the former will take off a much greater Degree of their Tendency to the Center than the Endeavour of the latter will; and ſince the ſame may be ſaid of thoſe which are at any other aſſignable equal Diſtances from the Center, all thoſe which lie between the Center of the Body, and ſuch, as are fartheſt from the Extremities of the Axis, will have their Tendency to the Center much more diminiſhed; than

than those will, which lie between the Center and the said Extremities; These latter Parts therefore will press in towards the Center, overbalance the former, and raise them to a greater Distance from it than they were at before, restoring thereby an *Equilibrium* of the Parts of the Body one among another. On which Account the Body will assume a flattish or oblate Form. That is, supposing Lines drawn through the middle of its Axis at right Angles therewith, those Lines will be lengthen'd and the Axis itself will be shorten'd.

Hence the Figures of the Heavenly Bodies.

PROPOSITION XV.

The Impetus or Force wherewith a Body in Motion endeavours to proceed forwards, depends not only on the Quantity of Matter in that Body, but likewise

likewiſe on the Swiftneſs the Body moves with: Thus, the Stroke of an Hammer is not only according to the Bulk or Weight of its Head, but is alſo according to the Swiftneſs of the Motion it ſtrikes with. If therefore two Bodies of equal Quantities of Matter be ſuſpended at the Ends of a Lever of equal Arms, each of them when the Lever turns on its Center, having equal Degrees of Swiftneſs or Velocity, will therefore have the ſame Impetus or Force whereby they endeavour to proceed (being in like Circumſtances with reſpect to both thoſe things, which alone can give the one a Force or Tendency to move with, ſuperior to the other) and conſequently neither of them will preponderate. If one of the Bodies be larger than the other, the larger Body having the ſame Velocity with the other, but more Matter, will have the Advantage, and preponderate. If the Arms of the Lever are unequal, and

and the Bodies equal, that Body which is at the greateſt Diſtance from the Center of Motion, moving the quickeſt, will have the Advantage over the other that way, and overpoiſe it. So that the leaſt Body or Power, imaginable, may be made to equiponderate, overpoiſe, or keep in Motion the greateſt, by being applied to ſuch a Machine, and in ſuch Manner, that when the Machine moves, what it wants in Weight or Force, may be made out by the Velocity it has, compared with the Velocity the Body has at the ſame Time, which is to be equiponderated, over-poiſed, or moved by it.

This holds equally in all Machines, *and is the* Foundation *of their* Theory.

PROPOSITION XVI.

Imagine the Surface of a large round Body to be covered every where, or in

Part, with ſmaller ones to an equal Height, and that theſe ſmaller ones tend towards the large Body by the firſt Principle; imagine alſo the whole Maſs of ſmaller ones divided into Columns reaching from top to bottom; thoſe Columns, if their Baſes be equal, will equiponderate, or be an equal Counterpoiſe to one another; and ſo they will, if their Baſes be unequal: For in this Caſe the Columns being of unequal Size in Diameter, if a larger Column ſubſides, the lower Parts of that Column (to find Room for themſelves) will raiſe a ſmaller Column farther than the larger one ſettled in the ſame time, and in ſuch Proportion that, what the little Column wants in Weight, will be made out to it in Velocity; and conſequently, according to what was ſhewn in the foregoing Propoſition, the little Column will be a juſt Balance to the greater.

Farther, if in the abovementioned Suppoſition, there be a Body among thoſe ſmaller ones, heavier than a Bulk of them equal to its own Bulk, a Column of which that Body is a Part, will be heavier than any other Column of an equal Baſe; it will therefore ſubſide, permitting the Body to come to the Bottom: if the Body be lighter than a Bulk of the ſmaller ones equal to its own Bulk, a Column, of which it makes a Part, will be lighter than any other; the Body therefore will be buoy'd upwards, till it riſes ſo far out above the Surface, that it, together with the Column below it, may be a Counterpoiſe to another Column of equal Baſe.

Hence the Effects of the Preſſure of Fluids *upon one another, and upon* Solids *immerſed in them.*

Proposition XVII.

Imagine the Surface of a large round Body covered every where, or in Part, with ſmaller ones to an equal Height; and that theſe ſmaller ones tend towards the large one by the firſt Principle, and that they are at the ſame time diſpoſed to fly from one another by Virtue of the third, conſtituting thereby an elaſtic Subſtance ſurrounding that large Body, as in Propoſition the ſecond; and let them be divided into Columns, as in the laſt Propoſition. And let it be farther ſuppoſed, that the Diſpoſition in thoſe ſmaller Bodies, whereby they endeavour to depart from each other, is capable of being increaſed by Heat; and that at the Bottom of ſome of theſe Columns, that Diſpoſition is actually increaſed, but no where elſe, or at leaſt not in ſo great a Degree: then

then will the Bodies, where that Diſpoſition is increaſed, diffuſe themſelves into a larger Space, and ſo taking up more Room than an equal Number in the neighbouring Columns, a Column of which they are a Part, will become lighter than a neighbouring one of an equal Baſe. For, ſince the Bodies in the lower Parts of this Column are more diſtant from each other, than ſuch as are in other Columns, this Column cannot contain ſo many of them; that is, it cannot be ſo heavy as another of equal Baſe, unleſs it be longer; that is, unleſs the uppermoſt Parts thereof ſtand out above the Tops of the neighbouring Columns; but this they will not do; for by Virtue of the Tendency thoſe Parts have to the large Body, they will immediately (like Water raiſed above the Banks, which before confined it) ſpread themſelves every Way. This Column therefore, which, ac-

cording to the foregoing Propoſition, before this happened, was a Counterpoiſe to thoſe, which are round about it, being now become lighter, is no longer ſo. The Conſequence of which is, that the lowermoſt Parts of the neighbouring Columns, will preſs in under this from all Sides to reſtore the Equilibrium. Neither can the Equilibrium be reſtored, ſo long as the Place we have been conſidering remains hotter, than thoſe which are round about it. For, ſince the Bodies, that come in, will ſpread themſelves into a larger Space by Means of the Heat they receive there, and fill up more Room, than the like Number in another Column of equal Baſe, the Column to which they belong, will, for the Reaſons abovementioned, always be lighter than another of equal Baſe. And conſequently, according to the Tenour of the foregoing Propoſition, the neighbouring Columns

will overpoise it, whatever Dimensions, as to their Bases, we suppose them to be of.

Imagine the like to happen to a Column or Columns of the Earth's Atmosphere, and the lower Parts of the neighbouring Columns rushing in accordingly at the Bottom from all round, and you have an adequate Idea of the Cause and Nature of the Winds; *every Stream of the Particles of the Atmosphere rushing in, as above, being a distinct Wind blowing from that Point of the Compass from which they come. And if you conceive the Center of that warmer Space to shift its Place variously upon the Surface of the Earth, you then get the Idea of the several Sorts of them, as the* Trade Winds, Monsoons, *&c. For Instance, if it shifts regularly along the same Path, it causes Trade Winds; if now*

 forwards,

forwards, and then backwards, Monsoons, &c.

These are the Principal Phænomena *in Natural Philosophy that are independent of each other; the rest are for the most Part, no other than so many particular Cases, Circumstances, or Consequences of these, or, in short one way or other related to them. For the Solution of which, I refer the Reader to the Book it self.*

From a due Consideration of the Propositions here laid down, the Reader will be able to form a true Judgment *of the Nature and Business of* Natural Philosophy; *will see the* Uniformity *and* Consistency *of the several Parts thereof with each other, and therein the wonderful* Wisdom *and* Contrivance *of the* supreme Being, *in choosing so short and easy a Method of producing so great a Variety of Effects.*

There

There is one Thing more I think proper to be taken Notice of, before I put an End to this Preface; viz. *That it has been a ſtanding Objection againſt all* Natural Philoſophy *in general; that whereas it aſcribes Effects to natural or* mechanical Cauſes, *acting by* fixed *and* unalterable Laws, *it therefore excludes a Providence and the immediate Care and Protection of the* ſupreme Being, *making him no other than an Idle Spectator of Things here below.*

In Anſwer to this, it is to be conſidered in the firſt Place, that the Principles of the Philoſophy *which is now received, are ſo far from being* mechanical Cauſes, *at leaſt thoſe which are here made Uſe of, that, as above demonſtrated, they are the very Reverſe; and conſequently can be no other than the continual acting of God upon Matter, either mediately or immediately. Conſe-*

quently

quently Natural Philosophy, *by endeavouring to account for the* Phænomena *of Nature by those Principles, it is evident that it is so far from excluding the* Deity *from being concerned in the Affairs of this World; that it tends to shew that none are performed without his Order and Direction. Neither, secondly, does* Natural Philosophy *inculcate, that the Laws by which those Principles act, are* fixed *and* unalterable: *The Accusation is therefore foreign. But to consider this Matter a little more particularly.*

When, in Natural Philosophy, *a Principle is said to act according to a particular Law, the Meaning is not, that it acts necessarily and unalterably so; but only, that it does so* ordinarily, *and in common Cases. Doubtless the* Author, *both of Matter and of those very Principles by which it acts, can, notwithstanding those Principles, cause it to be not differently from what it would*

would do in conſequence of them alone, and ſo by that means produce Effects contrary to the common Courſe of Nature, whenever he ſhall think proper. That he has done ſo, when wiſe Ends required it, appears from Hiſtory. That it may be done a Thouſand Ways, unperceived by us, is evident. For Inſtance, though Lightning may be accounted for by theſe Principles; and in all Probability is ordinarily *the Reſult thereof; yet who will affirm, that in any one* particular *Caſe, that thoſe Principles formed that very Lightning, or that its Courſe was directed by them? Upon the whole therefore, to preſume, that the ordinary and common Courſe of Nature is not ſometimes altered, is* haſty *and* unwarrantable.

MATERIAL ERRATA.

PREFACE. First Edition.

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APPENDIX to Part I. First Edition.

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PART III. Second Edition.

198	—	24	—	situated at	—	at, situated to
202	—	23 and 25	—	LM, BL	—	LMq, BLq

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A COMPENDIOUS SYSTEM OF Natural Philosophy.

With NOTES
Containing the MATHEMATICAL DEMONSTRATIONS and ſome occaſional REMARKS.

PART I.
OF
The Properties of BODIES.
Their Laws of MOTION.
And
The Mechanical POWERS.

CAMBRIDGE,
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MDCCXXXIV.

A COMPENDIOUS SYSTEM OF Natural Philosophy.

PART I.

The INTRODUCTION.

SO wild and extravagant have been the Notions of a great part of Philosophers, both ancient and modern, that it is hard to determine, whether they have been more distant in their sentiments from truth, or from one another; or have not exceeded the fancies of the most fabulous Writers, even Poets and Mythologists. This was owing to a precipitate proceeding in their searching into Nature, their neglecting the use of Geometry and Experiment, the most necessary helps to the finding out Causes and proportioning them to their Effects.

THEIR manner of Philosophizing was to give bodies certain *arbitrary* properties, such as

beſt ſerv'd their purpoſe in accounting for the Phænomena * of Nature; from whence proceeded ſo many various Sects of Philoſophers; every one aſcribing a different cauſe to the ſame appearance, as his particular genius and imagination led him.

The chief agreement obſervable among moſt of the Ancient Philoſophers conſiſts in this, *viz.* that they conceived all bodies as compoſitions of Air, Earth, Fire, and Water, or ſome one or more of theſe, from whence they acquired the name of Principles or Elements, which they ſtill retain.

Epicurus advanc'd a little farther, and aſſerted, that though bodies conſiſted of ſome one or more of theſe, yet that they were not ſtrictly Elements, but that they themſelves conſiſted of Atoms; by an accidental concourſe of which, (as they were moving through infinite ſpace in lines nearly parallel) all things received their Form and manner of Exiſtence †.

Des Cartes has contrived an Hypotheſis very different from the reſt, he ſets out with a ſup-

* By a Phænomenon of Nature is meant any motion or ſituation of bodies among one another, which offers it ſelf to the notice of our ſenſes, and is not the immediate reſult of the action of an intelligent Being.

† For the Opinions of the Ancient Philoſophers conſult *Diogenes Laertius* and *Stanley*'s Lives.

poſition

position that the Universe at first was entirely full of matter, that, from this matter when first put in motion, there would necessarily be rubbed off (by the grinding of the several parts one against another) some particles sufficiently fine to pass through the hardest and most solid bodies without meeting with any resistance: of these consists his *Materia subtilis*, or *Materia primi Elementi*. He imagined that from hence also would result other particles of a globular form, to which he gave the name of *Materia secundi Elementi*. Those which did not so far lose their first figure, as to come under the denomination of *Materia primi* or *secundi Elementi*, he call'd *Materia tertii Elementi*; and maintain'd that all the variety, which appears in natural bodies was owing to different combinations of those Elements.

He likewise supposes that God created a certain quantity of Motion and allotted it to this mass of Matter, which therefore (being created) can no more be annihilated without an omnipotent hand, than Body it self; in consequence of which he was obliged to teach, that the quantity of motion is always the same: so that if all the Men and Animals in the World were moving, yet still there would be no more motion, than when they were at rest; the motion lost

being transfer'd to the Æther. So unaccountable are the notions of this great Philosopher, that it is surprizing his doctrine should have met with such universal reception, and have got so strong a party of Philosophers on its side, that notwithstanding it was more absurd, than the Schoolmens *Substantial Forms*, they must all be exploded to make way for his ingenious Hypothesis.

Des Cartes has been said by a late Writer * to have joyned to his great genius an exquisite skill in Mathematics, and by mixing Geometry and Physics together to have given the World hopes of great improvements in the latter. But this Writer ought to have been told that what he look'd upon in *Des Cartes's* Book of Principles as Demonstrations, are only Illustrations, there not being a Demonstration from Geometry in all his Philosophical Works †.

The present method of Philosophizing establish'd by Sir *Isaac Newton* is to find out the Laws of Nature by experiments and observations. To this, with a proper application of Geometry, is owing the great advantage the

* Mr. *Wotton* in his Reflections on Ancient and Modern Learning.

† See this Subject discuss'd more at large in *Keil's* Introduction to his Examination of Dr. *Burnet's* Theory &c. Second Edition.

pre-

present System of Philosophy has over all the preceding ones, and the vast improvement it has received within the last Age. It is indeed in vain to imagine, that a System of Natural Philosophy can be framed by any other method: for without observations it is impossible we should discover the Phænomena of Nature, without experiments we must be ignorant of the mutual actions of bodies, and without Geometry we can never be certain whether the causes assign'd be adequate to the effects we would explain, as the various Systems of Philosophy built on other foundations evidently shew.

THIS way of searching into Nature was first proposed by my Lord *Bacon* *, prosecuted by the *Royal Society*, the *Royal Academy* at *Paris*, the honourable Mr. *Boyle*, Sir *Isaac Newton*, &c.

WHAT wonderful advancement in the knowledge of Nature may be made by this method of enquiry, when conducted by a genius equal to the work, will be best understood by considering the discoveries of that excellent Philosopher last mentioned. To him it is principally owing, that we have now a rational System of Natural Philosophy; 'tis he who, by pursuing the sure and unerring method of rea-

* See his *Novum Organum.*

soning

ſoning from experiment and obſervation, joyned with the moſt profound skill in Geometry, has carryed his enquiries to the moſt minute and inviſible parts of matter, as well as to the moſt remote bodies in the Univerſe, and has eſtabliſh'd a Syſtem, not ſubject to the uncertainty of a mere Hypotheſis, but which ſtands upon the ſecure baſis of Geometry it ſelf.

CHAP.

CHAP. I.

The Properties of Body.

IT being the design of *Physics* or Natural Philosophy to account for the Phænomena of the Material World, it must be our method to begin with laying down the known properties of Body.

THESE are 1. Solidity. 2. Extension. 3. Divisibility. 4. A capacity of being moved from place to place. 5. A Passiveness or Inactivity. These are all the essential properties of Bodies that we are acquainted with, and that they are essential, appears from what follows.

1. SOLIDITY, called also Impenetrability, is that power which Body has of excluding all others out of its place.

THAT Body, as such, must be endued with this property follows from its nature, for otherwise two bodies might exist in the same place, which is absurd. The softest are equally solid with the hardest; for we find by experiment, that the sides of a Bladder filled with Air or Water, can by no means be made to come together *.

* At *Florence* a hollow Globe of Gold was fill'd with Water, and then exactly clos'd; the Globe thus clos'd was put into a Press driven by the force of Screws; the Water finding no room for a nearer approach of its particles, toward each other, made its way through the pores of that close Metal standing in drops like Dew on the outside, before the Globe would yield to the violent Pressure of the Engine. V. *Locke's* Essay B. 2. c 4.

2. THAT

2. That Body is extended, is self evident, it being impossible to conceive any Body which has not length, breadth and thickness, that is, Extension.

3. It is no less evident that Body is divisible, for since no two particles of Matter can exist in the same place, it follows that they are really distinct from each other, which is all that is meant by being divisible.

In this sense the least conceivable particle must still be divisible, since it will consist of parts, which will be really distinct *. To illustrate this by a familiar Instance: Let the least imaginable piece of matter be conceived lying on a smooth plane surface, 'tis evident the surface will not touch it every where, those parts therefore, which it does not touch, may be supposed separable from the rest, and so on as far as we please; and this is what is meant when we say matter is infinitely divisible.

* This Proposition is demonstrated Geometrically thus, suppose the line *AD* (*Fig.* 1.) perpendicular to *BF* and another as *GH* at a small distance from it also perpendicular to the same line; with the Centers *CCC* &c. describe Circles cutting the line *GH* in the points *e*, *e*, *e*, &c. Now the greater the Radius *AC* is, the less is the part *eH*. But the Radius may be augmented in infinitum, and therefore the part *eH* may be diminished in the same manner; and yet it can never be reduc'd to nothing, because the Circle can never coincide with the right line *AF*; consequently the parts of any magnitude represented by *GH* may be diminished in infinitum. *Q. E. D.* V. *Keil's* Introd. ad Phys. Præl. 3, 4, 5. *Gravesande's* Elem. Math. Phys. L. 1. c. 4. Schol.

How

How far matter may actually be divided, may in some manner be conceiv'd from hence *, that a piece of Wire, gilt with so small a quantity as 8 grains of Gold, may be drawn out to the length of 13000 feet, the whole surface of it still remaining cover'd with Gold †.

A quantity of Vitriol being dissolved and mix'd with 9000 times as much Water, will tinge the whole, consequently the Vitriol will be divided into as many parts, as there are visible portions of matter in that quantity of Water ‡.

THERE are Perfumes, which, without a sensible diminution of their quantity, shall fill a very large space with their odoriferous particles, which must therefore be of an inconceivable smallness, since there will be a sufficient number in every part of that space, sensibly to affect the organ of smelling.

4. THAT all matter is moveable follows from its being finite: and to suppose it positively

* We have a surprizing instance of the minuteness of some parts of Matter from the nature of Light and Vision. Let a Candle be lighted and placed in an open plane, it will then be visible two miles round, consequently was it placed two miles above the surface of the Earth, it would fill with luminous particles a Sphere, whose diameter was four miles, and that before it had lost any sensible part of its weight. The force of this Argument will appear better when the Reader is acquainted with the cause of Vision.

† *Keil's* Introd. ad Phys. Præl. 5. Religious Philos. Contempl. 25.

‡ Mem. de l' Acad. 1706.

infinite is abſurd, becauſe it conſiſts of parts *.

5. By the Paſſiveneſs or Inactivity of matter, (commonly call'd its *Vis Inertiæ*) is meant the propenſity it has to continue its ſtate of Motion or Reſt, till ſome external force acts upon it. This will be farther explain'd under the firſt Law of Nature.

CHAP. II.

Of Vacuum.

I. PLACE void of Matter is call'd empty Space or *Vacuum*.

II. It has been the opinion of ſome Philoſophers, particularly the *Carteſians*, that Nature admits not a *Vacuum*, but that the Univerſe is entirely full of Matter, in conſequence of which opinion they were oblig'd to aſſert, that if every thing contain'd in a veſſel could be taken out or annihilated, the ſides of that veſſel, however ſtrong, would come together; but this is contrary to experience, for the air may be drawn out of a veſſel by means of the Air Pump, which will nevertheleſs remain whole, if its ſides are ſtrong enough to ſupport the weight of the incumbent Atmoſphere.

III. Should it be objected here, that it is impoſſible to extract all the Air out of a Veſſel, and

* See Mr. *Law*'s Tranſlation of ABp. *King de Origine Mali* Note 3.

that

that there will not be a Vacuum on that account; the anſwer is, that ſince a very great part of the Air that was in the Veſſel may be drawn out, as appears by the quick deſcent of light Bodies in a Receiver*, there muſt be ſome vacuities between the parts of the remaining Air: which is ſufficient to conſtitute a *Vacuum.* Indeed to this it may be objected by a *Carteſian*, that thoſe vacuities are fill'd with *Materia ſubtilis* that paſſes freely through the ſides of the Veſſel, and gives no reſiſtance to the falling Bodies; but ſince the exiſtence of this ſame *Materia ſubtilis* can never be prov'd, we are not oblig'd to allow the objection, eſpecially as Sir *Iſaac Newton* has found, that all Matter affords a reſiſtance nearly in proportion to its denſity †.

There are many other Arguments to prove this, particularly the motions of the Comets through the Heavenly Regions without any ſenſible reſiſtance; the different weight of Bodies of the ſame bulk *&c.* but thoſe being not yet explain'd are not ſo proper to be inſiſted on in this place.

* By this Term is meant any Veſſel, out of which we extract the Air by the Air Pump.

† *Newtoni* Opt. p. 310.

CHAP. III.

Of Attraction and Repulsion.

BESIDES the forementioned properties of Matter, it has also certain powers or active Principles, known by the names of *Attraction* and *Repulsion*, probably not essential or necessary to its existence, but impressed upon it by the Author of its Being, for the better performance of the Offices for which it was design'd.

II. ATTRACTION is of two kinds. 1. Cohesion, or that by which the several particles whereof Bodies consist, mutually tend toward each other. 2. Gravitation, or that by which distant Bodies act upon each other.

III. THE Attraction of Cohesion is prov'd from abundance of Experiments, of which some of the most obvious are as follows.

1. LET a small glass Tube (commonly call'd a Capillary Tube) open at both ends, be dipt into a Vessel of Water, the Water will immediately rise up in the Tube to a certain height above the level. This rise of the Water is manifestly owing to the Attraction of those particles of the glass which lie in the inner surface of the Tube immediately above the Water: accordingly the quantity of Water raised

raised is always proportionable to the largeness of that surface *.

2. Let two spheres of Quicksilver be placed near each other and they will immediately run together and form one globule.

IV. The Laws of this Attraction are 1*st*. That it acts only upon contact or at very small distances, for the Spheres mentioned in the last experiment will not approach each other till they are plac'd very near. 2. It acts according to the breadth of the surfaces of the Attracting Bodies, and not according to their quantities of Matter. Let there be two polish'd glass Plates laid one upon another in such a manner, as to touch at one end, and there make a very small angle: if two unequal drops of oil be put between these plates at equal distances from the line of contact, so that the least may touch both glasses, they will then both move towards the ends that touch, because the Attraction of the

* The heights the Water rises to in different Tubes, are observ'd to be reciprocally as the diameters of the Tubes, from whence it follows that the quantities raised are as the surfaces which raise them.

Dem. Let there be two Tubes, the diameter of the first double to that of the second, the Water will rise half as high in the first as in the second, now was it to rise equally high in both, the quantity in the first would be four times as great as in the second, (Cylinders of equal heights being as the squares of their diameters; 11. *El.* 14.) therefore since it is found to rise but half as high, the quantity is but twice as much, and therefore as the diameter; but the surfaces of Cylinders are as their diameters, therefore the quantities of Water raised are also as the surfaces. *Q. E. D.*

See a Dissertation on this Subject. Part II.

surfaces

ſurfaces inclines that way; but the largeſt, touching the glaſſes in moſt points, will move the faſteſt. 3. 'Tis obſerv'd to decreaſe much more than as the ſquares, of the diſtances of the Attracting Bodies from each other increaſe: that is, whatever the force of Attraction is at a given diſtance, at twice that diſtance it ſhall be more than 4 times leſs than before *.

V. From hence it is eaſy to account for the different degrees of hardneſs in Bodies; thoſe whoſe conſtituent particles are flat or ſquare, and ſo ſituated as to touch in many points, will be hard; thoſe particles which are more round and touch in fewer points will conſtitute a ſofter Body; thoſe which are ſpherical will form a fluid †.

VI. Attraction of Gravitation is that by which diſtant Bodies act upon each other. This is ſeen every day in the falling of heavy Bodies toward the Earth.

VII. The Laws of this Attraction are 1. That it decreaſes as the ſquares of the diſtances between the Centers of the attracting Bodies increaſe. Thus a Body at the ſurface of the Earth (*i. e.* about the diſtance of 4000 miles from its Center,) which weighs 10 Pounds, if it was plac'd 4000 miles above the ſurface of the Earth *i. e.* twice as far diſtant from the Center as before,

* *V. Keilii* Opera Ed. 4to. p. 626.
† *Newtoni* Optic. p. 335.

would

would weigh 4 times less, if thrice as far, 9 times less *&c.* The truth of this Proposition is not to be had from Experiments, (the utmost distance we can convey Bodies to, from the surface of the Earth, bearing no proportion to their distance from its Center,) but is sufficiently clear from the Motions observ'd by the Heavenly Bodies. 2. Bodies attract one another with forces proportionable to the quantities of Matter they contain; for all Bodies are observ'd to fall equally fast in the exhausted Receiver, where they meet with no resistance. From whence it follows, that the action of the Earth upon Bodies is exactly in proportion to the quantities of Matter they contain; for was it to act as strongly upon a less Body as upon a larger, the least Body being most easily put into motion would move the fastest. Accordingly it is observable that the weight of a Body is the same, whether it be whole or ground to powder*.

VIII. From hence it follows, that was a Body to descend from the surface toward the Center of the Earth, it would continually become lighter and lighter, the parts above attracting it as well as those below, in which case it is demonstrated by the Mathematicians, that the Gravity would decrease with the distance of the Body from the Center †.

* *Gravesande* Lib. 4. Chap. 11. *Cotes*'s Preface to *Newton*'s Princip.

† *Dem.* Let there be a Body as *P*, (*Fig.* 2.) placed any where within

Scholium. It may be proper to observe here, that when Philosophers speak of Bodies gravitating to, or attracting each other, that Body is said to gravitate to another, which moves towards it, while the other actually is, or appears to be at rest; and this other is said to attract the former; though indeed the force be-

within a concave sphere, as *AB*; which let us suppose divided into an infinite number of thin concentric surfaces; I say the Body *P* will be attracted equally each way by any one of these, *v. g.* the interior *IHKL*. Let there be lines as *IL*, *HK*, &c. drawn through any point of the Body *P*, in such a manner as to form the surface of two similar figures, suppose Cones, the diameters of whose bases may be *IH*, *KL*, which let be infinitely small. These bases (being as the squares of the lines *IH*, *KL*) (20. *Elem.* 6.) will be directly as the squares of their distances from *P* (for the Triangles being infinitely small are similar.) But those bases include all the particles of matter in the interior surface, that are opposite to each other; the opposite attractions are therefore in the same ratio with those bases, that is as the squares of the distances *PH*, *PI*. But the attraction is inversely as the squares of the distances of the attracting Bodies, §. 7. *i. e.* inversely as the squares of the same distances *PK*, *PI*; these two ratios destroying each other, it is evident, that if the concavity of the Sphere was fill'd with Matter, that alone, which lies nearer the Center than the Body, can affect it, the respective actions of all the parts, that are more distant, being equal, and in contrary directions, since the same is demonstrable of any of the remaining concentric surfaces. Let us then see what effect that which lies nearer the Center than the Body will have upon it, which may be considered as a Sphere, on whose surface the Body is plac'd. The distances of each particle of Matter from the Body, (taken collectively) will be as the diameter of the Sphere, or as the Radius, *i. e.* as the distance of the Body from the Center; their action therefore upon the Body will be inversely as the square of that distance; but the quantity of Matter will be as the cube of that distance; (18. *Elem.* 12.) the attraction therefore will be also in that proportion. Now, these two ratios being compounded, the attraction will be only as the distance of the Body from the Center. *Q. E. D.*

ing

ing mutual and equal on both ſides (as will be explain'd under the *3d* Law of Nature) the ſame Term might be apply'd to either the gravitating or attracting Body.

It is farther to be obſerv'd, that when we uſe the Terms, Attraction or Gravitation, we do not thereby determine the Phyſical Cauſe of it, as if it proceeded from ſome ſuppoſed occult quality in Bodies; but only uſe thoſe Terms to ſignify an Effect, the Cauſe of which lies out of the reach of our Philoſophy. Thus we may ſay, that the Earth attracts heavy Bodies; or that ſuch Bodies tend or gravitate to the Earth: though at the ſame time we are wholly ignorant whether this is effected by ſome power actually exiſting in the Earth or in Bodies, or external to both: ſince it is impoſſible any error in our reaſonings can follow from hence; it being evident, that all the conſequences of ſuch tendency muſt be the ſame, let the cauſe be where or what it will.

X. Repulsion is that property in Bodies, whereby if they are placed juſt beyond the Sphere of each others Attraction of Coheſion, they mutually fly from each other.

Thus if an oily Subſtance lighter than Water be placed on the ſurface thereof, or if a piece of Iron be laid on Mercury, the ſurface of the fluid will be depreſs'd about the Body laid on it. This depreſſion is manifeſt-

ly occasion'd by a repelling power in the Bodies, which hinders the approach of the Fluid towards them.

But it is possible in some cases to press or force the repelling Bodies into the Sphere of one anothers attraction; and then they will mutually tend towards each other, as when we mix Oyl and Water till they incorporate *.

XI. Besides the general Powers forementioned there are some Bodies that are endued with another call'd *Electricity*. Thus Amber, Jet, Sealing-Wax, Agate, Glass and most kinds of Precious Stones attract and repel light Bodies at considerable distances.

The chief things observable in these Bodies are. 1. That they don't act but when heated. 2. That they act more forcibly when heated by rubbing than by fire. 3. That when they are well heated by rubbing, light Bodies will be alternately attracted and repell'd by them, but without any observable regularity whatever. 4. If a line of several yards in length has a Ball or other Body suspended at one end, and the other end be fixed to a glass Tube; when the Tube is heated by rubbing, the Electrical Virtue of the glass will be communicated from the Tube to the Ball, which will attract and repel light Bodies in the same manner as the

* We have an undeniable Proof of this Repulsive Force in Sir *Isaac Newton*'s Opticks. B. 3. and Query 31.

glass

glaſs it ſelf does. 5. If the glaſs Tube be emptied of Air, it loſes its Electricity *.

XII. LASTLY, the Loadſtone is obſerv'd to have Properties peculiar to it ſelf, as that by which it attracts and repels Iron, the Power it communicates to the Needle and ſeveral others †.

CHAP. IV.

Of the Laws of Motion commonly called Sir Iſaac Newton'*s Laws of Nature.*

I. ALL Bodies continue their ſtate of reſt or uniform motion in a right line, till they are made to change that ſtate by ſome external force impreſſed upon them.

THIS Law is no other than that univerſal property of Bodies, call'd Paſſiveneſs or Inactivity; whereby they endeavour to continue the State they are in, whatever it be. Thus a Top only ceaſes to run round on account of the reſiſtance it meets with from the Air, and the friction of the plane whereon it

* See *Hauksbee's* Experiments. *Philoſoph. Tranſact.* Numb. 326.

† Several ſolutions of theſe Properties of *Electricity* and *Magnetiſm* have been attempted by different Philoſophers, but all of them ſo unſatisfactory as not to deſerve a particular account in this Place. See *Chambers's* Dictionary in *Electricity*, and *Des Cartes* Opera Philoſophica. P. IV. §. 133. with ſeveral others cited in *Quæſtiones Philoſophicæ.* [illegible]

moves. And a Pendulum when left to vibrate in *vacuo*, where there is nothing to ſtop it but the friction ariſing from the motion of the pin on which it is ſuſpended, continues to move much longer than one in the open Air.

II. THE change of Motion, produc'd in any Body, is always proportionable to the force, whereby it is effected; and in the ſame direction, wherein that force acts.

THIS is an immediate conſequence of this Axiom, the Effect is always proportionable to its Cauſe. For inſtance, if a certain force produces a certain motion, a double force will produce double the motion; a triple force triple the motion *&c.* If a Body is in motion, and has a new force impreſſed on it in the direction wherein it moves, it will receive an addition to its motion proportional to the force impreſſed; but if the force acts directly contrary to its motion, the Body will then loſe a proportional part of its motion: again, if the force is impreſſed obliquely, it will produce a new direction in the motion of the Body, more or leſs different from the former in proportion to its quantity and direction. *

* This caſe is expreſſed more accurately by Mathematicians thus. If the proportion and direction of 2 forces acting upon a Body at the ſame time, be repreſented by the ſides of a Parallelogram, the Diagonal of that Parallelogram will repreſent the proportion and direction of their united forces.

Dem. Let the Body *A* (*Fig.* 3.) be impell'd with a force, which would carry it to *E*, in the ſame time that another, acting

upon

III. Reaction is always contrary, and equal to Action; or the actions of two Bodies upon each other are equal, and in contrary directions.

Thus, suppose a Stone or other Load to be drawn by an Horse; the Load reacts upon the Horse as much as the Horse acts upon the Load; for the harness, which is stretch'd equally between them both ways, draws the Horse towards the Stone, as much as the Stone towards the Horse, and the progressive motion of the Horse is as much retarded by the Load, as the motion of the Load is promoted by the endeavour of the Horse*. This will

upon it in the direction *AD*, would carry it to *D*. Imagine, that while the Body passes to *E*, the line *AD* (in which the Body moves by the other force) moves to *EB*, in a direction parallel to it self; when the Body has advanc'd to *G* in the line *AE*, the line *AD* will have got to *GF*, and the Body will have passed over such a part of it *GH*, as bears the same proportion to the whole line *GF*, as *AG* does to *AE*, that is *GH* (the shorter side of the Parallelogram *GM*, is to *GF*, or, which is the same thing, to *EB* (the shorter side of the Parallelogram *ED*,) as *AG* (the longer side of the former) is to *AE* (the longer side of the latter,) from whence the Parallelograms are similar, *El.* 6. *Def.* 1. and consequently, by 24. *El.* 6. the point *H* is in the Diagonal, that is, the Body will always be found in the line *AB*. Q. E. D.

Coroll. From hence we have an easy method of resolving a given motion into any two, or more directions whatever; *viz.* by describing a Parallelogram about the given direction as a Diagonal, the two sides of which will represent the directions sought. Thus, suppose a Body was impell'd in the Line *AB*, we may conceive it as acted upon by two forces at the same time, one towards *E*, the other towards *D*, or any other two whatever, provided the lines be drawn of such length, that, when the Parallelogram is compleated, the given line *AB* shall be its Diagonal.

* It may be thought perhaps that (two equal and contrary forces

be better explain'd from the following instance: let a Person sitting in a Boat draw another equally heavy towards him, they will both move towards each other with equal velocities let the Boat he sits in be the lightest, and it will move the fastest; because the action being equal on both sides, the same quantity of motion will be given to each Boat, that is the lesser will have the greater velocity *.

We have a farther confirmation of this from Attraction. Suppose two Bodies attracting one another, but hinder'd from meeting by some other Body placed between them: if their tendencies towards each other are not equal then the Body that is between them, will b pressed on one side more than on the othe and consequently the stronger pressure ove coming the weaker, they cannot remain rest, but will all move on continually in th direction wherein the stronger force acts; whic is both contrary to the first Law of Nature ar Experience. This may be try'd with a Loa stone and Iron; which, being put into prop Vessels contiguous to one another and made

forces destroying one another) the Horse will in this case be able to move at all, because the Load draws him back, as mu as he draws the Load forwards. But it is to be observ'd that strength of the Horse is not properly exerted upon the Load upon the Ground; consequently the Ground reacting and co nuing at rest pushes the Horse forward with just so much force the Horse exerts, above what is counteracted by the Load.

* See the distinction between Motion and Velocity. Chap.

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Plate I. Pag. 22.

P. 8.

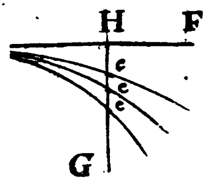


Fig. 2. P. 15.

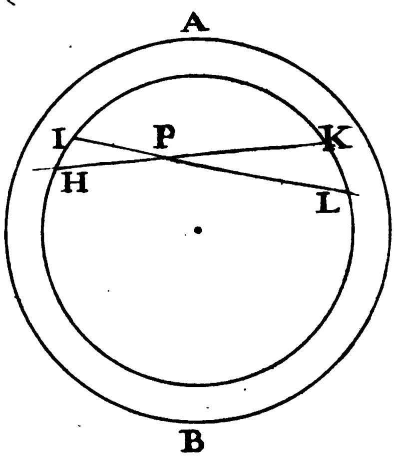
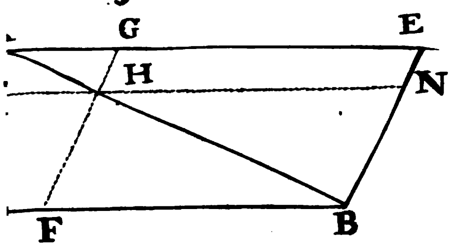


Fig. 3. P. 20.



float on water, will be an exact counterbalance to each other, and remain at rest, whatever be the attractive power of the Loadstone, or the proportion of their respective magnitudes.

THESE Laws receive an abundant additional proof from hence, *viz.* that all the conclusions that are drawn from them, in relation to the Phænomena of Bodies, how complicated soever their Motions be, are always found to agree perfectly with observation. The truth of which sufficiently appears in all parts of the *Newtonian* Philosophy *.

CHAP. V.

The Phænomena of Falling Bodies.

I. THE Laws of Nature being thus explained, we proceed to account for those Phænomena, which are solvable by them.

II. To begin with those of Falling Bodies. Constant experience shews, that Bodies have a tendency towards the Earth, which is call'd Gravity, the Laws of which were enumerated in Chap. 3. §. 7.

III. THE height, Bodies can be let fall from, bears so small a proportion to their distance from the Center of the Earth, that it cannot

* See these Laws explain'd more at large by *Cheyne* in his Principles of Philosophy. *Keil's* Introd. ad Phys. Præl. 11, 12.

sen-

ſenſibly alter their Gravity; which therefor may be conceiv'd, as acting conſtantly an uniformly upon them, during the whole tim of their fall: from whence they muſt neceſſa rily acquire at every inſtant, an equal degre of velocity, which on that account will cor ſtantly increaſe, in proportion to the tim the Body takes up in falling.

IV. The ſpaces Bodies fall through in di ferent times, reckoning from the beginnin of their fall, are as the ſquares of thoſe times thus, a Body will fall 4 times as far in 2 m nutes, as it does in one, and 9 times as f in 3, 16 times as far in 4 *&c.* *

* In order to demonſtrate this Propoſition, it will be neceſſa to lay down the following Theorem. *viz.*

That the ſpace a Body paſſes over, with an uniform motio is in a ratio compounded of the time and velocity. For t longer a Body continues to move uniformly, the more ſpa it moves over; and the faſter it moves during any inte val of time, the farther it goes; therefore the ſpace is in a rat compounded of both, that is, is had by multiplying one into t other.

Coroll. Therefore the area of a rectangle, one of whoſe ſid repreſents the celerity a Body moves with, and the other t time of its motion, will expreſs the ſpace it moves through.

This being premiſed, let the line *AB* (*Fig.* 4) repreſent t time a Body takes up in falling, and let *BC* expreſs the celeri acquir'd by its fall; farther, let the line *AB* be divided into a indefinite number of ſmall portions, *ei*, *im*, *mp*, &c. and let *e* *ik*, *mn*, *pq*, &c. be drawn parallel to the baſe. Now it is evide from §. 3. (*viz.* that the velocities are as the times in which the are acquir'd) that the lines *ef*, *ik*, *mn*, *pq*, &c. being to each oth (4. *El.* 6.) as the lines *Ae*, *Ai*, *Am*, *Ap*, &c. will repreſent th celerities in the times repreſented by theſe: that is, *ef* will be a the velocity of the Body in the ſmall portion of time *ei*, and *i* will be as the velocity in the portion of time *im*; in like manne

p

V. From this Propoſition it follows, that a Body falls 3 times as far, in the ſecond portion of time, as it does in the firſt; 5 times as far in the third; 7 times in the fourth, and ſo on in the ſeries of the odd Numbers: for otherwiſe, it could not fall 4 ſpaces in 2 minutes, and 9 in 3, as the Propoſition aſſerts.

VI. The ſpaces deſcrib'd by Falling Bodies in different times are as the ſquares of the laſt acquir'd velocities. For by §. 4. the ſpaces are as the ſquares of the times, and by §. 3. the velocities are as the times; therefore the ſpaces are alſo as the ſquares of the velocities.

VII. The ſpace a Body paſſes over from the beginning of its fall in any determinate time, is half what it would deſcribe in the ſame

pq will be as the velocity in the portion of time *po*, which portions of time being taken infinitely ſmall, the velocity of the Body may be ſuppos'd the ſame, during any whole portion; and conſequently, by the Corollary of the foregoing Theorem, the ſpace run over in the time *ei* with the velocity *ef* may be repreſented by the rectangle *if*: in like manner the ſpace run over in the time *im* with the celerity *ik*, may be expreſs'd by the rectangle *mk*; and that run over with the celerity *mn* in the time *mp*, by the rectangle *pn*; and ſo of the reſt. Thereforethe ſpace run over in all thoſe times will be repreſented by the ſum of all the rectangles, that is, by the triangle *ABC*, for thoſe little triangular deficiencies, at the end of each rectangle, would have vaniſhed, had the lines *ei*, *im*, *mp*, &c. been infinitely ſhort, as the times they were ſuppoſed to repreſent. Now as the ſpace, the Body deſcribes in the time *AB*, is repreſented by the triangle *ABC*, for the ſame reaſon the ſpace paſs'd over in the time *Ao* may be repreſented by the triangle *Aor*, but theſe triangles being ſimilar are to each other, as the ſquares of their homologous ſides *AB* and *Ao* (20. *El.* 6.): that is, the ſpaces repreſented by the triangles are to each other, as the ſquares of the times repreſented by the ſides. *Q. E. D.*

 time

time moving uniformly with its last acquir'd velocity *.

VIII. In like manner, when Bodies are thrown up perpendicularly, their velocities decrease, as the times they ascend in increase; their gravity destroying an equal portion of their velocity every instant of their ascent.

IX. The heights Bodies rise to, when thrown perpendicularly upwards, are as the squares of the times spent from their first setting out, to the moment they cease to rise. That is, if a Body is thrown with such a degree of velocity, as to continue rising twice as long as another, it shall ascend 4 times as high; if thrice, 9 times as high, *&c.*

These two are the converse of the 3*d* and 4*th* Sections †.

CHAP. VI.

Of the descent of Bodies on oblique Planes, and of Pendulums.

WHEN a Body descends on an oblique Plane, its motion is contiually acce-

* For let the time be *AB*, and the last velocity *BC*, the space the Body runs over, while it is acquiring that velocity, is *ABC*, but the space it would pass over in the time *AB*, was it to move uniformly with the celerity *BC*, is, by the Theorem, (Note p. 24.) the space *ABCD*, double the former. *Q. E. D.*

† See *Keil's* Introd. ad Physf. Præl. 11. *Gravesande* L. 1. Ch. 17.

lerated

lerated by the action of gravity, but in a leſs degree, than when it deſcends perpendicularly; its free deſcent in this caſe being hinder'd by the interpoſition of the Plane: from whence it follows, that what was ſaid in the laſt Chapter, concerning the perpendicular deſcent of Bodies, is true of ſuch as fall on oblique Planes, allowance being made for the difference of acceleration.

II. The effect of Gravity upon a Body falling down an oblique Plane, is as much leſs than the ſame acting on another falling freely; as the perpendicular height of the plane is leſs than its length *.

III. A Body falls through as much longer ſpace perpendicularly, than it does obliquely in the ſame time, as the oblique ſide of the Plane is longer than the perpendicular height †.

* *Dem.* Let *AC* (*Fig.* 5.) be the inclin'd Plane, the Body at *A*, and the action of gravity, whereby it endeavours to fall perpendicularly, repreſented by the line *AB*; let *AD* be perpendicular to *AC*, *AD* will then repreſent the direction by which the Plane acts upon the Body (for all Bodies act in lines perpendicular to their ſurfaces,) let then thoſe two forces be reſolved into one in the direction *AC*, (as ſhewn in Note to §. 4. Chap. 4) by compleating the Parallelogram *BD* whoſe Diagonal will be *AG*. In order to this *BG* muſt be let fall perpendicularly upon *AC* (that it may be parallel to the oppoſite ſide of the Parallelogram *AD*) conſequently (8. *Elem.* 6.) *AG* is to *AB* as *AB* to *AC*, that is, the tendency of the Body down the plane is to its perpendicular tendency, as *AB* is to *AC*. *Q. E. D.*

† That is, ſuppoſing *BG* (*Fig.* 5.) perpendicular to *AC*, the Body would fall to *G* in the ſame time it would fall to *B*, for, as was obſerv'd Note the laſt, *AB* is as much longer than *AG* as *AC* is longer than *AB*.

IV. The velocity a Body acquires by falling perpendicularly, as much exceeds that which it acquires by falling obliquely in the same time, as the space of its perpendicular descent in that time exceeds that of its oblique one *.

V. A Body takes up as much more time in falling down the oblique side of a Plane, than it does the perpendicular height of it, as the oblique side exceeds the height †.

Dem. It is evident that in all motions equally accelerated, the sum of the velocities produced in any time (that is the spaces run over in that time) are as the generating forces, that is, the space pass'd over by the Body on the oblique side *AC*, is to the perpendicular space a Body would pass over in the same time; as the action of gravity on the Body in the direction *AC*, to its perpendicular action in the direction *AB*: but these actions are to each other, (by §. the last,) as the oblique side to the perpendicular height, the spaces therefore pass'd over will be in the same proportion. *Q. E. D.*

* Since by § the last, a Body falls to *G*, (*Fig.* 5.) in the same time another falls to *B*, and by (Chap. 5. §. 7.) the space a falling Body passes over in any time, is half that which it would run over in the same time moving uniformly with its last acquir'd velocity, it follows that the Body falling down the oblique plane would pass over double the space *AG*, moving uniformly with its last acquir'd velocity, in a portion of time equal to that in which it was acquir'd; likewise double the space *AB* would be pass'd over by the other Body, moving uniformly with its last acquir'd velocity, in a portion of time equal to that in which it was acquir'd; but since the velocities of Bodies moving uniformly are as the spaces they run over in equal times, the velocities of the Bodies in G and B are to each other as double the lines *AG* and *AB*, that is as the lines themselves, which by §. 3. are as the spaces run through in the same time, from whence the Proposition is clear.

† *Dem.* The square of the time in which *AC* (*Fig.* 5.) is run over, is to the square of the time in which *AG* is run over, as *AC* to *AG*, (by Chap. 5. §. 4.) that is, since *AC*, *AB*, *AG* are continually proportional (8. *Elem.* 6.) as the square of *AC* to the square

VI. A Body acquires the ſame velocity in falling down the oblique ſide of a Plane, as if it fell freely through the perpendicular height of it *.

VII. A Body takes up the ſame time in falling through the Chord of a Circle, whether it be long or ſhort, as it does in falling perpendicularly through the diameter of the ſame Circle †.

VIII. Upon this is founded the Theory of Pendulums: for from hence it follows, that ſuppoſing a Pendulum could be made to vibrate in a Chord of a Circle, inſtead of an arch, all its vibrations would require the ſame time, whether they were large or ſmall ‡.

ſquare of *AB* (by *Def.* 10. *Elem.* 5.) therefore the times themſelves are as the lines *AC* and *AB*, that is, as the oblique ſide of the Plane to the perpendicular height. *Q. E. D.*

* *Dem.* The ſquare of the velocity a Body acquires by falling to *G*, is to the ſquare of the velocity it acquires by falling to *C*, as the ſpace *AG* to the ſpace *AC* (by Chap. 5. § 4.) that is (by 8. *Elem.* 6. and *Def.* 10. *Elem.* 5.) as *AGq* to *ABq*. But ſince *AG* is run over in the ſame time *AB* is, (ſee Note to §. 3) the velocity in *G* is to the velocity in *B*, as *AG* to *AB*, (by §. 4.) and conſequently ſince the velocities both in *C* and *B* bear the ſame proportion to that in *G*, they muſt be equal to each other. *Q. E. D.*

† *Dem.* It was demonſtrated (§. 3.) that a Body will fall from *A* to *G*, (*Fig.* 6.) on the inclin'd Plane *AC*, in the ſame time another would fall freely to *B*, provided *AGB* is a right angle, in which caſe *AG* (by 31. *Elem.* 3.) is a Chord of that Circle of which *AB* is the Diameter; therefore a Body falls through the Chord &c. *Q. E. D.*

‡ This may be illuſtrated by conceiving the laſt figure inverted (as in *Fig.* 7.), where ſuppoſing the Ball ſuſpended in ſuch a manner, as to ſwing in the right line *GA* inſtead of the Arch *GA*, it would always fall through it in the ſame time however long or ſhort

IX. From hence we ſee the reaſon, why the ſhorter arches a Pendulum deſcribes, the nearer its vibrations come to an equality, for ſmall arches differ leſs from their Chords than large ones. But if the Pendulum is made to vibrate in a Curve, which Mathematicians call a *Cycloid*; each ſwing will then be perform'd in the ſame time, whether the Pendulum moves through a larger or leſſer ſpace. For the nature of this Curve is ſuch, that the tendency of a Pendulum towards the loweſt point of it, is always in proportion to its diſtance from thence; and conſequently let that diſtance be more or leſs, it will always be run over by the Pendulum in the ſame time *.

ſhort it was, for the inclination of the line *GA* to the horizontal line *BC*, is not alter'd by inverting the figure.

* The Deſcription of a *Cycloid*.

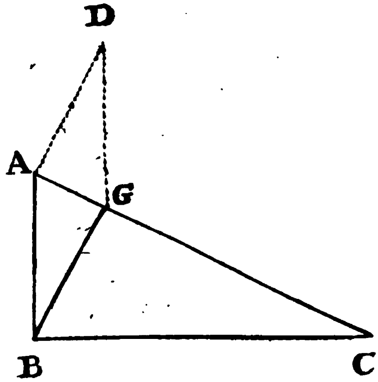
Upon the right line *AB*, (*Fig.* 8.) let the Circle *HDE* be ſo plac'd, as to touch the line in the point *H*, then let this Circle roll along upon it from *H* to *C*, as a wheel upon the ground, then will the point *H* in one revolution of the Circle deſcribe the Curve *HKC*, which is call'd a Cycloid. Now ſuppoſe two Plates of Metal bent into the form *HK* and *KC*, and placed in the ſituation *LH* and *LC*, in ſuch manner, that the points *H* and *C* may be apply'd to *L*, and the points anſwering to *K* be apply'd to *H* and *C*. This done, if a Pendulum as *LP*, in length equal to *LH*, be made to vibrate between the Plates or Cheeks of the Cycloid *LC* and *LH*, it will ſwing in the line *CKH*; and the time of each vibration, whether the Pendulum ſwings through a ſmall or a great part of the Cycloid, will be to the time a Body takes up in falling perpendicularly through a ſpace equal to *IK*, (half the length of the Pendulum;) as the Circumference of a Circle to its Diameter, and conſequently it will always be the ſame.

They that would ſee a Demonſtration of this and ſeveral other things relating to this Curve, may conſult *Huygens Horol. Oſcillatorium* or *Cotes's Harmonia Menſurarum*.

X.

24.

Fig. 5. *P.* 27.



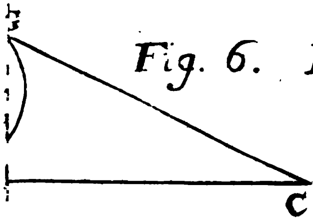


Fig. 6. *P.* 29.

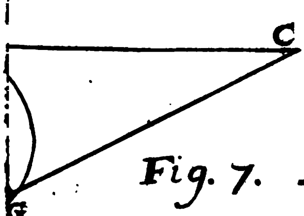


Fig. 7. *P.* 29.

X. The time of the deſcent and aſcent of a Pendulum, ſuppoſing it to vibrate in the Chord of a Circle, is equal to the time in which a Body falling freely would deſcend through eight times the length of the Pendulum.

For the time of the deſcent alone upon the Chord is equal to that in which a Body would fall through the Diameter of the Circle (by §. 7.); that is, twice the length of the Pendulum: but in twice that time (*viz.* during a whole vibration) the Body would fall four times as far (Chap. 5. §. 4.), that is, through eight times the length of the Pendulum.

XI. The times, that Pendulums of different lengths perform their vibrations in, are as the ſquare roots of their lengths *.

XII. The Center of *Oſcillation* is a point in which, if the whole gravity of a Pendulum was collected, the time of its vibration, would not be alter'd thereby †; this is the point from

* *Dem.* Let there be two Pendulums *A* and *B* (*Fig.* 9. and 10.) of different lengths, the time the firſt vibrates in (ſuppoſe through a Chord) is equal, to the time in which a Body would fall freely through *DA*, the Diameter of the Circle (as demonſtrated §. 7.); in like manner the time *B* vibrates in, is that in which a Body would fall through *FB*. Now the times in which Bodies fall through different ſpaces are as the ſquare roots of thoſe ſpaces, that is, of *DA* and *FB*, or of their halves *CA* and *CB*, *i. e.* of the lengths of the Pendulums. *Q. E. D.*

† The Rule for finding the Center of Oſcillation.

If the Globe *AB* (*Fig.* 11.) be hung by the ſtring *CD*, whoſe weight is inconſiderable, the Center of Oſcillation is found thus; ſup-

whence the length of a Pendulum is measur'd, which in our Latitude, in a Pendulum that swings seconds, is 39.2. inches.

XIII. THE squares of the times in which Pendulums, acted upon by different degrees of gravity, perform their vibrations in, are to each other, as the gravities *.

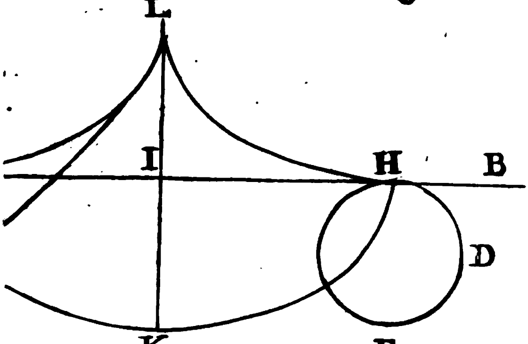
suppose *E* the Center of the Globe, take the line *G* of such a length, that it shall bear the same proportion to *ED* as *ED* to *EC*, then *EH* being made equal to $\frac{2}{5}$ of *G*, the point *H* shall be the Center of Oscillation

If the weight of the Rod *CD* be too considerable to be neglected divide *CD* (*Fig.* 12.) in *I*, so that *DI* may be equal to $\frac{1}{3}$ of *CD*, and make a line as *K*, in the same proportion to *CI*, that the weight of the Rod bears to that of the Globe, then having found *H* the Center of Oscillation of the Globe, as before, divide *IH* in *L*, so that *IL* may bear the same proportion to *LH*, as the line *CH* bears to the line *K*; then will *L* be the center of Oscillation of the whole Pendulum. See *Huygens* Horol. Oscillat. pag. 141, 142.

* *Dem.* The spaces falling Bodies descend through, are as the squares of the times, when the gravity by which they are impell'd is given (Chap. 5. §. 4); and as the gravity when the time is given (for the sum of the velocities produced in any time will always be as the generating forces): consequently when neither is given, they are in a ratio compounded of both; the squares of the times are therefore inversly as the gravities. [*For if in* 3 *quantities* a, b, c; a *is as* bc, *then* b : $\frac{a}{c}$, i. e. *if* a *is given, as* $\frac{1}{c}$ *or as* c *inversly.*] But if the squares of the times in which Bodies fall through given spaces are inversly as the gravities by which they are acted upon; then the squares of the times in which Pendulums of equal lengths, perform their vibrations, will be also in the same ratio, on account of the constant equality between the time of the vibration of a Pendulum, and of the descent of a Body through eight times its length (§. 12.).

FROM

Plate III. *Pag.* 32.



P. 31.

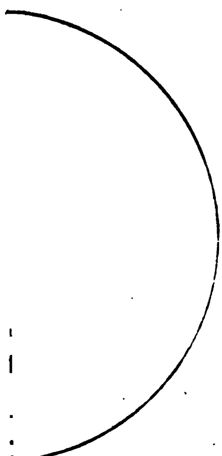
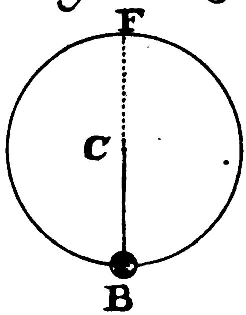


Fig. 10. *P.* 31.



FROM whence it follows, that a Pendulum will vibrate ſlower when nearer the Equator, than the ſame when nearer the Poles; for the gravity of all Bodies is leſs, the nearer they are to the Equator; *viz.* on account of the ſpheroidical Figure of the Earth, and its rotation about its Axis, as will be explain'd hereafter. To which we may add the increaſe of the length of the Pendulum occaſion'd by the heat in thoſe parts; (for we find by experiment that Bodies are inlarged in every dimenſion, in proportion to the degree of heat that is given them:) for which reaſon (Chap. 6. §. 11.) the vibrations of the Pendulum will alſo be ſlower.

CHAP. VII.

Of Projectiles.

1. A BODY projected in a direction parallel or oblique to the Horizon would proceed on in infinitum in a right line, (by the firſt Law of Nature) but being continually accelerated towards the Earth by its Gravity, it will deſcribe a Curve called a *Parabola* *.

* *Dem.* Let us ſuppoſe the Body thrown from *A*, in the direction *AB* horizontally (*Fig.* 13.) or obliquely (*Fig.* 14.) it would (if not attracted towards the Earth) in equal times deſcribe equal parts of the line *AB*, as *AC*, *CD*, *DE*, &c. but if in the firſt portion of time, while it moves from *A* to *C*, it deſcends by its Gravity as far as *G*, by a compoſition of theſe two Motions (Chap. 4. §. 2.)

II. THE greatest distance, to which a Body can be thrown with a given velocity, is at the elevation of 45 degrees*.

it will be found in *H*, and while it moves from *A* to *D* twice as far, it will move downwards to *M*, 4 times as far as before (Chap. 5. §. 4) and will therefore be found in *I* supposing *DI* 4 times as long as *CH*. Again, while it moves to *E* three times as far from *A* as *C* is, it will have moved downwards 9 times as far as it did in the first portion of time, and therefore will be found in *K*, provided *EK* be 9 times as large as *CH* &c. that is the lines *CH*, *DI*, *EK*, &c. will be to each other as the squares of the lines *AC*, *AD*, *AE*, &c. which is the property of the parabolic Curve, (De L' *Hospital* B. I. Prop. 1. Cor. 2. and Prop. 3. Cor. 1.) and consequently the line *AHIK*, &c. which the Body moves in, whether thrown horizontally or obliquely, is a Parabola. *Q. E. D.*

* It is demonstrated by the Writers on Conic Sections, that the Quotient which arises from the division of the square of the line *GH* by the line *AG* *viz.* the quantity $\frac{GHq}{AG}$ (in either of the parabolic curves, (*Fig.* 13. or 14.), or of the square of *MI* by the line *AM* viz. $\frac{MIq}{AM}$ or of the square of *NK* by *AN* viz. $\frac{NKq}{AN}$ &c. provided those lines are all parallel to *AB* which touches the curve in the point *A*, is always the same: which Quotient is call'd the *Parameter* of the point *A*.

Now the velocity, with which the Body is projected from *A*, being (*ex hypoth.*) such as would carry it to *C*, in the time it would fall by its own gravity to *G*; and to *E* in the time it would fall to *N*; and since it would move over twice the space *AN* in that time, had it moved uniformly with the velocity acquir'd at *N*; it follows, that the velocity it moves with from *A* to *E*, is to that which a Body acquires by falling to *N*, as *AE* to twice *AN* (Chap. 5. §. 7.) or as $\frac{1}{2}$ *AE* to *AN*. But the velocity a Body would acquire by falling through a fourth part of the Parameter of the point *A* viz. $\frac{\frac{1}{4}NKq}{AN}$ is to the velocity it would acquire by falling to *N*, as *AE* to 2*AN*: (see this demonstrated in Note †) therefore the velocity a Body ought to be projected with from *A* to make it describe the given *Parabola*

AHIK

AHIK, is equal to the velocity it would acquire by falling through a fourth part of the Parameter belonging to that point of the Parabola from whence it is projected.

† The square of the velocity acquir'd by a Body in descending through a fourth part of the Parameter, or $\frac{\frac{1}{4}NKq}{AN}$ is to the square of that which is acquir'd by falling through the line *AN*; as $\frac{\frac{1}{4}NKq}{AN}$ to *AN*, (Chap. 5. §. 6.), that is, multiplying both terms by *AN*, as $\frac{1}{4}NKq$ to *ANq*, and by extraction of their square roots; as $\frac{1}{2}NK$ to *AN*. Q. E. D.

Corol. This affords us an easy method of finding what direction it is necessary to throw a Ball in with a given velocity, in order to strike an object in a given situation. *v. g.* Let it be requir'd to strike an object as *K* with a ball thrown from *A* with a given velocity. Here it is only necessary to make the triangle *ANK* (suppose a right line drawn from *A* to *K*) such, that $\frac{NKq}{AN}$ or which is the same thing $\frac{AEq}{EK}$ in the triangle *AEK*, may be equal to four times the space a Body must fall through, to acquire such a degree of velocity as that with which it is intended to be thrown, and then *AE* will be the direction sought. In order to this we must lay down the following Lemma.

Let there be a Circle as *ABC (Fig. 15.) AK* a Tangent in the point *A*, *AB* perpendicular to the Horizon and parallel to *KE* or *KI*, I say $\frac{AEq}{EK} = AB$. For the angle *ABE* is equal to the angle *EAK (32. Elem. 3.)*, and the angle *BAE* is equal to the angle *AEK* as alternate, therefore the triangles *ABE* and *AEK* are similar; consequently *AB* is to *AE*, as *AE* to *EK*, and multiplying the extreme terms together, and middle terms together, $AB \times EK = AEq$ and dividing both sides of the equation by *EK*, $AB = \frac{AEq}{EK}$. Q. E. D.

By the same method of arguing $\frac{AIq}{IK}$ may be proved equal to *AB*.

The Problem.

Let it be requir'd to strike an object as *K (Fig. 16.)* with a Ball projected from *A* with a given velocity.

III. If 2 Balls are thrown at different elevations (but with equal degrees of velocity), the one as much above 45 degrees as the other below, the horizontal distances (or Randoms) where they both fall will be the same *.

Solution. Erect *AB* perpendicular to the Horizon, and equal to four times the height a Body must fall from, to acquire the velocity with which the Ball is to be thrown; bisect this in the point *G*, through which draw *HC* perpendicular to *AB*, and meeting the line *AC* (perpendicular to *AK*) in *C*. On *C* as a Center with the Radius *CA*, describe the Circle *ABD*; lastly through *K* draw the line *KEI* perpendicular to the Horizon, cutting the Circle in the points *E* and *I*; I say *AE* or *AI* will be the direction sought.

For by the Lemma $AB = \frac{AEq}{EK}$ or $\frac{AIq}{IK}$, but (*ex constructione*) *AB* is equal to four times the height a Body must fall from, to acquire the velocity with which it is to be thrown, therefore its equal $\frac{AEq}{EK}$ or $\frac{AIq}{IK}$ is the same, which by the Corollary was the thing requir'd to determine the direction sought, consequently the Parabola, which the Body will describe, will pass through the point *K*. Q. E. D.

Coroll. 1. From hence it is evident, that if the object to be struck, be placed any where in the horizontal line *AO* (*Fig.* 17.) beyond *Q*, the Problem is impossible; for then *QH* will not touch the Circle, and the Ball will not reach that point with any direction whatever. And that when the Ball is directed towards *H*, it will fall on *Q* the greatest distance it can possibly be thrown to; but the angle *QAH* being equal to *ABH* in the opposite segment (32. *Elem.* 3.) is equal to half *AGH* at the Center (20. *Elem.* 3.) which is a right one; consequently *QAH* is an angle of 45 degrees.

* *Coroll.* 2. If the object is situated in the horizontal line *AQ* (*Fig.* 18.) but nearer to *A*, than the greatest horizontal distance at which it may be struck, suppose in *K*; the two directions *AE* and *AI* with which it may be hit, are equally distant from the direction *AH*; for the angles *IAH* and *HAE* are equal, as insisting on equal arches *IH* and *HE* (28. *Elem.* 3.)

IV. The height a Body will rise to, when thrown perpendicularly upwards, is equal to half the greatest horizontal distance it can be thrown to with the same velocity *.

From hence we may easily know how far a Mortar-Piece, or other such Machine, will carry a Ball. Let the Ball be thrown perpendicularly upwards, note the time of its ascent and descent, half that is the time of descent, from whence we learn the height, to which the Ball is thrown, for Bodies are observ'd to fall in the first second of time 16 feet, consequently in 2 seconds they fall 4 times 16 feet (Chap. 5. §. 4.) in 3, 9 times as much *&c.* but (§ 4.) the perpendicular height being doubled will give the greatest horizontal distance to which that Machine will carry the Ball with an equal Charge.

V. The Randoms of two Projectiles, having the same degrees of elevation, but thrown with different velocities, are as the squares of the velocities: for by the last, the Randoms are as the heights to which the Bodies thrown perpendicularly upwards will ascend, but the

* *Coroll.* 3. The altitude of a perpendicular projection is equal to a fourth part of the height *AB*; for the velocity with which the Body is projected, is *(ex hypoth.)* such as it would acquire by falling through a fourth part of the line *AB*; but a fourth part of the line *AB* is equal to half the line *GH*, or *AQ* *(Fig. 17.)* that is half the greatest horizontal distance to which the Body can be thrown.

See *Cotes*'s Harmonia Mensurarum p. 87. *Keil*'s Introduct. ad Physſ. Præl. 16.

heights

heights are (Chap. 5. §. 6.) as the squares of the velocities.

VI. Supposing the motion of the Earth, all Bodies, when thrown perpendicularly upwards, describe *Parabola's*; notwithstanding they appear both to ascend and descend in the same right line.

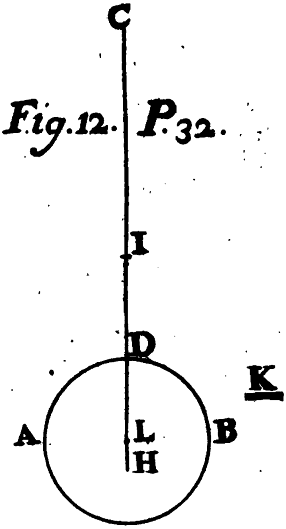
This may very easily be illustrated in the following manner; let there be a Body carryed uniformly along the line AB (*Fig.* 19.) by the motion of the Earth from A towards B; as it passes the point C let it be projected upwards by some force acting underneath it in the direction CO perpendicular to the former; the Body will not thereby lose its motion which it had in common with the Earth towards B (by the first Law of Nature), but will be carryed by two motions, one towards B the other towards O; let us then suppose, that in the time it would have advanced forwards to P in the line AB, it rises upwards to M in the line CO; it will then be found in D (Chap. 4. §. 2.): in like manner supposing it would have advanced forward to Q while it rises to N, it would then be found in E, afterwards in F, then in G *&c*: describing the Curve CGL which (from what was demonstrated under §. 1.) is a Parabola *.

* *Dem.* Suppose the motion the Body had in common with the Earth towards *B* (*Fig.* 20.) and that with which it is projected towards *O*, such, as being compounded (Ch. 4. §. 2.) would have

Plate IV. *Pag.* 38.

1.

Fig. 12. *P.* 32.



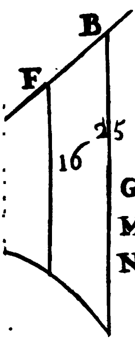
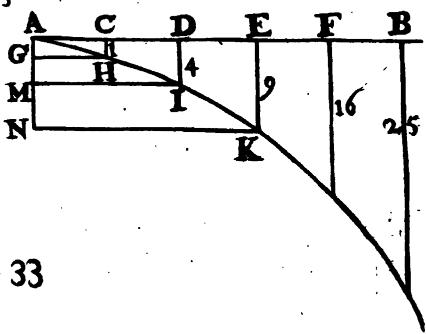


Fig. 14.



P. 33

THE reaſon why it appears to a Spectator to riſe and fall perpendicularly, is becauſe he is carryed uniformly along with it by the motion of the Earth in its firſt direction. *v. g.* Suppoſe the Spectator at C at the inſtant the Body is thrown from thence, when it arrives at D, he will be moved to P, when the Body is at E he will be at Q *&c.* as is evident from what was obſerved about the motion of the Body in the Curve; and they will both meet in L. Therefore ſince the Spectator imagines himſelf ſtanding ſtill, and ſees the Body always perpendicularly over his head, he muſt of courſe think that it riſes right up and falls right down.

IT may be proper to obſerve here, that Experiments relating to the motion of projected Bodies, do not exactly anſwer the Theory; the reſiſtance of the Air deſtroying part of their motion: for which a ſmall allowance is to be made.

have produced a motion in the direction *CX*; it will follow from thence, that the path deſcribed by it will be the ſame, as if it had been thrown in that direction from a point as *C* at reſt; but in that caſe it would have deſcrib'd a Parabola as *CGL* (§. 1.) therefore in this. *Q. E. D.*

CHAP.

CHAP. VIII.

Of Centripetal and Centrifugal Forces.

WHEN a Body is projected in an horizontal direction and by its Gravity made to describe a Parabola as demonstrated Chapter the last; the curvature of that Parabola will vary in proportion to the velocity with which the Body is thrown, and the Gravity which impels it towards the Earth. For the less its Gravity is in proportion to the quantity of matter it contains, or the greater the velocity is with which it is projected; the less will it deviate from a strait line, and the further it will go, before it falls to the Earth. For instance, if a Bullet be shot out of a Cannon from the top of a Mountain with a given velocity in an horizontal direction, and goes in a curve line, suppose to the distance of two Miles from the foot of the Mountain before it falls to the ground; the same Bullet shot with a much greater velocity would fly to a much greater distance before its fall. And by encreasing the velocity, the distance to which it is projected, may be encreased as much as you please; so that it will not fall to the ground, till it is arrived at the distance of ten, or thirty, or ninety degrees; or till it has even sur-

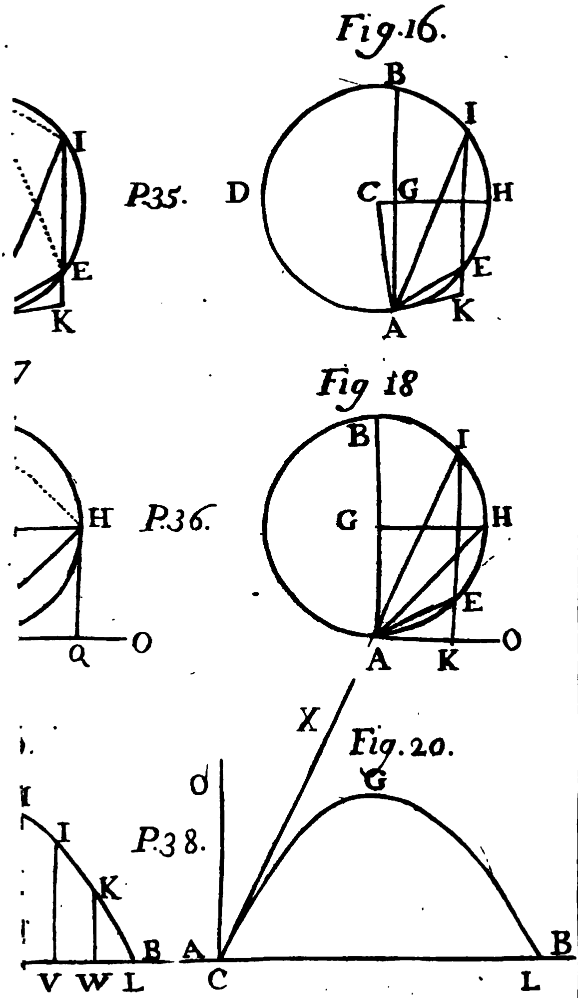


Fig.16.
B
I
P.35.
D
C
G
H
E
K
A
Fig 18
B
I
G
H
P.36.
E
Q
O
A
K
O
X
Fig.20.
G
P.38.
I
K
B
A
V
W
L
C
L
B

surrounded the whole Earth, and arrives at the very top of the Mountain from whence it was projected: in this case it will perform a second revolution, and so on *in infinitum* without a new projection, provided the resistance of the Air is taken away. Nay it may be projected with such violence, that it will continually recede from the Earth, moving in a Curve, till at length it gets out of the Sphere of the Earth's Attraction; after which it will go on in a straight line without ever returning. Which may thus be illustrated.

Let ABC (*Fig.* 21.) represent the Earth, M the point from whence the Body is projected in the direction MQ: it may be thrown with such force as to carry it to B before it falls, or to C, or even to go round to M, describing the Circle MDM; or lastly it may be made to describe the Curve MO, till it gets out of the Sphere of the Earth's Attraction, suppose at O, going on afterwards in the infinite strait line OX, there being nothing to stop or alter its course. Farther, it may be projected with such a force from M (*Fig.* 22.) as will cause it continually to recede from the Earth, till it arrives at the opposite point G, describing the curve MKG; and if the point G is within the Sphere of the Earth's Attraction, the Body will return to M, describing the Curve GLM exactly similar to MKG;

 and

and in moving nearer and nearer to the Earth till it comes to M, will regain what velocity it loſt in going from M to G, its Gravity conſpiring with its motion from G to M in the ſame degree in which it oppoſed it from M to G; conſequently the Body when at M having recovered the velocity with which it ſet out, will be inabled to perform a ſecond revolution in the ſame Curve as before, and ſo on.

Again, ſuppoſe it had been projected from the point M with a leſs degree of force than would have carryed it round in the Circle MDM (*Fig.* 21.), but greater than would have ſuffered it to have fallen to the Earth at the oppoſite point F (*Fig.* 22.); it would alſo in this caſe have arrived at the point M from whence it ſet out; for the exceſs of velocity it would have gained in F, by its tendency towards the Earth in its way thither, over and above that with which it was projected from M, would be ſufficient to carry it off again from the Earth, till it arrived at M; and to make it deſcribe the path FPM exactly ſimilar and equal to the former, loſing in its way from F to M juſt ſo much velocity, as it gained by paſſing from M to F; and thereby it would be inabled to perform an infinite number of revolutions in the ſame Curve without requiring a ſecond projection.

From

FROM hence it follows, that ſuppoſing a Body projected from a point at any diſtance within the Sphere of the Earth's Attraction, with a force ſufficient to carry it half round without falling to the ſurface, it is impoſſible it ſhould fall upon any part of the other half; but will return to the point from whence it ſet out, making continual ſucceſſive revolutions in the ſame Curve; provided it meets with no reſiſtance from the Medium through which it paſſes, nor any other obſtacle to obſtruct its motion *.

FROM hence alſo it is clear, that, the nearer the revolving Body approaches to the Earth, the faſter it moves; its velocity being continually increaſed during the time of its acceſs towards the Earth, and as much retarded during its receſs from it. And this acceleration and retardation will always be ſuch, that the Body will deſcribe equal Areas in equal times: the meaning of which is, that if we imagine a line conſtantly extended from the Center of the Earth to the Center of the Body, that line will always deſcribe or paſs through equal ſurfaces or ſpaces in equal times, for it conſtantly

* Gravity is here ſuppoſed to be inverſely as the ſquares of the diſtances from the Earth, for 'tis poſſible that the force by which a Body tends towards another, may vary in ſuch a manner at different diſtances, that the projected Body ſhall deſcribe a Spiral line, continually approaching to or receding from that about which it revolves.

becomes ſhorter the faſter it moves, and *vice verſâ* *.

And for the ſame reaſon that a Body projected with a ſufficient velocity may by the force of Gravity be made to deſcribe a Curve round the Earth, and perform continual ſucceſſive revolutions in the ſame; it follows that the Moon, may by the ſame force of Gravity be made to revolve about the Earth, or any

* *Dem.* Let the time in which the Body performs one revolution be divided into equal parts, in the firſt of which let the Body deſcribe the right line *AB* (*Fig.* 23.): in the ſecond part of time, if not prevented, it would go ſtraight on to *c*, deſcribing the line *Bc* equal to *AB* by the firſt Law of Nature; the lines *SA*, *SB*, *Sc* being drawn, the triangles *SBA*, *ScB* will be equal to each other, their baſes *AB* and *Bc* being equal and their heights *S* the ſame (38. *Elem.* 1.). When the Body arrives at *B*, let the Centripetal force by one ſingle impulſe turn it out of the ſtraight line *Bc* into the line *BC*: in which let it move on uniformly without receiving a ſecond impulſe till it comes to *C*. Let *Cc* be drawn parallel to *SB* meeting *BC* in *C*; then at the end of the ſecond part of time the Body will be found in *C*, having deſcribed the Diagonal of the Parallelogram *Nc* (Chap. 4. §. 2.). Draw *SC*, and the triangle *SCB* will be equal to the triangle *ScB*, (each having the ſame baſe *SB* and being between the ſame parallels *Cc* and *SB*) and therefore alſo equal to the triangle *SBA*. For the ſame reaſon, if the Centripetal force acts in the points *C*, *D*, *E* ſucceſſively, ſo as to make the Body deſcribe the ſtraight lines *CD*, *DE*, *EF*, &c. in ſo many equal parts of time, the triangles *SCD*, *SDE*, *SEF*, &c will be all equal to one another and to the triangle *SAB*. Conſequently equal Areas are deſcribed in equal times. Let us then ſuppoſe the baſes of thoſe triangles, *viz.* *AB*, *BC*, *CD*, *DE*, &c. diminiſhed *in infinitum*, and likewiſe the times in which they are deſcribed; then will the Perimeter *A*, *B*, *C*, *D*, *E*, *F*, &c. become a Curve, and any number of thoſe triangles taken together, (or their Areas) will be proportionable to the times in which they are deſcribed. *Q. E. D.*

other

other Planet by the like force about the Sun, if the velocities with which they move are duly adjusted to the forces by which they are acted upon.

When a Body revolves about another in this manner, that force or power by which it is prevented from flying off (as it otherwise would do in a Tangent to the Curve which it describes) is call'd the *Centripetal*; the counter-action of this, by which it endeavours to fly off, the *Centrifugal*; these, by the 3*d* Law of Nature being equal to each other, are called by one common name *Central Forces*; that with which the Body is at first projected, or continues its motion from any point, is the *Projectile* force; and the time in which it performs one revolution, the *Periodical* time.

These forces properly relating to the motions of the Heavenly Bodies will be more largely treated of in another place.

CHAP. IX.

Of the Communication of Motion.

I, BEFORE we proceed to explain the Laws, by which Bodies communicate their motion, from one to another, it is very necessary to make a distinction between Motion and Velocity; which ought to be well observ'd and is as follows,

By

By the Motion of a Body (ſometimes called its quantity of motion, ſometimes its *Momentum*) is not to be underſtood the velocity only, with which the Body moves; but the ſum of the motion of all its parts taken together: conſequently the more matter any Body contains, the greater will be its motion, though its velocity remains the ſame. Thus, ſuppoſing two Bodies, one containing ten times the quantity of matter the other does, moving with equal velocity; the greater Body is ſaid to have ten times the motion or Momentum that the other has: for 'tis evident that a tenth part of the larger has as much as the other whole Body. In ſhort that quality in moving Bodies which Philoſophers underſtand by the term Momentum or motion, is no other than what is vulgarly call'd their *Force*, which every one knows to depend on their quantity of matter as well as their velocity. This is that power a moving Body has to affect another in all actions that ariſe from its motion, and is therefore a fundamental Principle in Mechanics. See it farther explained in the next Chapter.

II. Now ſince this Momentum or Force depends equally on the quantity of matter a Body contains, and on the velocity with which it moves; the method to determine how great it is, is to multiply one by the other. Thus ſuppoſe two Bodies, the firſt having twice the

quan-

quantity of matter and thrice the velocity which the other has; any two numbers that are to each other as two to one, will expreſs their quantities of matter (it being only their relative velocities and quantities of matter which we need conſider), and any two numbers that are as three to one, their velocities; now multiplying the quantity of matter in the firſt *viz.* 2 by its velocity 3, the product is 6; and multiplying the quantity of matter in the ſecond by its velocity, *viz.* 1 by 1, the product is one; their relative forces therefore or powers will be as 6 to 1, or the Moment of one is ſix times greater than that of the other. Again if their quantities of matter had been as 3 to 8 and their velocities as 2 to 3, then would their Moments have been as 6 to 24, that is, as 1 to 4.

THIS being rightly apprehended, what follows concerning the Laws of Communication of Motion by Impulſe, and the Mechanical Powers will be eaſily underſtood.

The Communication of Motion.

I. In Bodies not Elaſtic.

III. THOSE Bodies are ſaid to be not *Elaſtic*, which when they ſtrike againſt one another do not rebound, but accompany one another after Impact as if they were joyned. This proceeds from their retaining the impreſſion made upon their ſurfaces after the impreſſing force ceaſes to act. For all rebound-

ing

ing is occasioned by a certain spring in the surfaces of Bodies, whereby those parts, which receive the impression made by the stroke, immediately spring back and throw off the impinging Body; now this being wanting in Bodies void of Elasticity there follows no separation after Impact.

IV. When one Body impinges on another which is at rest, or moving with less velocity the same way, the quantity of the motion or Momentum in both Bodies taken together remains the same after Impact, as before; for by the 3*d* Law of Nature, the reaction of one being equal to the action of the other, what one gains the other must lose.

Thus, suppose two Bodies one impinging with 12 degrees of velocity on the other at rest: the quantities of Matter in the Bodies being equal, their Moments and velocities are the same; the sum in both 12; this remains the same after Impact (§. 4.), and is equally divided between them (§. 3.); they have therefore 6 a piece, that is the impinging Body communicates half its velocity and keeps half.

V. When two Bodies impinge on each other by moving contrary ways, the quantity of motion they retain after Impact, is equal to the difference of the motion they had before; for by the 3*d* Law of Nature, that which had the least motion, will destroy an equal quantity in the other, after which they will

will move together with the remainder, that is the difference.

THUS for inſtance, let there be two equal Bodies moving towards each other, the one with 3 degrees of velocity, the other with 5, the difference of their Moments or velocities will be 2; this remains the ſame after Impact (§. 5.) and is equally divided between them (§. 3.) they have therefore one a piece: that is, the Body which had 5 degrees of velocity, loſes 3 or as much as the other had, communicates half the remainder, and keeps the other half*.

II. In Elaſtic Bodies.

VI. BODIES perfectly *Elaſtic* are ſuch as rebound after Impact with a force equal to that with which they impinge upon one another: thoſe parts of their ſurfaces, that receive the impreſſion, immediately ſpringing back, and throwing off the impinging Bodies with a force equal to that of Impact.

* From theſe poſitions it is eaſy to deduce a Theorem, that ſhall ſhew the velocity of Bodies after Impact in all caſes whatever. Let there be two Bodies A and B, the velocity of the firſt a of the other b; then (§. 2.) the Moment of A will be expreſſed by Aa, and of B by Bb: therefore the ſum of both will be $Aa+Bb$; and $Aa-Bb$ will be the difference when they meet. Now theſe quantities (by §. 4. and 5.) remain the ſame after Impact; but knowing the quantities of motion and quantities of matter, we have the velocity (which §. 3. is the ſame in both) by dividing the former by the latter, (as follows from §. 2.) therefore $\frac{Aa+Bb}{A+B}$ or $\frac{Aa-Bb}{A+B}$ will in all caſes expreſs the velocity of the Bodies after Impact.

VII. From hence it follows that the action of Elastic Bodies on each other (that of the spring being equal to that of the stroke), is twice as much as the same in Bodies void of Elasticity. Therefore when Elastic Bodies impinge on each other, the one loses and the other gains twice as much motion as if they had not been Elastic; we have therefore an easy way of determining the change of motion in Elastic Bodies, knowing first what it would have been in the same circumstances, had the Bodies been void of Elasticity.

Thus if there be two equal and Elastic Bodies, the one in motion with 12 degrees of velocity impinging on the other at rest, the impinging Body will communicate twice as much velocity as if it had not been Elastic, that is, (by §. 4.) 12 degrees or all it had; consequently it will be at rest, and the other will move on with the whole velocity of the former.

VIII. It sometimes happens that in Bodies not Elastic, the one loses more than half its velocity, in which case supposing them Elastic it loses more than all; that is, the excess of what it loses above what it has, is negative, or in a contrary direction; thus suppose the circumstances of Impact such, that a Body which has but 12 degrees of velocity, loses 16; the overplus 4 is to be taken the contrary way, that is, the Body will rebound with 4 degrees of velocity. *v. g.* Let it be required to de-

termine

termine the velocity of a Body after Impact against an immoveable object. Let us first suppose the Object and Body both void of Elasticity: 'tis evident the impinging Body would be stopt or lose all its motion, and communicate none; if they are Elastic, it must lose twice as much (by §. 7.) and consequently will rebound with a force equal to that of the stroke.

IX. It is sufficient if only one of the Bodies is Elastic, provided the other be infinitely hard; for then the impression in the Elastic Body will be double of what it would have been, had they both been equally Elastic: and consequently the force with which they rebound will be the same as if the impression had been equally divided between the two Bodies.

X. There are no Bodies that we know of, either perfectly Elastic or infinitely hard; the nearer therefore any Bodies approach to perfection of Elasticity, so much the nearer do the Laws they observe in the mutual communication of their motion, approach to those we have laid down.

XI. Sir *Isaac Newton* made trials with several Bodies, and found that the same degree of Elasticity always appeared in the same Bodies, with whatever force they were struck; so that the Elastic power in all the Bodies he made trial upon, exerted it self in one con-

ſtant proportion to the compreſſing force. He found the celerity with which Balls of Wool bound up very compact, receded from each other, to bear nearly the proportion of 5 to 9 to the celerity wherewith they met; and in Steel he found nearly the ſame proportion; in Cork the Elaſticity was ſomething leſs; but in Glaſs much greater; for the celerity with which Balls of that material ſeparated after percuſſion, he found to bear the proportion of 15 to 16 to the celerity wherewith they met *.

XII. We have hitherto ſuppoſed the direction in which Bodies impinge upon one another to be perpendicular to their ſurfaces: when it is not ſo, the force of Impact will be leſs, by how much the more that direction varies from the perpendicular; for it is manifeſt that a direct impulſe is the greateſt of all others that can be given with the ſame degree of velocity †.

* Newt. Princip. Phil. pag. 21.

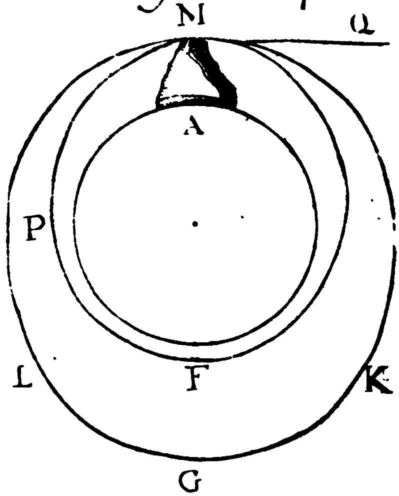
† The force of oblique Percuſſion is to that of direct, as the Sine of the Angle of Incidence to the Radius.

Dem. Let there be a plane as *AD* (*Fig,* 24.) againſt which let a Body impinge in the point *D* in the direction *BD*; which line may be ſuppoſed to expreſs the force of direct Impulſe, and may be reſolved into two others (Chap. 4. §. 2.) *BC* and *BA*; the one parallel, the other perpendicular to the Plane; but that force which is exerted in a direction parallel to the Plane can no way affect it; the ſtroke therefore ariſes wholly from the other force expreſſed by the line *BA*; but this is to the line *BD*, as the Sine of the Angle of Incidence *ADB* to the Radius; from whence the Propoſition is clear.

If

Plate VI. Pag. 52.

Fig. 22. P. 42.



P. 44.

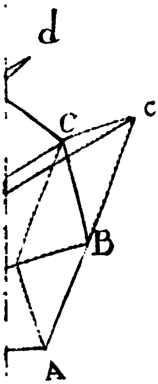
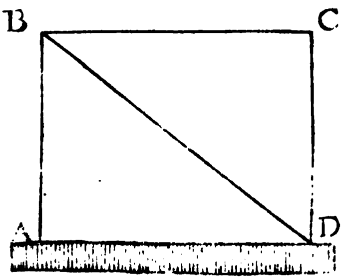


Fig. 24. P. 52.



XIII. THIS is the case when Bodies impel one another by acting upon their surfaces; but in forces where the surfaces of Bodies are not concerned, as in Attraction *&c.* we must not consider the relation which the direction of the force has to the surface of the Body to be moved, but to the direction in which it is to be moved by that force. Here the force of action will be less, by how much the more these two directions vary from each other*. My meaning in both cases will be understood from the instance of a Ship under Sail. The force by which the Wind acts upon the Sail, will be less, by how much the more its direction varies from one that is perpendicular to its surface: but the force of the Sail to move the Ship forward, will be less, by how much the more the direction of the Ship's Course varies from that in which she is impelled by the Sail.

If the surface of the Body to be struck is a Curve, then let *AD* be made a Tangent to *D* the point of Incidence, and the Demonstration will be the same.

* The force of oblique action is to that of direct, as the Co-Sine of the Angle comprehended between the direction of the force, and that wherein a Body is to be moved thereby, to the Radius.

Dem. Let *FD* (*Fig.* 25.) represent a force acting upon a Body as *D*, and impelling it towards *E*; but let *DM* be the only way in which it is possible for the Body to move; the force *FD* may be resolved (Chap. 4. §. 2.) into two others *FG* and *FH*, or which is equal to it *GD*; but 'tis evident that only the force *GD* impels it towards *M*. Now, *FD* being the Radius, *GD* is the Co-Sine of the Angle *FDG* comprehended between the two directions *FE* and *GM*; from whence the proposition is clear.

XIV.

XIV. To this we may add the following Propoſition, relating to oblique forces, *viz.* that if a Body is drawn or impelled three different ways at the ſame time by as many forces acting in different directions; and the quantity of thoſe forces is ſuch that the Body is kept in its place by them: then will the forces be to each other as the ſeveral ſides of a triangle drawn reſpectively parallel to the directions in which the forces act *.

* *Dem.* Let the lines *AB, AD, AE, (Fig. 26.)* repreſent the 3 forces acting upon the Body *A* in thoſe directions, and by that means keeping it at reſt in the point *A*. Then the forces *EA* and *DA* will be equivalent to *BA* otherwiſe the Body would be put into motion by them *(contra Hypoth.)* But theſe forces are alſo equivalent to *AC* (Chap. 4. §. 2) conſequently *AC* may expreſs the other force, and *EC*, which is parallel and equal to *AD*, may expreſs that force: but *ACE* is a triangle whoſe ſides are all parallel to the given directions, therefore the ſides of this triangle will expreſs the relation of the forces by which the Body is kept at reſt. *Q. E. D.*

CHAP.

CHAP. X.

Of the Mechanical Powers.

I. HAVING in the foregoing Chapter accounted for the Communication of Motion by Impulſe; we proceed next to conſider motion as communicated without Impulſe; which is done by means of certain Inſtruments, commonly known by the names of *Mechanical Powers.* The uſe of theſe Powers conſiſts chiefly in managing great Weights or performing other Works with a determinate force.

II. THEY are uſually reckoned five. *viz.* The Lever, the Wheel and Axis, the Pully, the Screw, and the Wedge; to which ſome add the Inclined Plane. To theſe all Machines how complicated ſoever are reducible.

III. THESE Inſtruments have been of very ancient uſe; for we find that *Archimedes,* was well acquainted with the extent of their Power, as may be inferred from that celebrated ſaying of his, Δὸς ποῦ στῶ, καὶ τὴν γῆν κινήσω. By which he meant that the greateſt imaginable Weight might be moved with the ſmalleſt Power.

IV. THAT Body which communicates motion to another, is called the *Power*; that which receives it, the *Weight.*

V.

V. THAT point in a Body which remains at reſt, while the Body is turning round, is called the *Center of Motion.* Beſides this, there are two other Centers in Bodies, 1. that of *Magnitude*, which is a point, as near as poſſible, equally diſtant from all the external parts of the Body; 2. that of *Gravity*, or that about which all the parts of the Body, in whatever ſituation it is placed, exactly balance each other.

VI. WHEN a Body communicates motion to another, it loſes juſt ſo much of its own, as it communicates to that other; the action of one being equal to the reaction of the other, See Chapter the laſt §. 4. and 5.

VII. WHEN two Bodies have ſuch relation to each other (ſuppoſe them fixed to different parts of the ſame Machine) that if one be put into motion, the other will thereby have neceſſarily ſuch a degree of velocity given it, that their Moments * will be equal; it will then be impoſſible that one ſhould begin to move without communicating to the other a

* It was propoſed (Chapter the laſt) to give ſome farther explication of the term *Momentum* in this place, and to ſhew that the Force or Power any Body has (except ſuch as does not proceed from motion) wholly depends upon it: it being then intended to treat this Subject in the uſual way. But the method here made uſe of renders ſuch explication unneceſſary; 'tis ſufficient if the Reader underſtands by it the quantity of motion in a Body, or its quantity of matter multiplied by its velocity, as defined in that Chapter.

quantity of motion equal to its own; 'tis evident therefore from the laſt Propoſition, that if we ſuppoſe it to begin to move; in that very inſtant it muſt loſe all its own motion by communicating it to the other Body: and conſequently will remain at reſt, communicating none at all. Now the Moments of two Bodies are equal (Chap. 9. §. 2.) when the velocity of the firſt is to that of the ſecond, as the quantity of matter of the ſecond to that of the firſt; for if we ſuppoſe their quantities of matter as 1 to 3, then by the ſuppoſition their velocities are as 3 to 1; and if we multiply the quantity of matter in the firſt *viz.* 1, by its velocity 3, and that of the other *viz.* 3 by its velocity 1; their products are equal; their Moments are therefore by the Definition (Chap. 9. §. 1. and 2.) equal. They will alſo be equal, when the ſpaces the Bodies paſs over are in that proportion; for the times they both move in being the ſame, the ſpaces will always be as the velocities.

VIII. From hence it follows, that in any Machine whether ſimple or compound, the Power however ſmall may have a Moment equal to that of the Weight; provided the Machine be ſuch, that when it is in motion, the velocity of the Power ſhall exceed that of the Weight, as much as the Weight is larger than the Power; for then what the Power wants in quantity of matter or weight

will be made up in velocity; consequently their Moments will be equal by §. the last, and therefore by §. 7. they will exactly balance each other; or be in *Æquilibrio.*

IX. But if the Power bears a greater proportion to the Weight, than the velocity of the Weight to that of the Power; it will then have a greater Momentum than the other, and consequently may communicate such a Momentum to it as it will receive, without losing all its own; the remainder therefore, if sufficient to overcome the friction of the Machine, will put it into motion. We proceed now to treat of each Mechanical Power in its order, and

I. Of the Lever.

X. The Lever is a right line (or bar whose weight in Theory is not considered) moveable on a Center, which is called its *Fulcrum*, or *fixed Point*.

XI. The Æquilibrium in this Machine is, when the distance of the Power from the fixed point is to that of the Weight from the same, as the quantity of matter in the Weight to that in the Power.

For supposing the Lever placed on its Fulcrum with the Weight to be raised at one end, and the Power applied to the other; 'tis evident the farther the Power is placed from the Fulcrum or center of motion, the larger will be its sweep when the Machine is put in motion; that is, it will move over so much

much more ſpace in the ſame time than the Weight to be raiſed: now if it is placed juſt ſo much farther from the Fulcrum, as it is leſs than the Weight, it will move juſt ſo much faſter; their Moments therefore will be equal (§. 7.) and conſequently the Power and Weight will exactly balance each other, or be in Æquilibrio*. And if the Power is ſufficiently augmented to overcome the friction of the Machine, it will put it in motion.

The Lever is of 3 kinds. 1. When the fixed point is between the Weight and the Power, as in the laſt caſe. 2. When the Weight is between the fixed point and the Power. 3. When the Power is between the fixed point and the Weight.

In all which caſes the Æquilibrium will be, when their diſtances from the fixed point are ſuch, that their velocities ſhall be inverſely as their quantities of matter; for then by §. 7. being at reſt, neither of them will communicate any motion to the other.

* Geometrically thus. Let *AB* (*Fig.* 27.) repreſent the Lever, *F* the Fulcrum, *W* the Weight, *P* the Power, the one ſuſpended at the extremity of the Lever *A*, the other at *B*, and let *BF* be to *FA* as *W* to *P*; then while the Lever moves from the ſituation *AB* into that of *CD*, the point *B* which ſuſtains the Power will move as much farther than *A* which ſuſtains the Weight (and conſequently as much faſter ſince they perform their motions in the ſame time) as the arch *BD* is longer than *AC*; that is, the triangles *BFD* and *AFC* being ſimilar, as the arm *BF* is longer than *AF*, which (*ex Hypoth.*) is as much as the Weight exceeds the Power, they will therefore (§. 7.) be in Æquilibrio. *Q. E. D.*

THE common Scales may be consider'd as a Lever of the first kind, where the Weight and Power are applied at equal distances from the fixed point.

THE Steelyard is also a Lever of the first kind, whose arms are unequal.

THE difference between the use of the Scales and the Steelyard consists in this; that as in one you make use of a larger Power (or more Weights) to estimate the weight of an heavier Body; in the other you use the same Power, but give it a greater velocity with respect to that of the Weight by applying it farther from the fixed point, which by §. 7. will have the same effect.

II. The WHEEL and AXIS.

XII. THIS Machine is a Wheel, that turns round together with its Axis; the Power in this is applied to the Circumference of the Wheel, and the Weight drawn up by means of a Rope wound about the Axis.

XIII. IN this there will be an Æquilibrium, when the Weight is to the Power, as the Diameter of the Wheel to the Diameter of the Axis.

'TIS evident, the velocity of the Power will exceed the velocity of the Weight, as much as the Circumference of the Wheel exceeds that of its Axis; because the spaces they pass over in one revolution will be as those Circumferences; that is, as much as the Diameter

of

of one exceeds that of the other, (the Circumferences of Circles being as their Diameters;) what therefore in this case the Power wants in weight will be made up in velocity, from whence (§. 7.) there will be an Æquilibrium *.

THE use of this Machine is to raise Weights to greater heights than the Lever can do, because the Wheel is capable of being turned several times round, which the Lever is not; and also to communicate motion from one part of a Machine to another; accordingly there are few compound Machines without it.

III. The PULLEY.

XIV. A Pulley is an Instrument composed of one or more Wheels moveable on their Axes.

XV. A simple Pulley, if its Axis is fixed, is of no other use, than to alter the direction of the Power; for the Power and Weight will both move through an equal space in the same time. But in a Pulley not fixed, as in *Fig.* 29. where the Rope runs under it, or in a combination of Pullies as in *Fig.* 30. the Æquilibrium will be, when the

* Geometrically thus. Let *AB* (*Fig.* 28.) be the Diameter of the Wheel, *DE* that of the Axis, *W* the Weight, and *P* the Power; when the Wheel begins to move, the point *B* and *D* will describe similar Arches about the Center *C*, in the same manner the point *A* and *B* in the Lever were shewn to do about the fixed point *F* (*Fig.* 27.) that is the point *B* will move as much faster than *D*, as *CB* is longer than *CD* or *AB* than *DE*, the motion therefore of *P* (§. 7.) will be equal to that of *W*. From whence the Proposition is clear.

Power

Power is to the Weight, as one to the number of Ropes, that paſs between the upper and lower Pullies.

SUPPOSE one end of the Rope fixed in B (*Fig.* 29.) the other ſupported by the Power P, it is evident, that in order to raiſe the Weight W one foot, the Power muſt riſe two, for both Ropes *viz.* BC and CP, will be ſhortened a foot apiece, whence the ſpace run over by the Power, will be double to that of the Weight; if therefore the Power is to the Weight as 1 to 2, their Moments will be equal: for the ſame reaſon if there be 4 Ropes paſſing from the upper to the lower Pullies as in *Fig.* 30. the velocity of the Power will be quadruple to that of the Weight, or as 4 to 1. *&c.* In all caſes therefore when the Power is to the Weight, as one to the number of Ropes paſſing from the upper to the lower Pullies, (§. 7.) there will be an Æquilibrium.

XVI. IF the Pullies be diſpoſed as in Figure the 31*ſt*, each having its own particular Rope, the action of the Power will be very much increaſed; for here every Pully doubles it, wherefore the Power is 4 times greater with 2 Pullies, 8 times with 3, 16 times with 4 *&c.* For it is evident from the conſideration of the Figure, the firſt will move half as faſt as the Power, the ſecond half as faſt as that, and ſo on; wherefore (§. 7.) the Power is doubled by each Pulley.

THE

Plate VII. *Pag.* 62.

3.

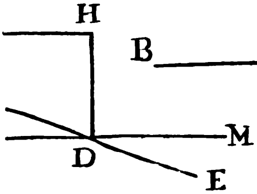
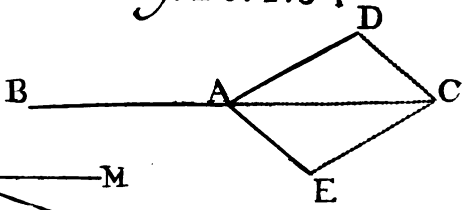


Fig. 26. *P.* 54.



P. 59.

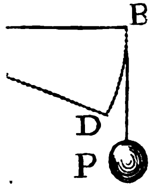
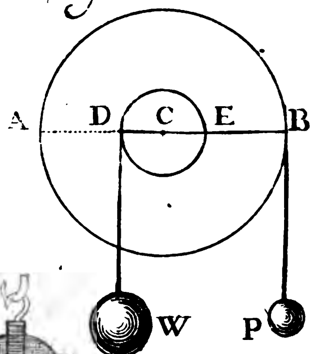
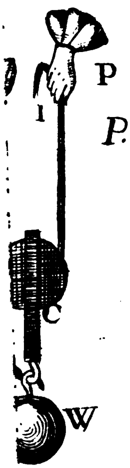


Fig. 28. *P.* 61.





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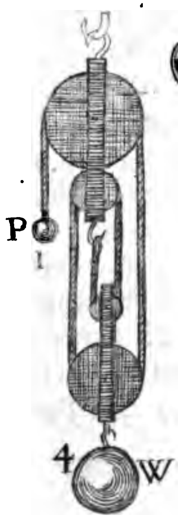


Fig. 30.

The use of the Pulley is nearly the same with that of the Wheel and Axis, but it is more portable and easier to be fixed up.

IV. The Screw.

XVII. In this Machine the Æquilibrium will be, when the Power is to the Weight, as the distance between any two contiguous threads or spirals in the Screw, to the way described by the Power in one whole revolution. It is manifest from the form of the Machine *(Fig. 32.)* that in one revolution of the Screw, the Weight will be moved through a space equal to the distance of two contiguous threads, and that the Power will run through a space equal to the compass it takes in one revolution, therefore (§. 7.) if the Weight exceeds the Power in this proportion, there will be an Æquilibrium.

This Machine is of great force, and very useful in retaining Bodies in a compressed state, because it will not run back, as the three foregoing will when the Power is removed. This arises from the great friction of those parts in the Screw, which during its motion slide upon those, that are at rest.

V. The Wedge.

XVIII. This Instrument is formed by two equal rectangles joined at their lower bases, and separated at their upper ones, by a third; which is called the *Back* of the *Wedge*; the other two, its *Sides*,

XIX.

XIX. In the foregoing Mechanical Powers we have all along conſidered the Weight, as moved in the ſame direction with that in which it is acted upon by the Machine, as is commonly the caſe; but in this, the Weight is generally applied in ſuch a manner as to be made to move in a direction different from that in which it is protruded by the Wedge; hence it is, that Mathematicians have differed in their determination of the Power of this Machine, ſome conſidering the Weight as moved by it in one direction and ſome in another. Nay there are ſome (I ſpeak of late Writers) that have differed from Truth it ſelf. We will therefore lay down the ſeveral Proportions they have given us for the determining the Power of this Machine and examine them one by one. 1. It is demonſtrated by ſome, that the Power will be equivalent to the reſiſtance of the Weight, when it bears ſuch proportion to it, as the breadth of the Back of the Wedge, does to the ſum of its Sides; or, which is the ſame thing, as half that breadth to one of its Sides. 2. Others make it ſomewhat larger, and demonſtrate that it ought to be as half the breadth of the back to the perpendicular height of the Wedge. 3. Some are of opinion, that there will not be an Æquilibrium in this Machine, unleſs the Power is to the Weight, as the whole breadth of the Back to the perpendicular height. *Wallis, Keil* &c. 4.

Grave-

Gravesande in his *Elements* (L. I. Ch. 13.) gives us the same proportion with the last; and in his *Scholium de ligno findendo*, tells us, that when the parts of the wood are separated no farther than the Wedge is driven in, the Æquilibrium will be, when the Power is to the Resistance, as half the breadth of the Back of the Wedge to one of its Sides.

THOSE who lay down the first Proportion for determining the Power of this Machine, suppose the parts, which are separated from each other thereby, to recede from their first situation in directions perpendicular to the sides of the Wedge. Thus let ACB (*Fig.* 33.) represent a Wedge; P, P, two Bodies to be separated by it, the one to be moved towards I, the other towards F, in the directions CI and CF perpendicular to AC and CB; then 'tis evident that when the Wedge is driven in to the situation MNO, the two Bodies will be moved to Q and Q; that is, one will have passed through the space CK the other through CL, but these spaces being equal, their velocities are the same as if they had both passed over one of them. *v.g.* CL, or which is equal to it DG (drawn perpendicular to CB); therefore the Power which we suppose applied at D moves through DC, while the obstacle moves through DG, consequently (§. 7.) when the Power is to the Weight as DG to DC,

 that

that is, as DB to CB *, or half the Back of the Wedge to one of its Sides, they will be in Æquilibrio. This proportion therefore, when the parts of the Weight are moved by the Wedge in the directions CI and CF, may be admitted as true.

2. THE second proportion is also true, supposing the Bodies P, P, to recede from each other in the directions CN, CM, parallel to AB the Back of the Wedge; for when the Wedge is driven in between them, to the situation MNO, the Bodies will have moved through a space as CN, or which is equal to it DB, half the Back of the Wedge, and the Power through a space equal to its height as before; consequently (§. 7.) in this case, the Æquilibrium will be, when the Power is to the Weight, as half the Back of the Wedge to its height †.

* For (8. *Elem.* 6.) the triangles *DCG* and *DCB* are similar, and consequently *DG* : *DC* : : *DB* : *BC*.

† The same may be otherwise demonstrated from Section 14. Chapter 9. thus. Let there be a Body as *L* (*Fig.* 34.) drawn against the Wedge *ABC* by the Weight *W*, in the direction *LF*; parallel to the back of the Wedge *AB*; but prevented from sliding down towards *C* by a Plane (whose upper surface we may suppose represented by *EF*) lying under it. I say, the Power will be to the Weight, when they are in Æquilibrio, as *DB* to *DC*.

Dem. The Body *L* is here acted upon in three directions, *viz.* by the force of the Weight *W* in the direction *LF*, by the two Planes *CB* and *EF*, in the directions *LG* and *LI*, perpendicular to their surfaces; let *GE* be drawn parallel to *LI*, then will the triangle *LGE* have all its sides respectively parallel to those directions; consequently (Chap. 9. §. 14.) if we suppose *LE* to express

3. Those, who imagine there will not be an Æquilibrium, unless the Power be to the Weight, as the whole breadth of the Back of the Wedge to its height, suppose as in the last case, that the Bodies to be separated, recede from each other in directions parallel to the Back of the Wedge; and endeavour to support their opinion by the following Argument: *viz.* that, when the Wedge is driven in to the situation MNO *(Fig. 33.)* as before, each part of the Weight having moved through a space equal to half the Back of the Wedge, the whole Weight has therefore moved through twice so much, or a space equal to the whole Back: as much as to say, the whole has moved farther than its parts; which is absurd.

express the force of the Weight *W*, *GL* will represent the pressure of the Body *L* against the Wedge; and if that is resolved into *GE* and *GH* the one perpendicular to the direction of the Power, the other parallel and contrary to it; the last, viz. *GE*, will express the whole force of the Weight to resist the motion of the Power; but *GE* is to *EL*, as *DB* to *DC* (for the triangle *EGL* and *DBC* are similar, the sides of one being *ex Construct.* respectively perpendicular to those in the other; v. g. *LG* to *CB*, *EL* to *DC* and *GE* to *DB*); consequently the Power is to the Weight, when they balance each other, as half the breadth of the Back of the Wedge to its height. *Q. E. D.*

Corol. Suppose the Body *L* had been drawn against the Wedge in the direction *GL* perpendicular to its surface, and to be moved by the Wedge in the contrary direction towards *G*, as in the first case; then if *GL* expresses the force with which it is drawn towards the Wedge, *GE* will be that with which it resists the Power; but *GE* is to *GL* as *DB* to *BC*, the triangles *EGL* and *DBC* being similar; consequently in this case, the Power will be to the Weight, as half the breadth of the Back of the Wedge to one of its sides; as was before demonstrated.

4. THIS is *Gravesande*'s miſtake in his *Elements*, the ſame he has alſo made in his *Scholium de ligno findendo*, and thereby determined the Power in both places to be twice as big, as it ought to be. If he had proceeded in the following manner, his Argument would have been eaſier, as well as the Concluſion juſter. Suppoſe the Wedge ABC driven into the Wood QLQ, (as repreſented *Fig.* 35.) which is ſplit no farther than the point of the Wedge, or however no farther than is juſt ſufficient to give it room to move, I ſay that in this ſituation of the Wedge, the Power is to the Weight, as one fourth part of the Back of the Wedge to one of its Sides. For it is evident, that when the upper ends of the Wood, which preſs againſt the Wedge in the points G, H, are put into motion by the Wedge, they will move in the directions HI and GF, perpendicular to the ſides of the Wedge, becauſe they turn as it were upon a joynt at L, which we ſuppoſe contiguous to C: again, ſince only the upper ends of the Wood are put into motion, and not the lower ones, which remain at L; 'tis evident that the motion of each piece (ſuppoſing their thickneſs the ſame from end to end, and their ſubſtance uniform) will be but half, what it otherwiſe would have been. Now were all the parts of the Wood to have the ſame degree of velocity, the Power would be to the Weight, as in the firſt caſe, *viz.*

as

Plate VIII. *Pag.* 68.

Fig. 32. *P.* 63.

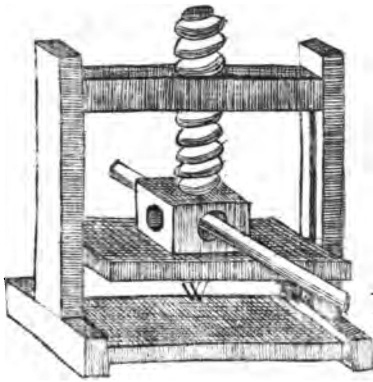
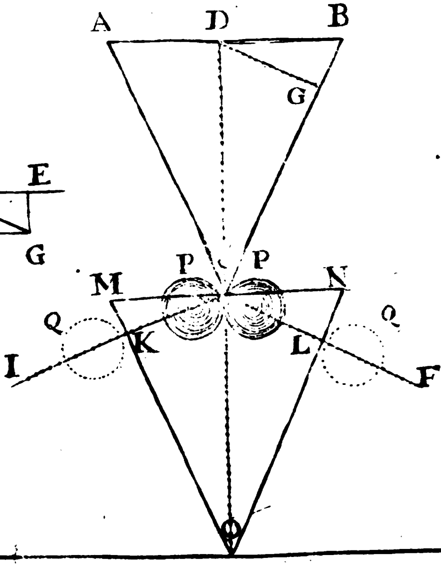


Fig. 33. P. 65.



66.

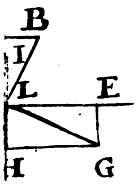


Plate. IX.

Fig. 36. P. 69.

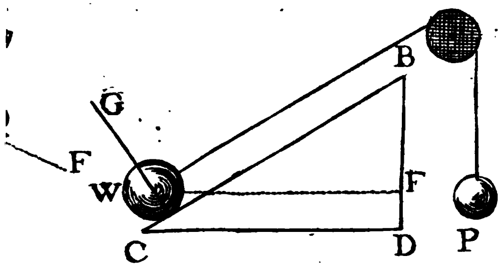
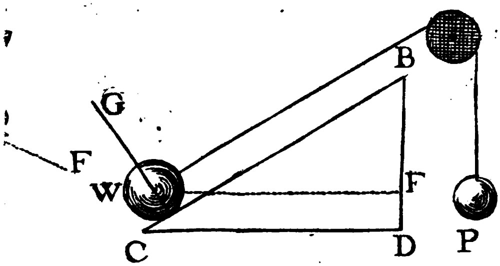


Plate. IX.

Fig. 36. P. 69.



as DB to BC (*Fig.* 33.); therefore in this case, it is as half DB to BC, or as one fourth part of the Back of the Wedge to one of its Sides. Which was to be proved.

XX. The form of the *Inclined Plane* being no other than that of half a Wedge, as is manifest from the representation of it (*Fig.* 36.) it follows that what has been demonstrated of the one, may be applied to the other, and the properties of both will be the same. For instance, if the Weight W is to be raised up the Plane CB, by the Power P, in a direction parallel to the Plane; instead of that, we may suppose the Weight prevented from running off the Plane by the String WB, and the inclined Plane driven under it like a Wedge in the direction DC: then will the Weight rise towards G in a direction perpendicular to CB, for we must always suppose the String CB parallel to the Plane, as it would have been, if the Weight had been drawn up by it; then will the action of the Plane upon the Weight be similar to that of the Wedge in the first case; and consequently the Power will bear such proportion to the Weight, as DB to BC; that is, as the height of the Plane to its length. Again, suppose the Weight was to have been drawn up the Plane by a String in the direction WF parallel to the base of the Inclined Plane CB; then if the Plane be driven under the Weight as before, it must rise in a direction

direction perpendicular to CD, that is parallel to DB: then the case will be analogous to the 2*d* of the Wedge; consequently the Power will be to the resistance of the Weight, when there is an Æquilibrium, in the proportion of DB to DC, as there demonstrated.

XXI. These are the Powers or Machines, which under different forms, constitute all others how complicated soever; and as the Æquilibrium in any one of these is, when the Power and Weight are inversely as their velocities; so in a Machine however compounded, the Power and the Weight will exactly balance each other, when they are in this proportion; for by §. 7. their Moments will then be equal, and the Machine, if at rest will continue in that state; and if put into motion by an external force, will gradually lose it, when that force ceases to act; on account of the unavoidable friction of the Machine, and the resistance of the Air, which it must necessarily meet with, unless its motion could be performed in a perfect Vacuum. From hence we see the impossibility of contriving an Engine, whose motion should be *perpetual*, that is, such as does not owe its continuance to the application of some external force; a Problem that has given birth to an almost infinite number of Schemes and Contrivances. For unless some method could be found out of gaining a force, by the artful disposition

and

and combination of the Mechanical Powers, equivalent to that which is continually deſtroyed by friction, and the reſiſtance of the Air, the motion which was at firſt given to the Machine muſt at length be neceſſarily loſt. But we ſee, that thoſe Inſtruments are only different means, whereby one Body communicates its motion to another; and not deſigned to produce a force which had no exiſtence before. A given force may be diſpoſed between the Power and the Weight an infinite number of ways; but can never be augmented by any Mechaniſm whatever: ſo much as we place in the Power will always be loſt to the Weight, and what we attribute to the Weight will never be found in the Power. Tis for want of a due conſideration of this, that ſo many Mechanical Deſigns have proved abortive, ſo many Engines unequal to the performance for which they were deſigned, and ſo many impoſſibilities attempted.

" If it were poſſible, ſays Bp. *Wilkins*, to " contrive ſuch an invention, whereby any " conceivable Weight may be moved by any " conceivable Power, both with equal velocity " (as it is in thoſe things which are immediately " ſtirred by the hand, without the help of any " other inſtrument) the works of Nature " would be then too much ſubjected to the " power of Art; and Men might be thereby " encouraged (with the Builders of *Babel*, or

" the

" the rebel Giants) to ſuch bold deſigns, as
" would not become a created Being. And
" therefore the Wiſdom of Providence has ſo
" confined theſe human Arts, that what an
" invention hath in the *ſtrength* of its motion,
" is abated in the *ſlowneſs* of it; and what it
" has in the extraordinary *quickneſs* of its mo-
" tion, muſt be allowed for in the great *ſtrength*
" requiſite in the Power, which is to move
" it *.

* *Wilkins*'s Mathem. Magick, p. 104.

Appendix to Part I.

CHAP. I.

Of the Vibration *of a* Pendulum *in a* Cycloid.

Proposition I.

IF a Pendulum be made to vibrate in a Cycloid, all its Vibrations, however unequal, will be *isocronous*; that is, they will be performed in equal Times (*a*).

(*a*) In order to demonstrate this Proposition, it will be proper to lay down the following Lemma's.

LEMMA I.

If a Body descends from A *along the Line* AX, (*Appendix Plate, Fig.* 1.) *by virtue of a Force which decreases in Proportion as the Distance of the Body from* X *decreases; that is, if when the Body comes to* M, N, O, &c. *the Action of that Force upon the Body, be as the Distances* XM, XN, XO, &c. *respectively: And if the last acquired Velocity of the Body; that is, its Velocity when it comes to* X, *be expressed, or set off, by the Perpendicular* XB, *equal in Length to the Line* AX, *and its Velocities at* M, N, O, *&c. be set off there by the Lines* MD, NP, OQ, &c. *in Length proportionable to each other and to the Line* XB, *as the Velocities of the falling Body at* M, N, O, &c. *are to each other and to its last Velocity at* X: *And if through the Extremities of these Lines, the Curve* ADB *be drawn; I say, that Curve will be a Portion of a Circle: And the Time in which the Body will descend through the whole Space or*

Line AX [*or any Part of it, as* MO] *will be such Time, as would be requisite for it to describe the whole Arch* AB [*or any Part as* DQ, *corresponding to* MO] *in, with its last acquired Velocity at* X.

Demonstration of the Lemma. Parallel and contiguous to the Line MD, draw NP, in which Case the Line MN becomes a Point, and the Arch DP a Tangent to the Curve: Produce PD till it meets XA produced, in T; draw the Line XD; and let fall the Perpendicular DL. Then the Lines DL and TM being parallel, the Angles PDL and DTM are equal, as being alternate (by 27 Elem. 1.); and the Angles at L and M as being right ones; the Triangles therefore PDL and DTM are similar, which for the Sake of referring to it afterwards, let us make the

first Step of the following Process	1	The Triangles PDL and DTM are similar
From the first Step we have this Proportion (5 Elem. 6.)	2	PL : LD : : DM : MT.
By the Figure	3	LD = MN
But MD being the Velocity when the descending Body comes to M, the Point MN is described with that Velocity; for there is no Acceleration during the Passage of a Body over a Point; consequently MN is proportionable to MD; that is,	4	MN is as MD.
Comparing the second, third, and fourth Steps	5	PL : MD : : DM : MT
But MD and NP being the Velocities of the descending Body at M and N, LP the Difference of those Lines, expressing the Increase of Velocity in the Body, will be proportionable to the moving Force at the Point MN; that is, by the Supposition, to the Distance XM; therefore	6	PL is as XM
Comparing the fifth and sixth	7	XM : MD : : DM : MT
Consequently (5 Elem. 6.)	8	The Triangles XMD and DMT are similar.

And therefore, since their Angles at M are right ones, the Triangle TDX is (by the Converse of Prop. 8. Elem. 6.) right angled at D. Consequently since the same is true of any other Point

Point of the Curve, as well as D, the Arch ADB is a Portion of a Circle (16 Elem. 3.) Which is the first Part.

Secondly, comparing the first and eighth Steps, the Triangles PDL and XMD are similar; therefore	9	LD : DM :: DP : DX
ADB being a Portion of a Circle, as already proved	10	DX = XB
Comparing the 3d, 9th and 10th Steps	11	MN : DM :: DP : XB.

Since then the Point MN bears the same Proportion to MD, or the Velocity it is described with by the falling Body, that the Point DP does to the last acquired Velocity XB, it follows that the former, MN, is described in the same Time with the Velocity the Body has when there, that the latter, DP, might be with the last acquired Velocity XB. And since the same is true of every other Part of the Arch ADB, it is obvious that the Time in which the Body will descend through any other Part of the Space AX, [or the whole of it,] will be such as would be required for it to describe any corresponding Part of the Arch ADB, [or the whole of it,] with the last acquired Velocity XB. Which was the other Part.

Coroll. Hence it follows, that if a Body descends along the Line AX, by Virtue of Forces acting upon it at A, M, N, O, &c. proportionably to the Length of the Lines XA, XM, XN, XO, &c. and if on X as a Center, and with the Radius XA a Portion of a Circle, as ADB, be described; and if the Radius or whole Sine XB, be put to represent the Velocity of the Body when it comes to X, the other Sines MD, NP, OQ, &c. will represent the respective Velocities of the Body at the several Points M, N, O, &c. And conversely, if one of the Sines, as MD, be put to express its Velocity at M, the other Sines NP, OQ, and the Radius or whole Sine XB, will express the Velocity of the Body at those other Points N, O and X.

LEMMA II.

If a Body moves along the Line AX, *(Fig. 2.) and be urged all the Way by Forces proportionable to its Distance from the Point* X; *whatever Point of that Line it sets out from, it will come to the Point* X *in the same Time. Which Time will bear such Proportion to the Time it would move over the whole Line* AX *in, with the Velocity it shall acquire by falling through the whole Line* AX, *as the Semicircumference of a Circle does to its Diameter.*

 Dem.

Dem. Let two Bodies A and P ſet out from the Points A and P at the ſame Time; and let them be urged by Forces proportionable to their Diſtances from the Point X: I ſay, thoſe Bodies will come to X at the ſame Inſtant of Time; that is, they will overtake one another at that Point. On X as a Center, and with the Radius's XA and XP deſcribe the two Quadrants AB and PQ; and draw the Line SX, and the Sines RS and MN; and let the whole Sine or Radius XB expreſs the Velocity the Body A will acquire by falling to X: Then by Corollary of Lemma 1. will the Sine RS, if taken as near as poſſible to A, expreſs the firſt Velocity of the Body A. But the Force, which urges the Body A is ſuppoſed to be to that which urges the Body P, as XA to XP (or becauſe the Archs AS and PN are ſimilar) as RS to MN: As therefore RS expreſſes the firſt Velocity of A, MN will expreſs the firſt Velocity of the other Body P: And therefore by the ſame Corollary, its Velocity when it comes to X, will be expreſſible by XQ. Farther, the Time the Body A falls to X in, is by Lemma 1. equal to the Time the Arch AB would be deſcribed in with the Velocity XB; and the Time the other Body falls from P to X in, is equal to the Time the Arch PQ would be deſcribed in with the Velocity XQ. But a Body will be as long in moving over the Arch PQ with the Velocity XQ, as over the Arch AB with the Velocity XB, the Lines XQ and XB having the ſame Proportion to each other, that the Archs have. Therefore the Time the Body A falls to X in, is equal to the Time the other Body P would fall to that Place in. Which was the firſt Part.

Again by Lemma 1.	1	The Time a Body would fall from A to X in, is equal to the Time it would move over the Arch AB in, with its laſt acquired Velocity at X.
Axiom, or ſelf evident Propoſition	2	The Time a Body would move over the Arch AB in with the laſt acquired Velocity at X, is to the Time it would move over AX in with the ſame Velocity, as AB is to AX.
Comparing the firſt and ſecond	3	The Time a Body would fall from A to X in, is to the Time it would move over AX in with the laſt acquired Velocity, as AB is to AX.

Axiom

Axiom	4	AB is to AX as twice AB is to twice AX.
By the Figure	5	Twice AB is to twice AX as the Semicircumference of a Circle is to its Diameter.
Comparing the 3d, 4th and 5th Steps	6	The Time a Body would fall from A to X in, is to the Time it would move over AX in with its last acquired Velocity, as the Semicircumference of a Circle is to its Diameter. Which was the second Part.

LEMMA III.

If from the lowermost Point of a Circle, as X, *(Fig.* 3.*) be drawn the Chords* XQ *and* XO, *the Power of Gravity whereby it shall cause a Body to descend along the former, will be to the Power whereby it shall cause it to descend along the latter, as the Length of the former is to the Length of the latter.*

Dem. Draw the Diameter XD, the Perpendiculars QR and OS; and join the Points QD and OD. Then (by 31 Elem. 3.) the Triangle XQD is right-angled at Q; and therefore (by 8

Elem. 6.)	1	XR : XQ :: XQ : XD.
And for like Reasons	2	XS : XO :: XO : XD
But by Part I. Chap. 6. § 2.	3	The Effect or Power of Gravity upon a Body descending along the Chord QX, is to that which it exerts upon another falling freely; that is, to its whole Power, as XR to XQ.
And also	4	The Power of Gravity upon a Body descending along the Chord OX, is to its whole Power, as XS to XO.
Comparing the 1st and 3d	5	The Power of Gravity upon a Body descending along the Chord QX, is to its whole Power, as XQ to XD.
Comparing the 2d and 4th	6	The Power of Gravity upon a Body descending along the Chord OX, is to its whole Power, as XO to XD.

Comparing the 5th and 6th Steps	7	The Power of Gravity upon a Body descending along the Chord QX, is to the Power of Gravity upon a Body descending along the Chord OX, as XQ to XO. Q.E.D.

The Description of a Cycloid, with the Definitions relating thereto. If a Circle as FCH (Fig. 4.) be rolled along the Line AB, till it has turned once round; the Point C in its Circumference, which at first touched the Line at A, will describe the Curve Line ACXB, which Curve is called a *Cycloid.* The right Line AB is its *Base*: The middle Point X is its *Vertex*: And a Perpendicular, as XD, let fall from thence to the Base, is its *Axis*: And the Circle FCH, or any other as XGD, equal thereto, is called the *Generating Circle.*

LEMMA IV.

If on XD, *the Axis of the Cycloid, as a Diameter, the generating Circle* XGD *be described; and if from a Point in the Cycloid, as* C, *the Line* CIK *be drawn Parallel to the Base, the Portion of it* CG, *will be equal to the Arch* GX.

Dem. Draw the Diameter HF, then the Circles FCH and

DGX being equal	1	KG = CI
Adding GI to each of them	2	KI = CG
By the Figure	3	KI = DF
Comparing the two last	4	CG = DF
By the Description of the Cycloid	5	The Arch CF = AF
By the Figure	6	The Arch CF = DG
Comparing the 5th and 6th	7	AF = DG
By the Description of the Cycloid	8	AFD = DGX
Comparing the 7th and 8th with the Figure	9	FD = GX
Comparing the 4th and the 9th	10	CG = GX. Q.E.D.

LEMMA V.

The same Things being supposed as in the foregoing Lemma, a Tangent to the Cycloid at the Point C, *is parallel to* GX *a Chord of the Circle* DGX.

Dem. It appears from the Description of the Cycloid, that since the Angle FCH is a right one, (as it is by 31 Elem. 3.) the Chord CH is a Tangent to the Curve at the Point C, but

CH

CH is parallel to GX; a Tangent therefore at the Point C, is parallel to GX, the Chord of the Circle DGX. Q. E. D.

LEMMA VI.

Things remaining as before, if from a Point of the Cycloid, as L, *the Line* LMK *be drawn parallel to the Base* AB, *the Arch* XL *of the Cycloid, will be double of* XM *the Chord of the Circle corresponding thereto.*

Dem. Draw the Line SQ parallel and contiguous to LK crossing the Circle in R, and the Chord XM produced, in P, then will LS, MR and MP become Points, the first having the Property of a Tangent to the Cycloid at LS, the second that of a Tangent to the Circle at MR, and the third, the Properties of a Production of the Chord XM. Join the Points X and R, and on MP let fall the Perpendicular RO: Produce also the Point RM, till it meets XN, a Tangent to the Circle at X. Then will the Lines XN and QS, being each perpendicular to the Diameter DX, be parallel; and the Triangles MNX and MPR will be similar; as having their Angles at M vertical, and at P and X alternate. But the Tangents NX and NM are equal (by 36 Elem. 3.) the corresponding Lines therefore PR and RM in the other Triangle, are so too: This last Triangle is therefore an Isosceles one; and therefore RO being perpendicular to its Base MP, MP is equal to twice MO. The Tangent LS is parallel to MP, (as being by Lemma 5. parallel to MX) and therefore equal to it, the Lines LK and SQ being parallel: It is therefore equal also to twice MO. But LS is the Difference between the cycloidal Archs XL and XS; and MO is the Difference between the Chords XM and XR, for since XO and XR are close together, RO which is perpendicular to one of them, may be considered as perpendicular to both: The Difference therefore between any two Archs of the Cycloid is twice that which is between two corresponding Chords of the Circle; and consequently any Arch, as XL, is double of the corresponding Chord XM. Q. E. D.

Coroll. Since when the Arch XL becomes XB, the corresponding Chord XM becomes XD the Diameter of the Circle DMX; it is obvious, that the Semicycloid BX, or AX, is equal to twice DX the Diameter of the generating Circle DMX.

LEMMA VII.

If a Body descends in a Cycloid, the Force of Gravity (so far as it acts upon it in causing it to descend along the Cycloid) will

PROPOSITION II.

The Time in which a Pendulum vibrating in a Cycloid, performs a Vibration, is to the

be proportionable to the Distance of the Body from the lowest Point of the Cycloid.

Dem. Let the Cycloid be AXB (Fig. 5.) whose Base is AB, and its Axis DX, on which last as a Diameter, describe the generating Circle DQX: Draw the Chords OX and QX; through the Points O and Q, and parallel to the Axis AB, draw the Lines LS and MR; draw also the Tangents LV and MY. Then because by Lemma 5. the Tangent LV is parallel to OX, and the Tangent MY parallel to QX, it's obvious that Gravity exerts the same Power or Force upon a Body descending in the Cycloid at L (because it then descends in the Tangent LV) as it would do upon the same Body descending along the Chord OX: And for the like Reason, it exerts the same Force upon it when it comes to M, that it would do if it were descending along QX: But (by Lemma 3.) the Power or Force of Gravity upon Bodies descending along the Chords OX and QX, are as the Lengths of those Chords; that is, by Lemma 6. (halves being proportionable to their wholes) as the Length of the Cycloidal Archs LX and MX. The Force therefore of Gravity upon a Body descending in the Cycloid at the Point L [or any other] is to its Force upon the same when at M [or any other Point] as the Space or Distance it has to move over in the former Case, before it gets to the lowest Point X, to that it has to run over in the latter, before it arrives at the same Point. Q. E. D.

Demonstration of the Proposition in the Text to which this Note refers.

By Lemma 7. The Force of Gravity so far as it causes a Body to descend in a Cycloid is proportionable to the Distance of that Body from the lowest Point, imagine then that Body to be a Pendulum vibrating in the Cycloid, then whatever Point it sets out from, it will by Lemma 2. come to the lowest Point in the same Time: And consequently since the like is true as to its ascending from that Point, all its Vibrations be they large or small, will be performed in the same Time. Q. E. D.

Time

2

Time in which a Body would fall freely thro' half the Length of the Pendulum, as the Circumference of a Circle is to its Diameter (*b*).

PROB.

(*b*) To demonstrate this, the following Lemma's will be of Use.

LEMMA VIII.

If in a right-angled Triangle, as BFG, *(Fig. 6.) the Perpendicular* FI *be let fall from the right Angle to the Hypothenuse* BG, *the Line* BI *multiplied by* BG *will be equal to* BFq.

Dem. By 8 Elem. 3. the Triangles BFI and BFG are similar, consequently BI is to BF, as BF is to BG, and therefore BI × BG = BFq. Q. E. D.

LEMMA IX.

If a Body descends along a Curve Line, as AX *(Fig. 7.) it will acquire the same Velocity that another, or the same Body, would do, by falling from an equal perpendicular Height in the Line* DX.

Dem. Parallel to the horizontal Line AD, draw the Lines BM and FN contiguous to each other; in consequence of which, the Lines MN and BG are capable of being considered as Points; and therefore the Velocity the descending Bodies pass over them with, as uniform; and the curve Line BG, as a straight Line also, and as a Tangent to the Curve AX at the Point BG. Things being thus, let it be supposed that the Bodies begin their Fall at B and M, or, which comes to the same Thing, that they have equal Velocities at those Points: Then the Velocities of the Bodies being uniform and equal to each other, (for there is no Acceleration in a Point) the Lines BG and MN may represent the Relation the Times they are passed over in bear to each other. Parallel to DX draw BF, and let the equal Lines BF and MN represent the Force of Gravity acting perpendicularly at those Points; and let the Force BF be resolved into two others, *viz.* BI and IF, the one parallel, the other perpendicular to the Course of the Body at B: It is only the former of these, *viz.* BI, that accelerates the Body along the Curve BG; the other, *viz.* IF, neither accelerates it nor retards it, but is wholly spent in pressing the Body close to the Surface BG, if it be a Surface; or in stretching the String which keeps the Body in the Course ABX, if it be a String. Now the Velocity a Body acquires by moving over any Space, is proportionable to the Force that acts upon

upon it, multiplied by the Time that Force acts. Since then BI represents the Force in one Case, and MN the Force in the other; and BG the Time in one Case, and MN the Time in the other; it follows that the Velocity generated in one Case, is as BI × BG; and in the other, as MN × MN; or since BF and MN are equal, as the Quantities BI × BG and BF × BF, (or BF^q) which Quantities by Lemma 8. are equal to each other. The Velocity therefore the one Body acquires by descending along BG, is equal to that which the other acquires by falling through MN: But the Lines BM and GN being parallel, it is obvious there is the same Number of BG's in the Curve AX, as of MN's in the perpendicular DX; the Velocity therefore which a Body would acquire by falling through one, is equal to that which it would acquire in falling through the other. Q. E. D.

Demonstration of the Proposition. Let AXB (Fig. 5.) be the Cycloid the Pendulum vibrates in. Then by Lemma 2. compared with Lemma 7. we have

	1	The Time a Body would descend from A to X in, is to the Time it would move over the same Space in with its last acquired Velocity, as the Semicircumference of a Circle is to its Diameter.
By the Coroll. of Lemma 6.	2	AX is equal to twice DX.
By Lemma 9.	3	The Velocity a Body acquires by falling from A to X, is equal to the Velocity it would acquire by falling from D to X.
From the three last compared	4	The Time a Body would descend from A to X in, is to the Time it would move over twice DX in, with the Velocity acquired by a Fall from D to X, as the Semicircumference of a Circle is to its Diameter.
By Part I. Chap. 5. § 7.	5	The Time a Body would move over twice DX in, with the Velocity acquired by falling from D to X, is equal to the Time it would fall from D to X in.
Comparing the 4th and 5th	6	The Time a Body would descend from A to X in, is to the Time it would fall from D to X in, as the Semicircumference of a Circle is to its Diameter.

From

PROBLEM.

To make a Pendulum *vibrate in a given* Cycloid.

Solut. Let AXB (Fig. 5.) be the given Cycloid; its Baſe AB, its Axis DX, and its generating Circle DQX, as before: Produce XD to C, till DC be equal to DX: Through C draw the Line EF parallel to AB, and take CE and CF, each equal to AD or DB; and on the Line CE as a Baſe, and with the generating Circle AGE equal to DQX, deſcribe the Semicycloid CTA, whoſe Vertex will therefore touch the Baſe of the given Cycloid in A. And on the Line CF alſo as a Baſe, deſcribe

From the Figure	7	The Time of Deſcent from A to X is half a Vibration.
From the Solution of the following Problem it will appear, that	8	DX is half the Length of a Pendulum, which in vibrating ſhall deſcribe the Cycloid AXB.
Comparing the three laſt Steps	9	The Time of half a Vibration is to the Time in which a Body would fall freely through half the Length of the Pendulum, as the Semicircumference of a Circle is to its Diameter.
Doubling the Antecedents of the laſt Step	10	The Time of an whole Vibration is to the Time in which a Body would fall freely through half the Length of the Pendulum, as the Circumference of a Circle is to its Diameter. Q. E. D.

an equal Semicycloid CB. On the Point C, hang the Pendulum CTP equal in Length to the Line CX: And let the upper Part of the String of it, (as CT, in its present Situation in the Figure) as it vibrates this way and that, apply itself to the cycloidal *Cheeks* CA and CB: Then will the Ball of it P oscillate in the given Cycloid AXB. Q. E. F. (*c*).

CHAP.

(*c*) Draw TG and PH, each parallel to the Base AB; and join the Points AG and DH. Then by the Corollary of

Lemma 6.	1	AC = 2 AE
By the Figure (DC being equal to DX)	2	2 AE = CX
Comparing the 1st and 2d Steps	3	AC = CX
By Construction	4	CTP = CX
Comparing the 3d and 4th	5	AC = CTP
From the 5th Step compared with the Figure	6	AT = TP
(The String touching the Cycloid at T) by Lemma 5.	7	GA is parallel to TK
By Construction	8	GT is parallel to AK
From the two last Steps compared, GATK is a Parallelogram, consequently	9	GA = TK, and GT = AK
By Lemma 6.	10	GA = $\frac{1}{2}$ TA
Comparing the two last Steps	11	TK = $\frac{1}{2}$ TA
Comparing the 6th and 11th	12	TK = $\frac{1}{2}$ TP
From the 12th Step compared with the Figure	13	TK = KP
Comparing the last Step with the Figure	14	The parallel Lines GT and PH are equally distant from AD
From the last compared with the Figure	15	The Arch GA = the Arch DH
Comparing the last with the Figure	16	The Chords GA and DH are parallel, and GE = HX.

From

CHAP. II.

Of the Center *of* Oſcillation *and* Percuſſion.

THE *Center of Oſcillation* is that Point in a Pendulum, in which, if the Weight of the ſeveral Parts thereof were collected, each Vibration would be performed in the ſame Time, as when thoſe Weights are ſeparate.

The Point or Center of Suſpenſion, is the Point on which the Pendulum hangs.

A general Rule *for finding the* Center *of* Oſcillation.

If ſeveral Bodies be fixed to an inflexible Rod ſuſpended upon a Point, and each Body

From the 7th and 16th Steps compared with the Figure	17	KP is parallel to DH.
And therefore (KD being by Conſtruction parallel alſo to PH) KDHP is a Parallelogram, conſequently	18	KD = PH
By Lemma 4.	19	GT = the Arch AG
Comparing the 9th and 19th	20	AK = the Arch AG
By the Deſcription of the Semicycloid CTA	21	AKD = AGE
From the two laſt compared with the Figure	22	KD = GE
Comparing the 18th and 22d	23	PH = GE
Comparing the 16th and 23d	24	PH = HX.

But by Lemma 4. if PH be equal to HX, P is a Point in the Cycloid AXB; the Ball of the Pendulum CTP therefore being at that Point, is in the given Cycloid. The Problem therefore was rightly ſolved. Q. E. D.

be

be multiplied by the Square of its Diſtance from the Point of Suſpenſion, and then each Body be multiplied by its Diſtance from the ſame Point; and all the former Products when added together, be divided by all the latter Products added together, the Quotient which ſhall ariſe from thence, will be the Diſtance of the Center of Oſcillation of thoſe Bodies from the ſaid Point.

Thus, if CF Fig. 8. be a Rod on which are fixed the Bodies A, B, D, &c. at the ſeveral Points A, B, D, &c. and if the Body A be multiplied by the Square of the Diſtance CA, and B be multiplied by the Square of the Diſtance CB, and ſo on for the reſt: And then if the Body A be multiplied by the Diſtance CA, and B be multiplied by the Diſtance CB, and ſo on for the reſt; and if the Sum of the Products ariſing in the former Caſe, be divided by the Sum of thoſe which ariſe in the latter, the Quotient will give CQ, the Diſtance of the Center of Oſcillation of the Bodies A, B, D, &c. from the Point C *(d)*.

(d) Dem. That the Proceſs may be leſs complicated, let us ſuppoſe but two Bodies, as A and F, fixed to the Rod CF; and let AI and FL be the Archs which the Bodies A and F deſcribe when the Pendulum vibrates, and let the Pendulum be removed into the Situation CL. Contiguous to the Line CL draw CR; then may the Archs IP and LR be conſidered as Tangents at the Points I and L, and thoſe Tangents as inclined Planes, down which the Bodies I and L are to roll: Theſe Tangents being each perpendicular to CL, are equally inclined to the Horizon, the Bodies therefore will endeavour to roll down with equal Velocities; but this they cannot do, becauſe being fixed to the inflexible

flexible Rod, they muſt deſcribe the unequal Archs IP and LR in the ſame Time. That is, the Body L will oblige the Body I to deſcribe a leſs Arch than it otherwiſe would have done; and the Body I will occaſion the Body L to deſcribe a larger Arch than it would have done. And the Effects of the Forces by which they act thus upon each other, like thoſe of Action and Reaction, will be equal. It remains to determine theſe Effects.

In order to which, parallel to LI draw MN, and let the equal Spaces LM and IN be thoſe the Bodies would move over in the leaſt Time poſſible, had they been independent of each other. And let the Archs LR and IP be thoſe which the Bodies joined to the Rod deſcribe in the ſame Time. For the Reaſon juſt mentioned, the former of theſe, *viz.* LR, will be larger, and the latter, *viz.* IP, will be leſs than LM or IN; and the Arch which the Center of Oſcillation deſcribes will be equal to LM or IN, becauſe the Center of Oſcillation deſcribes that Arch which the Bodies would deſcribe in the ſame Time, if they were both together, and neither of them an hindrance or furtherance to the other. Conſequently the Center of Oſcillation is at Y, where the Lines MN and PR croſs.

Now the Motion which the Body I loſes by being retarded, is its Motion over the Arch PN; and the Motion the other Body gains by being accelerated, is its Motion over MR: The Force or Moment of the firſt of theſe Motions, is the Product of the Body I multiplied by the Space PN; and the Force or Moment of the laſt is the Product of the Body L multiplied by the Space MR. Theſe are the Forces, Moments or Actions, which retard the one Body, and promote the Motion of the other. But obſerve, that theſe Forces or Moments, in as much as they act at different Diſtances from the Center C, about which the Bodies I and L, when the Pendulum ſwings, do revolve; have each their *Mechanical* Advantage; but the one a greater than the other: For inſtance, L has an Advantage which is as LC, its Diſtance from the *Fulchrum* C; and I only the Advantage IC. As then in determining the Effect of a Power applied to a Lever, we multiply it by its Diſtance from the *Fulchrum*; ſo the abovementioned Forces or Moments (*viz.* I multiplied by PN and L multiplied by MR) muſt be multiplied by their reſpective Diſtances from C; and then we have I multiplied by PN multiplied by IC, and L multiplied by MR multiplied by LC for the Effects, which, as things are circumſtantiated, thoſe Forces or Moments have upon the Bodies I and L. But, as obſerved above, thoſe Effects are equal, conſequently we have for the firſt Step

The Center of Percussion is that Point in a Pendulum, or in an inflexible Rod moving

	1	$I \times PN \times IC = L \times MR \times LC$
But the Triangles PNY and MRY are similar, consequently	2	$PN : MR :: PY : RY$
Comparing the two last	3	$I \times PY \times IC = L \times RY \times LC$
Or taking the Pendulum in the Situation CPR, in which I coincides with P, and L with R, we have	4	$P \times PY \times PC = R \times RY \times RC$
Or, which is the same thing	5	$A \times AQ \times AC = F \times FQ \times FC.$

That is, in Words, if one of the Bodies were multiplied by its Distance from the Center of Oscillation, and the Product arising from thence were multiplied by the Distance of the same Body from the Center of Suspension, this last Product would be equal to the Product of the other Body multiplied by its Distance from the Center of Oscillation, multiplied by its Distance from the Center of Suspension. And, since the same would be true if there were more Bodies, if each Body be multiplied by its Distance from the Center of Oscillation, and that Product by the Distance of the same Body from the Center of Suspension, all the Products relating to the Bodies on one Side the Center of Oscillation taken together, will be equal to all those which relate to the Bodies on the other Side thereof taken together. Let then the Distances of any Number of Bodies, as A, B, D, F, from the Center of Suspension be called a, b, d, f, respectively, and the Distance of the Center of Oscillation Q from the Center of Suspension C, be called x: And suppose the Distances of the Bodies A, B, D, less than the Distance CQ, or x; and that of the Body F greater, as in the Figure; then will the Distances of A, B and D from the Center of Oscillation be expressible by $x-a$, $x-b$, and $x-d$; and the Distance of F, by $f-x$; multiplying then each Body by its Distance from one Center, and the Product arising therefrom by the Distance of the same Body from the other Center, we shall have $Aax - Aaa + Bbx - Bbb + Ddx - Ddd = Fff - Ffx$, which reduced gives $x = \frac{Aaa + Bbb + Ddd + Fff}{Aa + Bb + Dd + Ff}$. Which latter Equation is the Sense of the Rule above laid down.

round

round a Point, with which, if the Pendulum or Rod ſtrikes againſt an Obſtacle, no Jar or Shock at the Point of Suſpenſion ſhall be occaſioned thereby.

Thus, let CF (Fig. 8.) be an inflexible Rod, having the Bodies A, B, D, *&c.* fixed in it at the Points A, B, D, *&c.* and let O be an Obſtacle againſt which, as it vibrates or ſwings round the Point of Suſpenſion C, it may ſtrike againſt: then, if there be no Jar or Shock occaſioned thereby at the Point C, the Point that ſtrikes againſt O, (as the Point Q ſuppoſe) is called the Center of Percuſſion.

PROPOSITION.

The Center of Percuſſion is the ſame with the Center of Oſcillation; and conſequently may be determined by the ſame Rule (*e*).

PROB.

(*e*) *Dem.* From the Definition of the Center of Percuſſion above laid down, it appears, that the Forces with which the Bodies A, B and D, which would paſs above O, move; muſt be a Counterbalance to the Forse of the Body F, which would paſs below it: and that the Force of F muſt be a Counterbalance to them. But the Forces wherewith thoſe Bodies move, are as their Maſſes multiplied by their Diſtances from C, their Velocities being as theſe Diſtances. Farther, when the Point Q comes to O, and is ſtopt there, the Bodies A, B and D, endeavouring to go on, ſway or bear againſt F, and F againſt them; juſt as if they were fixed to a Lever, as AF, having its Fulchrum at Q. Conſequently the Forces of the former Bodies, ſo far as they act againſt the latter, are as their Diſtances from the Point Q; and the Force of the latter, ſo far as it acts againſt the

the

PROBLEM.

Let it be required to find the Center of Oscillation, or Percussion of an inflexible Rod AB (Fig. 9. as a Bar of Iron, or the like) every where of equal Size, and vibrating in, or revolving round the Point A, as a Center of Suspension (*f*).

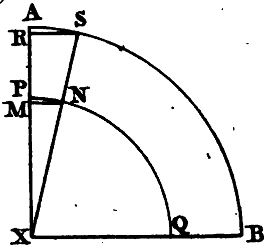
the former, is as its Distance also from Q: the abovementioned Forces must therefore be multiplied by the Distances of the Bodies from Q: but the former of them, as observed above, balances the latter; and the latter them. So many therefore of the last Products as relate to the Bodies above Q taken together, must be equal to that which relates to the Body (or Bodies) below it. But the like Products were equal to each other, when the Point Q was looked upon as the Center of Oscillation (as in the 5th Step of the foregoing Process) consequently the Center of Percussion is the same with that of Oscillation. Q. E. D.

(*f*) *Solut*. Imagine the Rod to be divided into the least possible Parts B, C, D, &c. each of which call *One*. These Parts we may consider as so many Bodies contiguous to one another; so that the Center of Oscillation or Percussion of these Bodies will be the Center of Oscillation or Percussion of the whole Rod. To find this, we are by the Rule above laid down in the Text, to multiply each of these Bodies by the Square of its Distance from A. The first of these Products then will be B (or One) multiplied by AB squared; but one multiplied by AB squared, is the same with AB squared; now AB squared is a square Area or Surface, one of whose Sides is AB. In like manner the Body C, when multiplied by the Square of its Distance from A, is a square Area, one of whose Sides is AC, somewhat less than the former. Imagine this Area laid upon the former; and the next, which will be less still, laid upon that; and so on till you come to the least of all. These will make a Pyramid, whose Base is the first Area, and its perpendicular Height will be equal to the Thickness of them all together; which Thickness will be as the Length of the Line BA. The Value or solid Content

Content of this Pyramid will be ABq (*viz.* its Base) multiplied by a third Part of AB (its perpendicular Height). In the next Place we are to multiply each of those Bodies by its Distance from A: Now the Body B (or One) multiplied by AB, give a Line, as AB; so the Product of C, multiplied by its Distance AC, give a Line, as AC; these Lines heaped one upon another (as the Areas were before) will make a Triangle, whose Base will be AB, and its perpendicular Height also AB; the Value, or Area of which, will be AB multiplied by $\frac{1}{2}$ AB. In the last Place, by the Rule, we are to divide the Sum of the Products in the first Case, by the Sum of the Products in the latter; that is, the Content of the Pyramid by the Area of the Triangle; that is, $AB^q \times \frac{1}{3} AB$, by $AB \times \frac{1}{2} AB$, which gives $\frac{\frac{1}{3} AB}{\frac{1}{2}}$; that is, $\frac{2}{3}$ AB, or two Thirds of AB: so that the Distance of the Center of Oscillation or Percussion, (as E suppose) from A the Center of Suspension, must be equal to two Thirds of AB, the whole Length of the Rod. Q. E. I.

A

Fig. 2.



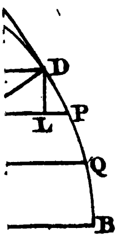


Fig. 3.

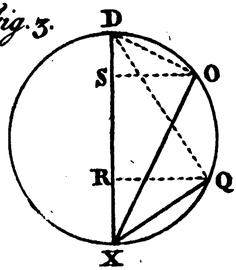


Fig. 4.

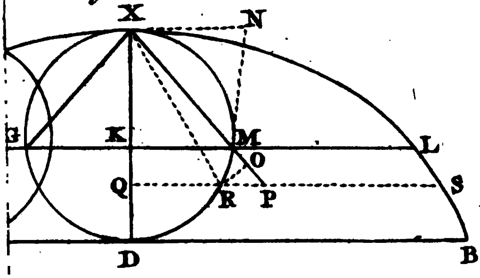
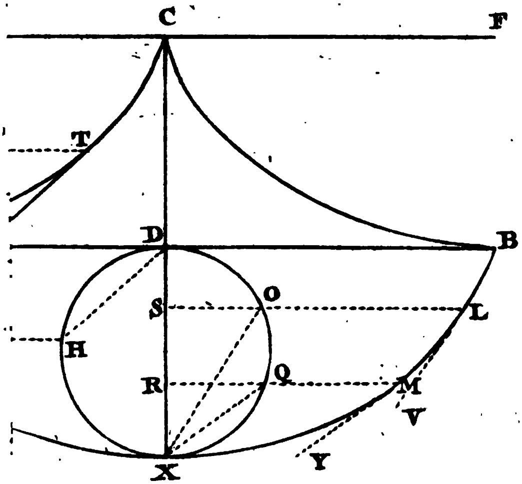
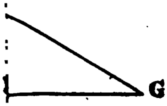


Fig. 5.





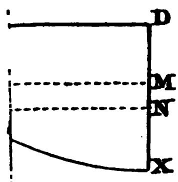


Fig. 8.

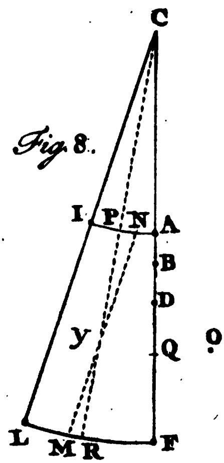


Fig. 9.

A

Compendious System

OF

Natural Philoſophy.

With NOTES

Containing the Mathematical Demonstrations and ſome occaſional Remarks.

PART II.

HYDROSTATICS

And

PNEUMATICS.

To which are added ſome Dissertations relating to theſe Subjects.

CAMBRIDGE.

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MDCCXXXV.

A COMPENDIOUS SYSTEM OF Natural Philosophy.

PART II. HYDROSTATICS.

CHAP. I.

Of the Phænomena which arise from the mutual Action of the Particles of Fluids upon one another.

I. IN the former Part of this Essay, I have laid down and explained the general *Laws of Nature*, and deduced from thence those Phænomena which are in a strict and proper Sense * denominated *Me-*

* In a larger Sense all the Effects and Operations of natural Bodies upon one another may be called Mechanical; as being all subject to the general Laws of Motion. In Hydrostatics Fluids are governed by the Laws of Mechanism, as much as the Mechanical Powers are; the same holds of the Rays of Light

chanical; I proceed now to an Explanation of ſuch, as Philoſophers have comprehended under the Name of *Hydroſtatics*; the Intention of which is to explain the Nature of *Fluids*, and the Manner in which they act upon one another and upon Solids.

II. THE Nature of a *Fluid**, as diſtinguiſhed from that of a *Solid* or hard Body, conſiſts in this, *viz.* that its Particles are ſo looſely connected together, that they readily move out of their Places, when preſſed with the leaſt Force one way more than another†.

Light, as will be ſeen when we come to Optics; and in the larger Bodies of the Planetary Syſtem, Mechaniſm equally prevails, as has been demonſtrated by Sir *Iſaac Newton*; which we ſhall endeavour to make out (when we treat of Aſtronomy) ſo far as the Nature of our Deſign will permit.

* Some Philoſophers make the following Diſtinction in Fluids; thoſe which flow or ſpread themſelves till their Surface becomes level or horizontal, they call *Liquid*; in contradiſtinction to Flame, Smoak, Vapour &c. which are alſo Fluids, but do not acquire ſuch a Surface. Thoſe which are capable of exciting in us the Idea of Moiſtneſs, as Water, &c. they call *Humid*, diſtinguiſhing them thereby from Air, Quickſilver and melted Metals. But thoſe Diſtinctions are quite unneceſſary in a Philoſophical Senſe: all Fluids being equally Liquid, when not prevented from putting on that Appearance by the Bodies about them; and as to Humidity, that is only a *relative* Quality; for though Quickſilver will not moiſten, or ſtick to a Man's Fingers, it will to Silver or Gold.

† The common Definition, *Fluidum eſt cujus partes impreſſioni cuicunque cedunt, & cedendo facillimè moventur inter* [illegible], though it expreſſes very well the Nature of a *compreſſible* F[illegible], as Air, yet does not anſwer to that of Water, whoſe Parts [illegible] been found to yield to no Force with which they have been c[illegible]preſſed, unleſs it was greater on one ſide than on th[illegible].

The

From whence Philosophers conclude, that they are exceedingly minute, smooth, and round *; it being otherwise impossible they

The Definition therefore seems imperfect as not expressing the *Inequality* of Pressure which is requisite to move the parts of some Fluids one among another.

* It is commonly observed, that the Roundness of the Particles conduces very much to Fluidity upon this Account, *viz.* because round Bodies touching one another in but few Points, the Force with which they mutually attract each other is the weaker. But upon this Supposition the Particles of a Fluid ought to move with less Freedom one among another, by how much the greater the Weight is with which they are compressed, (for it is the same thing in this respect, whether they press against each other by virtue of their own Attraction, or by some external Force) but that they do so, we have no Experience. A Diver, upon plunging out of his Bell at the bottom of the Sea, never finds the Water less fluid, notwithstanding the great pressure from above. Mr. *Boyle* having caused a Tadpole to be put into a Vessel of Water, and to be pressed with a very great Force, tells us that in appearance it found no Inconvenience from thence, but swam about with the same Freedom and Briskness as ever.

Quære, whether the Particles of which Fluids consist are in Contact with each other, or not? Perhaps they are prevented from approaching, nearer than to a certain Distance, by a repelling Power diffused around each single Particle. The Observation, that Water is not render'd less fluid by Pressure, seems to favour this Opinion; and the Property which the Air has of expanding or contracting it self, according to the Weight which it sustains (as shall be shewn Chap. 3.) proves beyond contradiction, that its Particles are endued with such a Power. But if the Particles of all Fluids have this Power, it will follow that they ought to be in some measure capable of being reduced into less Space by Pressure, as Air is, which they have not as yet appeared to be. Further since it has been proved (Part I. Chap. 3.) that if the parts of Fluids are placed just beyond their natural Distances from each other, they will approach and run together, and if placed farther asunder still, will

repel

should move with such Freedom one among another upon the least Inequality of Pressure.

repel each other; it follows upon the foregoing Supposition, that each Particle of a Fluid is surrounded with three Spheres of Attraction and Repulsion one within another: the innermost of which is a Sphere of Repulsion, which keeps them from approaching into Contact, the next a Sphere of Attraction diffused around this of Repulsion, and beginning where this ends, by which the Particles of Fluids are disposed to run together into Drops; the outermost of all a Sphere of Repulsion, whereby they repel each other, when removed out of that of Attraction.

If this were allowed, and we might go on, and suppose the Particles of all Bodies to attract and repel each other *alternately* at different Distances, perhaps we might be able to solve a great many Phænomena relating to small Bodies, which are now beyond the reach of our Philosophy. However upon the Supposition of the three Spheres of Attraction and Repulsion just mentioned, nothing is more easy than to see how Solids may be converted into Fluids, and Fluids into Solids (as is done in Liquefaction and Freezing); for allowing that the first or innermost Sphere of Repulsion, is capable, like that of the Particles of Air, of being augmented by Heat, and diminished or totally suspended by Cold, it follows that Bodies must be more or less fluid in proportion to the degree in which they are affected by Heat or Cold; for when the Action of the first Sphere of Repulsion is diminished or destroyed by Cold, the Particles of the Fluid must necessarily be brought into closer Contact with each other by the Force of the circumambient Attraction, and by that means constitute an harder Body than before. But we must not dwell too much upon an Hypothesis which wants Proof; I shall only add, that although some Fluids, as Water, have not been *as yet* contracted in their Dimensions, or made to take up less Space than they naturally do, by any Force with which they have been compressed by Art; yet there are none but are naturally contracted by Cold, from whence it seems reasonable to infer, that their Particles are at least *capable* of being brought into closer Contact, which is some Confirmation of this Doctrine.

It is an obvious Objection to this, that Water by freezing is augmented in its Bulk; but this may be owing to those Bubbles or Vacuities observable in the Water after it is frozen, which were

III. THOSE Particles considered separately are endued with all the common Properties of Matter, and subject to the same Laws of Motion and Gravitation with larger Bodies. To enquire therefore into the Nature of Fluids, is to consider what Appearances a Collection of very small round Bodies, subject to those Laws, will exhibit under different Circumstances. In order to which, it is usual with hydrostatical Writers to consider a Fluid, as divided into several perpendicular Columns contiguous to each other. Sometimes it is convenient to conceive it divided into thin Plates or *Strata* lying upon one another. In some Cases, the same Fluid is considered as distinguished both these Ways, *viz.* into perpendicular Columns, and also into thin Strata or Plates. Figure I. represents a Vessel filled with a Fluid to the Height EF, and divided into the Columns GH, IK, LM *&c.* and also into the Strata RS, TV, XY *&c.*

IV. FROM this Observation concerning the Properties of the Particles considered separately, immediately results the following Proposition, *viz.* that in a Vessel whose Form is such as is represented by ABCD, (*Fig.* I.) the Quantity of Pressure which each Stratum su-

were not in it before; and not to any general and uniform removal of the Particles of the Fluid from each other, which the Objection, if it is of any Force against what has been advanced, must suppose.

ſtains from the Weight of the Incumbent Fluid, is in Proportion to the Number of thoſe Strata which reſt upon it, that is, as the Height of the Surface of the Fluid; for if we ſuppoſe the Strata of equal Thickneſs, the Quantity ſuſtained is proportionable to the Number of Strata of which it conſiſts.

V. When the Surface of a Fluid is horizontal or level, each Particle thereof is diſpoſed to continue in its Place, being ſuſtained therein by the contiguous ones.

Let the Fluid be ſuppoſed to be divided into Strata, each of the Thickneſs of a Particle of the Fluid, and if the Truth of this Propoſition is denied, let the Particle *mn* be one of thoſe which is not ſuſtained in its Place by the contiguous ones, but is moving from thence towards ſome other Part of the Veſſel, *v. g.* towards D. Now ſince all the other Particles of that Stratum are at an equal Depth below the Surface of the Fluid with this, they alſo ſuſtain an equal Degree of Preſſure (by the laſt Propoſition,) conſequently for the ſame Reaſon that one of them is moving towards D, the reſt may all be ſaid to be moving in the ſame Direction: but this cannot be true of the whole Stratum, while the Veſſel is entire, and therefore of none of its Parts. Now the like Reaſoning will hold againſt the Motion of the Particle *mn* towards any other Part of the Veſſel, from whence it follows,

Fig. 18. P. 63.

Fig. 19. P. 64.

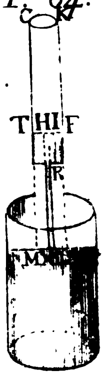
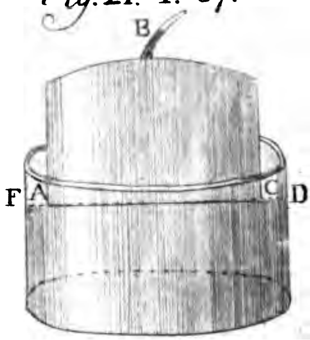


Fig. 20. P. 66.

Fig. 21. P. 67.



67.

Fig. 23.

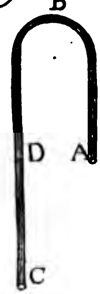


Fig. 24. P. 68.

Fig. 25.

Fig. 26. P. 71.

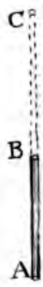
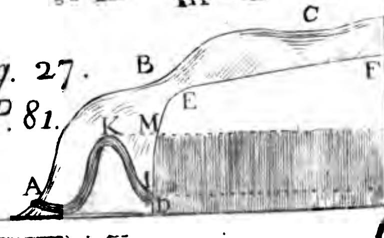


Fig. 27. P. 81.



llows, that each Particle of the Fluid, is
tained in its Place by the contiguous ones,
d therefore disposed to continue at Rest *.

VI. FROM hence is derived a fundamental
:oposition in Hydrostatics, *viz.* That when
.e Surface of a Fluid is level, whatever Pres-
.re any single Particle or small Portion of it
stains from the contiguous ones on one
rt, it sustains the same on all the rest, that
it is pressed by them with an equal Degree
Force on all Sides †.

FOR by the Definition of a Fluid (§. 2.)
:h Particle is disposed to give Way, and
ve out of its Place, when the Pressure is
t equal on all Sides; and (§. 5.) each Par-
le is pressed by the contiguous ones in such
nanner that it is sustained in its Place there-
, it is therefore pressed with an equal De-
:e of Force on all Sides.

Corol. FROM hence it follows that each

* This shews us the Absurdity of some Philosophers, who
e the sole Difference between Solids and Fluids in this,
that the Particles of these are ever in Motion, while those
: other are always at Rest.

† This Proposition with its Corollary is not strictly speaking
unless the Particle or Portion of Fluid we speak of is sup-
void of Gravity, for it presses downwards with a Force
to the Weight of those Particles which rest upon it added
own, whereas the Force with which it presses upwards is
equal to the Weight it sustains, *viz.* that of the incumbent
.. But the Particles of Fluids are so exceedingly minute,
the Gravity of each so very small, that the Error arising
hence can never be sensible.

Particle or ſmall Portion of a Fluid preſſes with the ſame Degree of Force in all Directions on thoſe which are contiguous to it. For by the third Law of Nature, every Particle preſſes upon the contiguous ones with the ſame Degree of Force, with which it is preſſed upon by them.

VII. THE Surface of a Fluid becomes level by its own Gravity, when no external Force prevents it from being ſo.

FOR the Particles of Fluids preſs in all Directions with Forces proportionable to the Height of their Surfaces (Cor. §. 6. and §. 4.) if then the Surface is not level, the different Parts of the ſame inferior Stratum will be preſſed not only downwards, but ſideways againſt each other with unequal Forces; the greater Preſſure therefore overcoming the weaker, the Particles which ſuſtain the leaſt Preſſure will be driven out of their Places and raiſed up, till the Surface becomes level; the Surface being level, each Particle will be equally preſſed in every Direction, (§. 8.) all therefore will remain at Reſt, and the Surface continue in that State*.

* This Demonſtration, as alſo the foregoing, is founded upon a Suppoſition, that Bodies tend downwards by their Gravity in Lines parallel to each other, which though *phyſically* true, is not ſtrictly ſo, their Tendency being towards the Center of the Earth, and conſequently in Lines which meet in a Point: and therefore if we would be accurate, the Fluid contained in a Veſſel ſhould be conſidered, as divided into Columns and

VIII.

VIII. FLUIDS gravitate in Fluids of the ſame Kind.

THIS Propoſition is a neceſſary Conſequence of what has been obſerved about the Nature of the Particles of which they conſiſt, *viz.* that they are ſolid, and endued with the ſame Properties with other Bodies. The Reaſon why their Gravity is not ſenſible in the Fluid, is becauſe the lower Parts ſuſtain the upper, and hinder them from deſcending. But it does not follow from thence, that their Gravity is entirely taken away, as

Strata, as repreſented Figure the ſecond, where *ABD* is the Earth, *C* its Center, *EFGH* a Fluid contained in a Veſſel, and divided into Columns, which if continued down to the Center of the Earth, would there terminate in a Point *C*; and into the concentric Strata *ab*, *cd*, &c. having the Center of the Earth for the Center of their Convexity. And then conſidering the Strata to be of this Form, and arguing from thence, in the ſame manner as before, we ſhall find that the Particles of the Fluid will not be in Æquilibrio with each other, till all the Parts of its Surface are at equal Diſtances from the Center of the Earth, forming thereby the Surface *EF*, concentric to that of the Earth. Conſequently the Surfaces of Fluids are not level or plain, but convex, having the Center of the Earth for the Center of their Convexity.

This Convexity by reaſon of the great diſtance of the Earth's Center approaches ſo near to a Plane, that in ſmall Portions of it the Difference is not ſenſible, and therefore may be neglected; but at Sea 'tis evident to Senſe: for when the Mariners put to Sea, the Shore firſt diſappears, then the lower Buildings, afterwards the Towers, Mountains *&c.*; in like Manner, when they approach a diſtant Ship, the top of the Maſt and Sails appear firſt, while the Ship it ſelf is intercepted from their View, by the Convexity of the Water between them, but when they aſcend the Maſt of their own Ship, it may eaſily be ſeen.

ſome * Philoſophers have imagined; for by ſo much as the lower Parts preſs upon thoſe which are above them, juſt ſo much additional Weight do they receive from the Reaction of the other upon them: Thus their Weight is communicated to the Veſſel, which upon this Account weighs according to the Quantity of the Fluid it contains.

IX. THE Preſſure of a Fluid is in Proportion to its perpendicular Height, and the Quantity of Surface againſt which it preſſes.

THIS Propoſition admits of four Caſes.

1. WHEN the Fluid is contained in a Veſſel of the ſame Dimenſions from top to bottom, and held in an erect Poſition, as that repreſented Fig. 1. it is evident the Preſſure of the Fluid upon the bottom will be in Proportion to its Magnitude, and the perpendicular Height of the Surface of the Fluid above it. For conceiving it divided into Columns, the Preſſure upon the Bottom by the fourth Propoſition, will be as the Length or Height of the Columns: and it will alſo be as the Number of them, becauſe the Quantity of Fluid which preſſes upon the Bottom is in that Proportion, that is, as the Magnitude of the Bottom preſſed upon. But when the Veſſel is inclined or irregular, the Truth of this Pro-

* This was the Notion of the *Carteſians*, who held, that when a Fluid is mixed with another of the ſame Kind, it loſes its own Weight thereby.

poſition

poſition is ſo far from being evident, that it has been commonly looked upon as a Paradox.

2. LET the Veſſel ABCD (*Fig.* 3.) be filled with a Fluid to the Height EF, and held in an inclined Poſition, as there repreſented; I ſay the Preſſure of this Fluid is proportionable to the Magnitude of the Baſe CD, and FG or HD the perpendicular Height of the Surface of the Fluid above it.

FOR ſuppoſing the Fluid divided into the Strata EI, KM, LO, *&c.* ſo far as the firſt Stratum EI is prevented from preſſing upon KI the Surface of the next inferior Stratum, by being in ſome Meaſure ſupported by the Side of the Veſſel FI, ſo far is its Preſſure augmented by the Reaction of the oppoſite Side EK upon it, which is exactly equal to the Action of the former, becauſe the Fluid preſſing every Way alike, at the ſame Depths below the Surface, exerts an equal Force againſt both theſe Sides. The Surface therefore of the ſecond Stratum is preſſed with the ſame Degree of Force with which it would be, if the Quantity of Fluid contained in the former Stratum was included within the Space HKQI, which is exactly equal to it, as having the ſame Baſe KI and the ſame perpendicular Height QI*. Now this being true of each Stratum, their Preſſure upon CD the Baſe of the Veſſel is the ſame as if they were all placed per-

* 31. *El.* 11.

pen-

pendicularly over it, and filled the Space RHCD; which they would do, since the sum of their perpendicular Heights QI, KS, LT and NV is equal to HD the perpendicular Height of this Space, and each of their Bases KI, LM, *&c.* is equal to CD its Base *. But by the foregoing Case, if the Space RHCD was filled with a Fluid, the Pressure of it would be proportionable to the Dimensions of the Base CD, and the perpendicular Height DH, therefore it is the same in the inclined Tube ABCD†.

3. LET the Vessel ABC be irregular, as represented *Figure* the 4*th*, and filled with a Fluid to the Height D, I say the Pressure of the Fluid upon the Base C, is proportionable

* 31. *El.* 11.

† Perhaps it may be thought more Geometrical to demonstrate this Proposition with the Generality of Authors from the Property of the inclined Plane. They consider *BD* the lower Side of the Tube, as an inclined Plane, on which the Fluid contained within it rests, and argue that it loses thereby a Part of its Weight in Proportion to the Length of the Plane, and therefore occasions no greater Pressure upon the Base, than if the Vessel was held erect, and filled only to the same perpendicular Height, as when inclined. But this Demonstration proves too much, for by this Way of Reasoning, one might shew, that the Pressure of the Fluid *EFCD* upon the Base *CD* is less than the Pressure of *RHCD* a Column of the same Fluid having an equal Base and perpendicular Height with it. For both the inclined and the perpendicular Column contain the same Quantity of Fluid, upon account of the equality of their Bases, and perpendicular Heights, but that rests upon an inclined Plane, which this does not, and therefore presses less upon the Base. But this is contrary both to Demonstration and Experience, this Argument therefore proves too much.

to

to the Magnitude of the Base, and CD the perpendicular Height of the Surface of the Fluid above it.

In Order that the Proof of this Proposition may be the better understood, we must premise the following *Lemma.* viz.

That when a Fluid passes through a Pipe, as AB, *(Fig. 5.)* which in some Parts is larger than in others, the Moment or Force with which it moves is every where the same. For while the Fluid is passing through the Tube, its Velocity in every Part will be reciprocally as the Quantity of Matter; for Instance it will be as much greater at C than it is at D, as the Quantity passing through C at any Instant of Time is less than at D, and so of the rest; because a less Quantity would be conveyed through the smaller Parts of the Tube in the same Time, unless it should move faster there in Proportion to the Smalness of them. Now the Momentum of Bodies is partly owing to the Quantity of Matter, and partly to the Velocity; (as explained Part I. Chap. 9. §. 1.) consequently what the Fluid, which is actually passing through the narrower Parts of the Tube, wants in Quantity, is compensated by its Velocity in those Parts, and what it wants in Point of Velocity in the other Parts is made up by the Quantity passing through them; so that the Moment is the same in every Part of the Tube, whether larger or nar-

narrower *. The ſame is true, whatever be the Poſition the Tube is held in.

Let us now conceive the Fluid in the Veſſel ABC (*Fig.* 4.) to be diſtinguiſhed into the Strata EF, GH, IK *&c.* Let us alſo imagine the Bottom of the Veſſel C to be moveable, that is, capable of ſliding up and down the narrow Part of the Veſſel, *v. g.* from C to GH, (without letting any of the Fluid run out.) Let it further be ſuppoſed that this moveable Bottom is drawn up or let down with a given Velocity, while the Veſſel it ſelf is fixed and immoveable; it is evident the lowermoſt Stratum which is contiguous to the Bottom will be raiſed or let down with the ſame Velocity, and will thereby have a Moment proportionable to that Velocity and the Quantity of Matter it contains: But by the Lemma, all the reſt of the Strata will have the ſame Moment, conſequently the Moment of all taken together, (that is, of the whole Fluid,) is the ſame, as if the Veſſel had been no larger in any one Part, than it is at the Bottom, (for then the Moment of each Stratum would alſo have been as great as that of the lowermoſt;)

* Thus we may obſerve in a River or Canal, that by how much the Breadth or Depth is leſs in any Part, ſo much the more rapid is the Stream in that Part; and on the contrary where it is wider and deeper, the Motion of the Water is more gentle and languid. So that the Moment with which it flows is the ſame in every Part.

Part II. Plate I. *Pag.* 16.

1. *P.* 7.

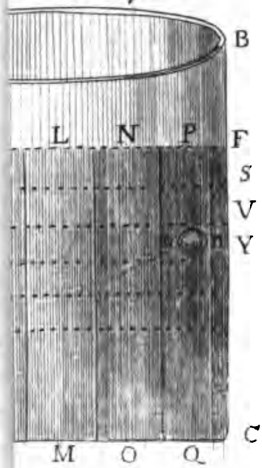
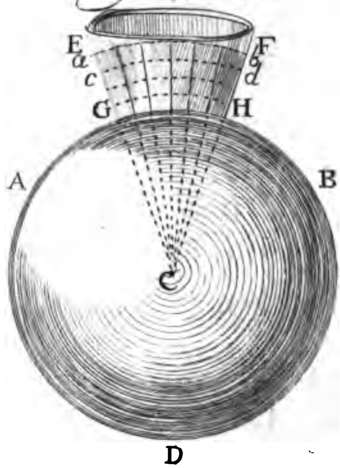


Fig. 2. *P.* 11.



3. *P.* 13.

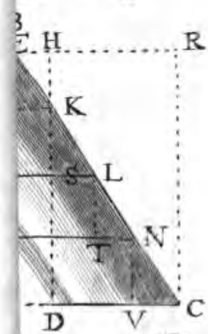


Fig. 4. *P.* 14.

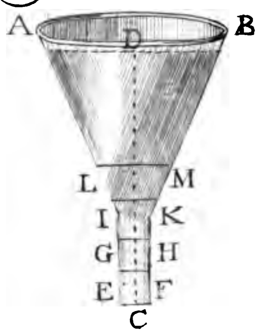
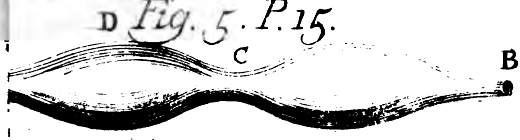


Fig. 5. *P.* 15.



most;) the Pressure therefore, or Action of the Fluid with which it endeavours to force the Bottom out of its Place, is as the Number of Strata, that is, the perpendicular Height of the Fluid, and the Magnitude of the lowermost Stratum, that is, the Bottom.

4. AGAIN, suppose the Vessel ABC (*Fig.* 6) filled with a Fluid to D; I say the Pressure upon the Bottom BC, is proportionable to the Dimensions of the Bottom, and DE the perpendicular Height of the Fluid.

FOR if we suppose the Bottom moveable, as before, and raised up or let down with a given Velocity, the Moment of every Stratum will be the same with the lowermost by the Lemma; therefore the Moment of all taken together, is the same as if the Vessel had been no less in any one Part than it is at the Bottom; consequently the Pressure is proportionable to the perpendicular Height, and the Magnitude of the Bottom. This Case is the Converse of the former *,

* Upon this Proposition is founded the Practice of conveying Water through Pipes from Place to Place, &c. For from hence it follows, that if one end of a Pipe is laid in a Reservoir of Water, the Fluid will run into the Pipe till it rises to a Level at the other End with its Surface in the Reservoir. Thus let *ABC* (*Fig.* 8.) represent a Reservoir or Bason of Water, *DGE* a Pipe laid from thence to *E*. If *E* the end of the Pipe is placed above the Line *ABF* the Level of the Water in the Reservoir, the Water will run into the Pipe till it rises in the other End to *F* the Level with *AB*, at which time the Water in the Pipe will be in Æquilibrio with that in the Reservoir, and remain at Rest. But if the End of the Pipe is

 below

FROM hence it follows, that if a Vessel is made of such Form, as is represented (*Fig. 7.*) by ABCDEFG, and filled with a Fluid to the Height C, the Weight which the Bottom sustains, is as great as it would be, had the Form of the Vessel been IKFG, which is every where of the same Dimensions, that the other is of at the Bottom, and filled to the Top IK. Because the Pressure by the Proposition, is proportionable to the Bottom and perpendicular Height, which in both Cases are the same.

THE Reason why the Vessel ABCDEFG, with the Fluid contained in it, does not weigh so much, as the Vessel IKFG, when full to the same Height IK, notwithstanding the Pressure upon the Bottom is the same in both, is because ABDE the upper Part or Cover

below the Surface of the Water in the Reservoir, it will continue to run out, till they are reduced to a Level. For let *GH* be the lowest Part of the Pipe, then since *F* the perpendicular Height of the Fluid on one Side, is equal to *B* the perpendicular Height of the Fluid on the other, and *GH*, which (being the Place where the Fluids press one against another,) may be considered as a Base to both, is common, it follows from this Proposition that the Pressures on each Side are exactly equal, and therefore being in contrary Directions will necessarily destroy each other, and the Fluid will remain in Æquilibrio. But while the End *E*, is below the Level, this Æquilibrium cannot be obtained, and therefore the Fluid will continue to run out.

For the same Reason, when two or more Tubes communicate with each other, the Surface of the Fluid they contain will stand at the same Level in all.

of

of the former Veſſel, is preſſed upwards by the Fluid below it with a Force equal to the Endeavour the Fluid in the ſmall Tube B C D has to deſcend, Which Endeavour is the ſame that it would be, if the Tube B C D comprehended alſo the two Spaces I C B A and C K E D, its Moment being the ſame in both Caſes by the Demonſtration; the Cover therefore is preſſed upwards with a Force equal to the Weight of as much Fluid as would fill the two Spaces I C B A and C K E D; conſequently the Veſſel, whoſe Form is A B C D E F G, is ſo much lighter than the other, that is, as much as the Fluid it contains is leſs.

FROM hence ariſes this Paradox, that the leaſt Quantity of Fluid may be made to raiſe any Weight how great ſoever it be.

FOR by the Propoſition the Cover A B D E is preſſed upwards with a Force equal to the Weight of as much Fluid as would fill the two Spaces I C B A and C K E D; now thoſe Spaces may be enlarged at Pleaſure in Height, by lengthing the Tube B C D (which at the ſame time muſt be made proportionably ſmaller, otherwiſe the ſame Quantity of Fluid will not fill it); it follows therefore that the ſame Quantity of Fluid may be made to preſs the Cover upwards with a given Force; if that Cover then is made moveable, any Weight that is laid upon it may be ſupported thereby.

X. The Velocity with which a Fluid spouts out at an Hole in the Bottom of a Vessel, is equal to that which a Body would acquire by falling freely from the Level of the Surface of the Fluid to the Hole.

Let there be a large cylindrical Tube ABCD (*Fig.* 9.) in the upper Part of which let us imagine a Cylinder of Ice FGHI exactly fitting it; let it further be supposed that HI the lower Surface of the Ice is continually melting, so as to afford a Stream of Water running down the Middle of the Tube. Now the Form of this Stream of Water will necessarily be such as is represented in the Figure by HLI, for the Water falling freely will descend faster and faster like other Bodies, causing thereby the Stream to become narrower and narrower. Now let it be supposed that the Tube has a Bottom as CD with an Hole in it at K just sufficient to let the Stream pass freely, it is evident there will be no Obstruction on this Account, but that the Fluid will pass through the Hole with such Velocity as it naturally acquires by falling from HI the lower Surface of the Ice. And if we suppose M and N the empty Parts of the Tube to be filled with Water, the Water will press equally upon the Sides of the Stream in every Direction (§. 6.) and therefore will be no Impediment to its Motion on that Account. Lastly let us suppose the Ice taken away, and the

the Stream supplyed from the Water at the Sides, as is the Case when a Fluid runs out through the Bottom of a Vessel; then will the Velocity with which the Water flows through the Hole continue the same; for so far as the Water coming from the Sides endeavours to descend it self, so far it obstructs the Descent of the Stream, and no farther; and consequently causes no Alteration in the Velocity or Quantity of Fluid running out. The Velocity therefore with which the Fluid passes through the Hole, is equal to that which a Body would acquire by falling freely from the Level of the Surface of the Fluid to that Place.

If the Hole is made in the Side of the Vessel at the same Distance below the Surface, the Velocity will be the same, on Account of that equal Tendency Fluids have to move every Way alike *.

* Upon this Principle is founded the Practice of making artificial Fountains. For if to a Vessel or Reservoir *ABCD* filled with a Fluid to the Height *EF*, be fixed the Pipe *CH*, (as represented *Fig.* 10.) with a small Aperture at *K*, the Fluid will spout up from thence to *FL* the Level of the Surface of the Fluid in the Vessel. For by this Proposition it will spout from *K* with such a Velocity, as a Body would acquire by falling from *FL* the Level of the Surface to the Aperture at *K*, that is, such as will carry it from the Aperture to the Level; because that Velocity which a Body acquires by falling from a certain Height is sufficient to carry it back to the same Height from whence it fell.

But in Practice the Height the Fluid rises to, is less than that of the Level of its Surface in the Reservoir; this is owing to the Resistance it meets with from the Air, its Friction against

XI. The Velocity with which a Fluid spouts out from the Bottom or Side of a Vessel is as the square Root of the Height of its Surface above the Hole *.

The Cause why a Fluid spouts out through an Hole made in the Bottom or Side of a Vessel is the Pressure or Weight of the Fluid incumbent upon the Hole; from whence it should seem, that the Velocity ought to be as the Pressure; but if so, then the Quantity run out would also be as the Pressure (for the faster the Liquor flows the greater is the Quantity thrown out in a given Time, and *vice versâ*) consequently upon this Supposition we should have two Effects, each depending on the same Cause, and equal to it, which is absurd. 'Tis not then the Quantity

against the Sides of the Pipe *&c.* It is found impossible to make it much exceed the Height of an hundred Feet: For when it spouts out of the Aperture with a Velocity necessary to carry it higher, the Stream is immediately dashed to Pieces by the Resistance of the Air, whereby it loses its Force, and is prevented from rising to any considerable Height.

* This Proposition may be otherwise demonstrated from the last in the following Manner. For since the Velocity with which a Fluid spouts out through an Hole in the Bottom or Side of a Vessel, is equal to that which a Body would acquire by falling from the Level of the Surface of the Fluid to the Hole, and the Velocities Bodies acquire by falling are as the square Roots of the Heights they fall from (Part I. Chap. 5. §. 5.) it follows that the Velocity, with which a Fluid spouts out from an Hole in the Bottom or Side of a Vessel, is as the square Root of the Height of the Level of the Surface of the Fluid above the Hole.

of

of Fluid run out, nor the Velocity with which it flows, but its Moment or both these multiplied together, (Part I. Ch. 9. §. 1.) that is the true and adequate Effect of the Pressure. These therefore being ever in the same Ratio, will each of them be as the square Root of the Pressure: For then being multiplyed together, their Product or the Moment of the spouting Fluid is adequately as the Pressure which occasions it; but the Pressure is as the perpendicular Height (§. 4.) therefore the Velocity and also the Quantity of Fluid spouting out is as the square Root of the Height of its Surface above the Hole.

To give an Instance or two; suppose two Holes made in the Side of a Vessel, the one an Inch below the Surface of the Fluid it contains, the other four; the Velocity with which the Liquor flows out of the lower Hole, will not be four times as great, as that with which it flows through the upper, notwithstanding the Pressure is four times greater: for if it should, the Quantity run out in a given Time would also be four times greater, consequently the Effect produced would be sixteen times greater than it is at the upper Hole, that is, four times greater than the Cause, which is absurd. Whereas the Velocity and Quantity of Matter will each be only twice as great as they are above, producing thereby a Force or Moment only four times as great, which is propor-

tionable to the Cause. So if an Hole were made sixteen times lower than the first, the Velocity and Quantity of Matter will not be each sixteen times greater than at the other, but only four times greater apiece, and so the Moment sixteen times greater, as the Pressure is *.

* From hence we may see the Error some of the foreign Mathematicians have fallen into with Regard to the Forces of moving Bodies, who contend that they are as the Squares of the Velocities multiplied by the Quantities of Matter. For from this Proposition it is, that one of the principal Arguments brought in Favour of this Opinion is derived. They argue thus, *Effects* are ever proportionable to their *Causes*, the *Pressure* of the incumbent Fluid is the *Cause* of its spouting out, the *Force* with which it spouts out, is the *Effect*, but by this Proposition the *Pressure* is as the *Square* of the *Velocity* it flows with, therefore the *Force* is likewise as the *Square* of the *Velocity*. True, it is so; but let us see the Consequence. The Force with which the Fluid spouts out is not only owing to the Velocity, but the Quantity run out in a given Time, they have each their Share in producing the Force, consequently the Force is in a Ratio compounded of both, or as the Product of one multiplied by the other, or, which comes to the same Thing, (since as was observed before, they are in the same Ratio with each other) as the Square of either of them; from hence it is, that the Forces of Fluids in Motion are said to be as the Squares of their Velocities; not that they are so in Virtue of those Velocities, as such, but in Virtue of them, and the Quantities of Matter taken together, or because the Squares of the Velocities is the same Thing with the simple Velocities multiplied by the Quantities of Matter. Therefore when it is said, the Forces of Fluids are as the Squares of the Velocities, that Part of the Force which arises from the Quantity of Matter is really taken into Consideration. How ridiculous then must it be in those Gentlemen to fetch an Argument from hence to prove, that the Forces of Bodies in Motion are as the Squares of the Velocities and Quantities of Matter too, when they are as the Squares of the Velocities, only because the Quantities of Matter are implied in them.

P. 17.

Fig. 7. P. 18.

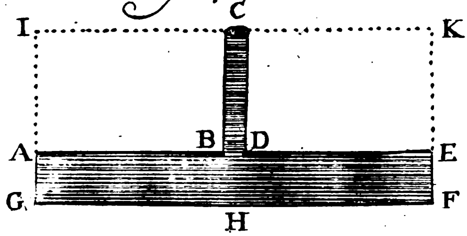
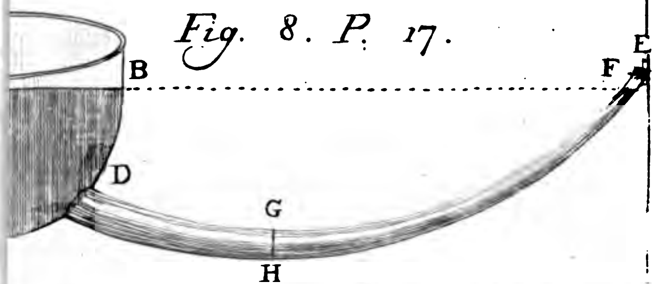


Fig. 8. P. 17.



9. P. 20.

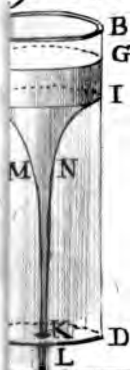
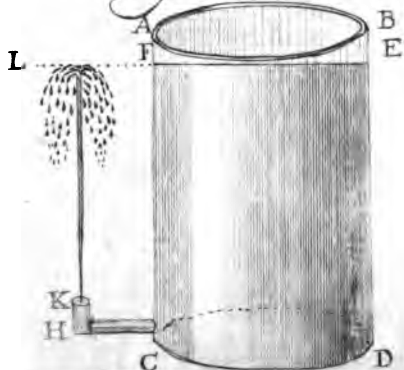


Fig. 10. P. 21.



XII. When a Current of Water or other Fluid falls perpendicularly upon the Surface of a Plane, or flows againſt it, (as the Wind againſt the Sail of a Ship, or the like) the Force, which the Fluid exerts upon it, is equal to the Weight of a Column of the ſame Fluid, whoſe Baſe is equal to the Plane, and its Height ſuch, that a Body falling freely through it would acquire the ſame Degree of Velocity with which the Fluid moves *.

In Order to demonſtrate this Propoſition, let us ſuppoſe the Veſſel ABCD *(Fig.* 11.) filled with a Fluid, and having a large Hole EF in the Bottom, then will the Preſſure of the Fluid cauſe a Stream to flow out, which in the Hole it ſelf will have ſuch a Degree of Velocity, as a Body would acquire by falling freely from the Surface of the Fluid in the Veſſel to the Hole (as demonſtrated §. 10.) In the midſt of this Hole, and conſequently in the Stream, let us ſuppoſe a Plane as PQ ſuſpended, but ſomewhat leſs than is ſufficient to fill the Hole, leaſt it ſtop the Current of the Water. Now 'tis certain this Plane ſupports a Column of the Fluid equal to that which preſſes upon any other Part of the Bottom of the Veſſel of equal Dimenſions with

* From this Propoſition is deduced the Method of computing the Power of a Machine, which is to be moved by Wind or Water &c. See an Inſtance of ſuch a Calculation in the Memoirs of the Royal Academy of Sciences for the Year 1725.

it ſelf (for being thus placed, it may be look-ed upon as a Part of the Bottom) but every Part bears a Column, whoſe Baſe is equal to its own Dimenſions, and its Height the ſame with that of the Surface of the Fluid in the Veſſel: Conſequently this Plane ſupports ſuch a Column, that is, it is reſiſted by the Stream with a Force equal to the Weight of a Column, whoſe Baſe has the ſame Dimenſions with it ſelf, and whoſe Height is equal to that of the Surface of the Fluid in the Veſſel, that is, ſuch an Height as a Body by falling freely from, would acquire a Velocity equal to that with which the Fluid moves.

XIII. The Preſſure of a Fluid againſt a perpendicular Bank or Sluice *&c.* is equal to the Weight of a Column of the ſame Fluid, whoſe Baſe is equal to ſo much of the Bank as is below its Surface, and which has half the Depth of the Fluid for its Height *.

If the Preſſure upon every Part of the Bank from the Surface to the Bottom was as great as it is at the Bottom, the Preſſure againſt it would be equal to the Weight of a Column whoſe Baſe is equal to ſo much of the Bank as is under the Surface of the Fluid, and

* From hence we ſee the Reaſon, why the Water of the Sea or great Lakes is as eaſily kept within their Banks (ſetting aſide the Force which ariſes from the Motion of the Waves *&c.*) as that of the narroweſt Canal, *viz.* becauſe the Preſſure of Fluids is not in Proportion to their Surfaces, but their Depths.

which

which has the *whole* Depth of the Fluid for its Height; for the Preſſure upon every Part of the Bank at the Bottom is equal to the Weight of a Column, whoſe Baſe correſponds to the Part preſſed upon, and its Height is that of the Depth of the Fluid; conſequently if the Preſſure was the ſame every where from Top to Bottom, it would be equal to the Weight of as many ſuch Columns as would anſwer to all the Parts of the Bank: But the Preſſure every where diminiſhes in Proportion as we approach the Top, where it is Nothing; it is therefore but *half** what it would be in the other Caſe; from whence the Propoſition is clear.

CHAP. II.

Of the Effects Fluids have on Solids immerſed therein.

I. THE *Specific* Gravity of a Body is that, by which it is ſaid to be heavier or lighter than another of a different Kind: Thus Lead is ſaid to be ſpecifically heavier than Cork; becauſe ſuppoſing an equal Bulk of

* Becauſe the Sum of a Number of Terms in Arithmetical Progreſſion beginning from Nothing, is half the Sum of an equal Number of Terms, each of which is equal to the laſt in the Progreſſion.

each, the one would be heavier than the other. From hence it follows that a Body ſpecifically heavier than another is alſo more denſe, that is, contains a greater Quantity of Matter under the ſame Bulk, becauſe Bodies weigh in Proportion to the Quantities of Matter they contain (Part I. Chap. 3. §. 7.)

II. If a Solid is immerſed in a Fluid of the ſame ſpecific Gravity with it ſelf, it will remain ſuſpended therein, in whatever Part of the Fluid it is put.

Let the Body FGHI *(Fig. 12.)* be immerſed in the Fluid ABCD to the Depth MN, or any other whatever; I ſay, it will continue in the ſame Part of the Fluid when left to it ſelf, without either riſing towards the Surface, or ſinking towards the Bottom.

For the Body being (by the Suppoſition) of equal Gravity with the Fluid, the Weight of the Column KLHI, which conſiſts partly of Fluid and partly of the Body, is the ſame as if it had been all Fluid; conſequently HI, that Part of the Surface of the Stratum MN which lies immediately under the Body, is preſſed with the ſame Degree of Force, that any other Part of the ſame Dimenſions is, and therefore the whole Column KLHI will be ſupported in its Place. Now the ſame being true of the Column KLHI whatever be its Length, 'tis evident the Body will be ſuſpended in its Place at any Depth.

III.

III. But if the Body is ſpecifically heavier than the Fluid in which it is immerſed, it will ſubſide to the Bottom: For then in whatever Part of the Fluid it is put, the Column KLHI, will always be heavier than an equal Column, that conſiſts all of Fluid; conſequently HI, that Part of the Stratum MN, which lies immediately under the Body will ſuffer a greater Preſſure, than any other Part of the ſame Dimenſions; and therefore will give way and permit the Body to ſubſide continually till it reaches the Bottom.

IV. On the contrary if the Body is ſpecifically lighter than the Fluid, it will riſe to the Top in what Part of the Fluid ſoever it is put. For then the Column KLHI will always be lighter than an equal Column which is all Fluid; conſequently HI will be leſs preſſed downwards than any other Part of the ſame Stratum of equal Dimenſions, and will therefore continually riſe up carrying the Body with it, till it, arrives at the Top.

V. A Body being laid on the Surface of a Fluid ſpecifically heavier than it ſelf, ſinks into it, till the immerſed Part takes up the Place of a Quantity of Fluid, whoſe Weight is equal to that of the whole Body.

Let EFGH *(Fig. 13.)* be a Body floating on a Liquor ſpecifically heavier than it ſelf, it will ſink into it till the immerſed Part IKGH takes up the Place of ſo much Fluid,

as

as is equal to it in Weight. For in that Case GH that Part of the Surface of the Stratum upon which the Body rests, is pressed with the same Degree of Force as it would be, was the Space IKGH full of the Fluid; that is, all the Parts of that Stratum are pressed alike, and therefore the Body after having sunk so far into the Fluid is in *Æquilibrio* with it, and will remain at Rest.

FROM hence it follows, that the Body is as much specifically lighter than the Fluid on which it floats, as the immersed Part is less than the Whole. For by how much the less the immersed Part is, so much the less Fluid is equal in Weight to the whole Body; that is, the Body is so much the lighter in Respect of the Fluid. And if the same Body is made to float successively in Fluids, whose specific Gravities differ among themselves, (but all exceed that of the Body), the lighter the Fluids are, so much greater will be the Part immersed *.

VI. A BODY suspended in a Fluid specifically lighter than it self, loses a Part of its Weight (or rather communicates it to the

* This Phænomenon is what gave Rise to the Hydrometer, an Instrument of great Use in ascertaining the Genuiness of Liquors; for it rarely happens, that the adulterated and the genuine Liquor (however they may agree in Appearance) are of the same specific Gravity.

Fluid)

Fluid) equal to that of a Quantity of Fluid of the ſame Bulk.

LET us inſtead of ſuppoſing the Body ſuſpended in the Fluid, imagine it to be away, and its Place filled with the Fluid; now 'tis evident, this being of the ſame ſpecific Gravity with the circumjacent Fluid, will be entirely ſupported by it, or if we ſuppoſe the Body to be of the ſame ſpecific Gravity with the Fluid, it will be wholly ſuſpended by it; we ſee therefore the Preſſure of the circumambient Fluid, whereby it endeavours to buoy up the Body, is equivalent to the Weight of ſo much Fluid, as would fill the Place the Body takes up. But ſince the Fluid preſſes only on the Surfaces of the Body, that Preſſure is the ſame, whatever be the ſpecific Gravity of the Body; the Body therefore loſes ſo much of its Weight as the Fluid would naturally buoy up; that is, ſo much as is the Weight of a Quantity of Fluid of the ſame Bulk.

THIS Propoſition affords us a Method of determining the Relation which the ſpecific Gravities of Bodies, whether Fluid or Solid, bear to each other. For whereas by weighing a Solid in a Fluid, ſpecifically lighter than it ſelf, we find the abſolute Weight of a Quantity of the Fluid equal to it in Bulk (*viz.* the Weight the Solid loſes) the Relation, that Weight bears to the Weight of the Solid, is the Relation of their ſpecific Gravities; becauſe

cause the Weights of Bodies, whose Bulks are equal, are as their specific Gravities: consequently if the same Solid is weighed successively in different Fluids (all lighter than it self) we gain the Relation which the specific Gravity of each bears to that of the Solid, and therefore to one another. Again, if different Solids are weighed in the same Fluid, the Relation which the specific Gravity of that Fluid bears to each Solid is had, and therefore also the specific Gravities of the Solids among themselves*.

* Upon this is founded the Use of the hydrostatical Balance for determining the specific Gravities both of Solids and Fluids The Practice is thus. First let the Solid be weighed in Air, that is, out of the Fluid; afterwards in it (this ought to be done by suspending it at one End of the Balance by a String that is as nearly of the same specific Gravity with the Fluid made Use of as possible, and letting it sink into the Fluid, till it is wholly immersed below the Surface; if the Fluid is Water, an Horse Hair is most convenient to hang the Body at the End of the Balance by) then substract its Weight in the Fluid from its Weight in Air, the Difference is what it loses in the Fluid. This done, say, by the Rule of Proportion, as the Weight lost in the Fluid is to its Weight in Air, so is Unity, or any Number taken at Pleasure, to a Fourth, which by its Relation to the former, will express the Relation of the specific Gravity of the Fluid to that of the Solid. Thus the Relation which the specific Gravity of the same Fluid bears to that of various Solids, or of the same Solid to that of various Fluids, and consequently the Relation of the specific Gravities of all among themselves may be obtained.

CHAP.

CHAP. III.

Of the Air.

I. THAT Part of Natural Philosophy, which treats of the Properties of the *Air*, and the Effects of its *Pressure* and *Elasticity*, is called *Pneumatics*.

II. AIR is a thin transparent elastic Fluid surrounding the Earth to a certain Height, which taken together, is called the *Atmosphere*.

III. THAT Air is a Fluid, is evident from the easy Passage it affords to Bodies moving in it: for this shows it to be a Body, whose Parts easily yield to a Pressure that is greater on one Side than on the other, which is the Definition of a Fluid.

IV. AIR gravitates towards the Earth, or is heavy like other Bodies.

TO prove this, we have Abundance of Arguments both from Sense and Experiment. Thus, when the Hand is applied to the Orifice of a Vessel, it readily perceives the Weight of the incumbent Atmosphere, as soon as the Air included in the Vessel begins to be drawn out. Thus, Glass Vessels exhausted of their Air (if not strong enough to sustain the Pressure of the incumbent Atmosphere) are crushed to Pieces by the Weight of the Air with-

out. When the Air is exhausted out of a Vessel, the Vessel weighs less than before. With a great many more Experiments generally mentioned by Authors on this Subject *.

V. The exact Weight of the incumbent Air is determined by filling a Tube with Mercury, and immerging the open End in a Vessel of the same Fluid: for then the Mercury will run down the Tube, till its Surface is fallen only to the perpendicular Height of about twenty nine or thirty Inches above the Surface of the Mercury in the Vessel; if the same Experiment is made with Water, the Surface of it will stand at about the Height of thirty two Feet above the Surface of that in the Vessel; the Column of Mercury in one Case, and the Column of Water in the other exactly balancing the Weight of a Column of Air, which reaches to the Top of the Atmosphere, and presses upon the Surface of the Fluid in the Vessels. This is what is called the Torricellian Experiment, from *Torricelli* the Inventor, and is the same with the common Barometer.

From hence it follows, (Chap. I. §. 9.) that all Bodies at the Surface of the Earth sustain as great a Weight from the Pressure of the Air, as is that of a Column of Water, whose Height is thirty two Feet, and its

* See *Boyle's* Tracts, or *Gravesande* Lib. II. P. III.

Base

Base equal to the Surface of the Body pressed upon *.

VI. THAT the Suspension of the Mercury in the Barometer depends on the Pressure of the external Air, is beyond all Doubt; for if the Barometer is included in the Air Pump, the Mercury falls in the Tube in Proportion as the Air is exhausted out of the Receiver; and if the Air is let in again gradually, the Mercury reascends proportionably, till it reaches its former Height.

VII. THAT the Atmosphere is extended to a determinate Height, appears from hence; *viz.* that when the Torricellian Tube is removed to a more elevated Place, the suspended Column of Mercury becomes shorter, which is, because a shorter Column of Air presses upon it, or, that the Tube in this Situation is nearer the Top of the Atmosphere.

* The Pressure of the Atmosphere upon every square Inch near the Surface of the Earth is about fifteen Pounds, being equal to the Weight of a Column of Mercury, whose Height is thirty Inches and its Base one square Inch. Now such a Column of Mercury would weigh about fifteen Pounds. The Weight of the Atmosphere therefore which presses upon a Man's Body is equal to so many Times fifteen Pound, as the Surface of his Body contains square Inches.

The Reason why a Person suffers no Inconvenience from so great a Pressure is owing to the Air included within the Pores and Fluids of the Body, which by its Reaction is a Counterpoise to the Pressure of the external Air: as we shall explain more fully, when we come to speak of the Diving Bell, and the Manner of using it.

VIII. The Elasticity of the Air is that Property by which it contracts it self into less Space, when an additional Pressure is laid upon it, and recovers its former Dimensions, when the Pressure is taken off. This is accounted its distinguishing Property, all the rest being common to it with other Fluids.

Of this we have numerous Proofs. Thus, a Bladder full of Air being compressed by the Hand, the included Air gives way; but when the Pressure is taken off, the Air expands it self, and readily fills up the Cavity or Impression made in the Surface of the Bladder. And if a larger Quantity of Air, than is naturally pressed into a Vessel by the Weight of the incumbent Atmosphere, is forced into it by the *Condenser* (an Engine for that Purpose) and if that Air is afterwards let out by opening the Vessel, the Remainder is found to be of the same Weight as at first; from whence it follows, that the Air by means of its Elasticity or Spring drives out all that which was forced in by the Condenser, recovers its former Dimensions, and fills the Vessel as before.

IX. From hence, together with what has been observed about the Pressure of the Atmosphere, it follows, that the Air near the Surface of the Earth, is compressed into a much narrower Space by the Weight of the Air above, than that which it would naturally take up, was it freed from that

Pressure;

Preſſure; accordingly it is found by Experiment, that when the Preſſure of the Atmoſphere is taken off from any Portion of Air, it immediately expands it ſelf into a vaſt Extent. Hence it is, that thin Glaſs Bubbles or Bladders filled with Air, being included in the Receiver of the Air-Pump, are broke in Pieces by the Spring of the Air they contain within them, when the Preſſure of the external Air is taken off. Thus a Bladder quite flaccid containing only a ſmall Quantity of Air in it, ſwells upon the Removal of the external Air, and appears diſtended as if it contained as great a Quantity as poſſible. The ſame Effect is found in carrying a Bladder ſomewhat flaccid to a more elevated Place, for there the external Preſſure being leſs, the Air included in the Bladder is in ſome meaſure freed from the Preſſure of the Atmoſphere, it therefore dilates it ſelf, and diſtends the Bladder as in the former Caſe.

X. It is found by Experiment, that the greater the Force is with which a Quantity of Air is compreſſed, ſo much leſs is the Space into which it is contracted. From whence it follows, that the Denſity of the Air is proportionable to the Preſſure which it ſuſtains. As to the utmoſt Degrees of Expanſion and Contraction which the Air is capable of, they are as yet unknown. In ſeveral Experiments made by Mr. *Boyle*, Air in its natural State, that

that is, preſſed only with the Weight of the incumbent Atmoſphere, dilated it ſelf, when that Preſſure was taken off, into more than thirteen thouſand Times the Space it took up before. And he was able ſo far to compreſs it, that it ſhould take up more than five hundred and twenty thouſand Times leſs Space than that into which it would dilate it ſelf when freed from its Preſſure *.

XI. From this Property it follows, that the Air in the inferior Parts of the Atmoſphere is more denſe, than that which is at great Heights in the ſame; or, that the Denſity of the Air decreaſes continually as we approach the Top of the Atmoſphere. For the Denſity of the Air is proportionable to the Force with which it is compreſſed, and that Force continually decreaſes as we approach the Top.

* See *Boyle*'s Tracts and Experiments on the Spring and Preſſure of the Air.

Various have been the Opinions of Philoſophers concerning the Cauſe of this prodigious Spring in the Particles of Air; ſome holding it to depend on their Figure, which they ſuppoſe to reſemble in ſome Manner little Bundles of Twigs or the Branches of Trees; ſome think them like Fleeces of Wool, others conceive them as rolled up like Hoops, or the Springs of Watches, and endeavouring to expand themſelves by Virtue of their Texture. But Sir *Iſaac Newton* is of Opinion, that ſuch a Texture is by no Means ſufficient to account for that vaſt Power of Expanſion obſerved above: but that each Particle is endued with a repelling Force, which encreaſes as they approach one another, and accordingly keeps them aſunder at Diſtances reciprocally proportionable to the Preſſure they ſuſtain.

See *Hales*'s Statical Eſſays. Vol. I. Chap. 6.

WAS

WAS the Denſity of the Atmoſphere every where the ſame, as it is near the Surface of the Earth, its Height (as is computed from the Quantity of Preſſure it exerts in raiſing the Mercury in the Barometer) would be about five Miles. But whereas its Denſity continually decreaſes, as we approach the Top, and it is uncertain how far the Particles may expand themſelves where there is little or no Preſſure, the true Height cannot be obtained. It is computed to continue of a ſenſible Denſity to the Height of about forty five or fifty Miles.

XII. THE Elaſticity of the Air produces the ſame Effects with its Preſſure.

FOR Action being equal to Reaction, the Force which the Spring of the Air exerts in endeavouring to expand it ſelf, is equal to the Force with which it is compreſſed; juſt as it is in the Spring of a Watch, which exerts no Force, but in Proportion as it is wound up; conſequently a Quantity of Air in ſuch a State of Contraction, as it would be compreſſed into by the Weight of the incumbent Atmoſphere, exerts a Force equal to that Weight. If a Quantity of Air therefore is included in a Veſſel, and is of the ſame Denſity with the circumambient Air, its Preſſure againſt the Sides of the Veſſel is equal to the Weight of the Atmoſphere. Thus Mercury is ſuſtained to the ſame Height by the elaſtic

Force

Force of Air included in a Glass Vessel no way communicating with the external Air, as by the Weight of the Atmosphere it self.

XIII. THE Elasticity of the Air is augmented by Heat and diminished by Cold *. For if a Bladder, which is about half filled with Air, is laid before the Fire, it will, when it is sufficiently heated, be distended and burst. Thus, Glass Bubbles being laid upon the Fire immediately burst with great Violence by the augmented Spring of the included Air.

XIV, THE Density of the Air thus contitinually varying, according to the different Degrees of Heat and Cold, to which it is exposed, makes it difficult to ascertain its true specific Gravity. *Ricciolus* estimates it to be to

* This Property is found in all Bodies both Solid and Fluid, but in a much less Degree, than it is in Air. Thus, if a Flask is filled with Water only to the lower Part of the Neck, and is then set upon the Fire, the Water, when it begins to grow warm, will rise into the Neck, and continue to ascend, as the Heat is increased. And when a Wire or Bar of Iron is heated, it is augmented both in Length and Diameter.

Upon this Property depend the Phænomena of the Thermometer, which is a Glass Bubble with a small hollow Stem arising from it This Bubble and Part of the Stem is usually filled with Mercury, or Spirit of Wine, which will rise or fall in the Stem as they are affected by the Heat or Cold of the external Air. If a sufficient Degree of Heat is suddenly applied to this Instrument, the Liquor is observed to descend a little before it rises, because the Glass distending it self, the Capacity of the Bubble is augmented, before the included Liquor is affected by the Heat.

that

that of Water, as one to a Thousand: *Mersennus* as one to one Thousand three Hundred: Mr. *Boyle* by more accurate Experiments found it to be, as one to nine Hundred and Thirty eight, and thinks, that, all Things considered, the Proportion of one to a Thousand may be taken as a medium; for there is no fixing any precise Proportion, because not only the specific Gravity of Air, but that of Water also is continually varying. However by some Experiments made since with more Accuracy before the *Royal Society*, the Proportion has been fixed at about one to eight Hundred and Eighty.

XV. AIR is necessary for the Preservation of Animal and Vegetable Life; neither will Fire subsist without it. The Reason of this is as yet unknown to Philosophers. Mr. *Hales* by several curious Experiments in his Statical Essays makes it probable, that 'tis owing to its Elasticity, See his *Analysis* of it. Statical Essays Vol. I. Chap. 6.

CHAP. IV.

Of the Resistance of Fluids.

I. THE Resistance a Body meets with in moving through a Fluid, is of three Kinds. The first arises from the Friction of the Body against the Particles of the Fluid;

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the second, from their Cohesion or Tenacity among themselves: the third, from their Inactivity, or the Tendency they have, in common with other Bodies, to keep the Places they possess.

THE first, *viz.* that which arises from the Friction of the Body against the Particles of the Fluid is very inconsiderable; for whatever the Weight is, which presses the Particles of a Fluid together, the Freedom, with which a Body moves through it, is not sensibly diminished thereby. As was observed Chap. 1. §. 2. in the Notes.

THE second, or that which arises from the Tenacity of the Particles of the Fluid, is as the Time the Body takes up in passing through it *; for the shorter the Time is in

* We have a very curious Argument in Confirmation of this, and which at the same time illustrates the Manner in which a Body makes its Way through a tenacious Fluid, by Sir *Isaac Newton* himself, in a Postcript to a Letter in the Philosophical Transactions N°. 371. It is as follows. 'Suppose Pieces of 'fine Silk, or the like thin Substance, extended in parallel Planes, 'and fixed at small Distances from each other. Suppose then a 'Globe to strike perpendicularly against the Middle of the 'outermost of the Silks, and by breaking through them to lose 'Part of its Motion. If the Pieces of Silk be of equal Strength, 'the same Degree of Force will be required to break each of 'them; but the Time, in which each Piece of Silk resists, will 'be so much shorter as the Globe is swifter; and the Loss of 'Motion in the Globe consequent upon its breaking through 'each Silk, and surmounting the Resistance thereof, will be pro-'portional to the Time in which the Silk opposes it self to 'the Globe's Motion; insomuch that the Globe by the Re-'sistance of any one Piece of Silk, will lose so much less of its 'Motion as it is swifter. But on the other Hand, by how 'much

which the Force of Cohesion is broke through, the less Effect it has in resisting the Motion of the Body. This Species of Resistance is also very small except in glutinous and viscid Fluids, whose Parts are not easily separated.

THE third Species is the principal Resistance, that Fluids give to Bodies, and arises from their Inactivity or the Tendency the Particles, of which they consist, have to continue at Rest. The Quantity of this Resistance depends on the Velocity the Body moves with on a double Account. For in the first Place, the Number of Particles put into Motion by the moving Body in any determinate Space of Time, is proportionable to the Velocity wherewith the Body moves; and in the next Place, the Velocity with which each of them is moved, is also proportionable

' much swifter the Globe moves, so many more of the Silks it ' will break through in a given Space of Time; whence the ' Number of the Silks, which oppose themselves to the Mo' tion of the Globe in a given Time, being reciprocally pro' portional to the Effect of each Silk upon the Globe, the Re' sistance made to the Globe by these Silks, or the Loss of Mo' tion the Globe undergoes by them in a given Time, will be ' always the same.

' Now if the Tenacity of the Parts of Fluids observes the ' same Rule, as the Cohesion of the Parts of these Silks; name' ly, that a certain Degree of Force is required to separate and ' disunite the adhering Particles, the Resistance arising from the ' Tenacity of Fluids must observe the same Rule, as the Re' sistance of the Silks; and therefore in a given Time the Loss ' of Motion, a Body undergoes in a Fluid by the Tenacity of ' its Parts, will in all Degrees of Velocity be the same; or ' in fewer Words, that Part of the Resistance of Fluids, which ' arises from the Cohesion of their Parts, will be Uniform.

to the Velocity of the Body; this Species therefore of Resistance is in a duplicate Proportion, or as the Square, of the Velocity, with which the Body moves through the Fluid *.

II. FARTHER the Resistance a Body moving in a Fluid meets with from thence, may be considered with Regard to the Fluid; and then it will be found to be more or less, according to the Density of the Fluid. For by how much denser the Fluid is, so much the greater Number of Particles are to be put into Motion by the Body in Order to make its Way through it.

III. THE next Thing to be considered is, the Effects of the Resistance of Fluids upon Bodies moving in them; that is, the Retardation which they cause in the Motion of a Body by their Resistance, or the Quantity of Motion they destroy.

* This may be otherwise demonstrated from the twelfth Section of the foregoing Chapter; for from thence it follows, that the Resistance a Fluid gives to a Solid against which it moves, is proportionable to the Height a Body must fall from, to acquire such a Degree of Velocity as the Fluid moves with: but the Heights Bodies fall from are as the Squares of the Velocities they acquire by falling; consequently the Resistance a Fluid gives to a Solid, against which it moves, is also as the Square of its Velocity, Now it matters not, as to the Resistance, whether the Fluid moves against the Solid, or whether it be at Rest, and the Solid moves in it; the Resistance therefore which a Fluid gives to a Solid moving in it, is as the Square of the Velocity, with which it moves.

AND

AND this in similar Bodies of equal Magnitudes is inversely as their Densities, or the Quantity of Matter they contain; for by how much the greater the Quantity of Matter in any Body is, so much the more easily does it overcome the Resistance it meets with from the Fluid. Thus we see the Resistance of the Air has a much less Effect in destroying the Motion of an heavy Body, than of a light one which has the same Dimensions.

IV. IN similar Bodies of equal Densities, but different Magnitudes, the Retardation is inversely as their homologous Sides. For the Resistance Bodies meet with in a Fluid, is inversely as the Quantity of Matter they contain (by the last,) that is inversely as the Cubes of their homologous Sides; and it is also directly as their Surfaces, because 'tis by them that they move the Fluid out of its Place, that is, directly as the Squares of their homologous Sides; consequently the Retardation is inversely as their homologous Sides *.

HAVING given the fundamental Principles of *Hydrostatics*, and shewn how Fluids both compressible and incompressible are disposed to act upon each other, and upon Solids by their Pressure, Motion, Elasticity and

* Because the inverse Ratio of the Cubes of any Numbers being compounded with the direct Ratio of the Squares of the same, gives the inverse Ratio of the Numbers themselves.

Resist-

Reſiſtance; I proceed now to account for ſome of the more remarkable Phænomena of Nature, in which they are in Part or altogether concerned: and this I deſign for the Subject of the following Diſſertations.

DISSERTATION I.

Of Sound.

WHEN the Parts of an elaſtic Body are put into a tremulous Motion by Percuſſion or the like, ſo long as the Tremors continue, ſo long is the Air included in the Pores of that Body, and likewiſe that which preſſes upon its Surface, affected with the like Tremors and Agitations: now the Particles of Air, being ſo far compreſſed together by the Weight of the incumbent Atmoſphere, as their repulſive Forces permit, (as has been explained Chap. 3.) it follows, that thoſe which are immediately agitated by the reciprocal Motions of the Particles of the elaſtic Body, will, in their Approach towards thoſe which lie next them, impel them towards each other, and thereby cauſe them to be more condenſed, than they were by the Weight of the incumbent Atmoſphere, and in their Return ſuffer them to expand themſelves again; whereby the like Tremors and Agitations will be propagated to the next, and ſo on, till having arrived at a certain Diſtance from the Body, they ceaſe, being gradually deſtroyed by a continual ſucceſſive Propagation of Motion to freſh Particles of Air throughout their Progreſs.

THUS

THUS it is that Sound is communicated from a tremulous Body to the Organ of Hearing. Each Vibration of the Particles of the ſounding Body is ſucceſſively propagated to the Particles of the Air, till it reaches thoſe which are contiguous to the *Tympanum* of the Ear, (a fine Membrane diſtended acroſs it,) and theſe Particles in performing their Vibrations impinge upon the Tympanum, which agitates the Air included within it, and that being put into a like tremulous Motion, affects the auditory Nerve, and thus excites in the Mind the Senſation or Idea of what we call *Sound.*

NOW ſince the repulſive Force of each Particle of Air is equally diffuſed around it every Way, it follows, that when any one approaches a Number of others, it not only repels thoſe which lie before it, in a right Line; but all the reſt, laterally according to their reſpective Situations; that is, it makes them recede every Way from it ſelf, as from a Center: and this being true of every Particle, it follows, that the aforeſaid Tremors will be propagated from the ſounding Body in all Directions, as from a Center: and farther, if they are confined for ſome Time from ſpreading themſelves by paſſing through a Tube or the like, will when they have paſſed through it, ſpread themſelves from the End in every Direction. In like Manner, thoſe which paſs through an Hole in an Obſtacle they meet with

with in their Way, will afterwards ſpread themſelves from thence, as if that was the Place where they began; ſo that the Sound which paſſes through an Hole in a Wall or the like, is heard in any Situation whatever, that is not at too great a Diſtance from it. Something analogous to this we may obſerve in the Motion of Waves upon the Surface of a Fluid, which are propagated equally through all Parts of the Surface in a Circle, though occaſioned not by a circular, but reciprocal Motion and Agitation of the Finger in a ſtraight Line.

SINCE the repulſive Force with which the Particles of Air act upon each other, is reciprocally as their Diſtances, (Chap. 3. §. 10.) it follows that when any Particle is removed out of its Place by the Tremors of a ſounding Body, or the Vibrations of thoſe which are contiguous to it, it will be driven back again by the repulſive Force of thoſe towards which it is impelled, with a Velocity proportionable to the Diſtance from its proper Place, becauſe the Velocity will be as the repelling Force. The Conſequence of this is, that, let the Diſtance be great or ſmall, it will return to its Place in the ſame Time; (for the Time a Body takes up in moving from Place to Place will always be the ſame, ſo long as the Velocity it moves with is proportionable to the Diſtance between the Places.) The Time there-

fore in which each Vibration of the Air is performed, depends on the Degree of Repulſion in its Particles, and ſo long as that is not altered, will be the ſame at all Diſtances from the tremulous Body; conſequently, as the Motion of Sound is owing to the ſucceſſive Propagation of the Tremors of a ſounding Body through the Air, and as that Propagation depends on the Time each Tremor is performed in, it follows, that the Velocity of Sound varies as the Elaſticity of the Air, but continues the ſame at all Diſtances from the ſounding Body.

And as the Velocity, with which Sound is propagated, depends on the Elaſticity of the Air, ſo it does alſo on its Denſity; for when the Denſity of the Air is augmented, while its Elaſticity remains the ſame *, a greater Num-

* Perhaps it will not appear to every one, how the Denſity of the Air can be augmented without a proportional Increaſe of its Elaſticity, becauſe *cæteris paribus*, the nearer the Particles approach each other, the ſtronger is the Action of their repulſive Force.

But it is to be conſidered, that when the Air becomes colder, its Elaſticity is diminiſhed, and then the Particles are brought cloſer together by the Preſſure of the Atmoſphere, till they acquire an Elaſticity equal to what they had before, *viz.* ſuch as anſwers to the Preſſure they ſuſtain (Chap. 3. §. 12). From hence we may infer, that the Propagation of Sound is ſlower in Winter than in Summer, when the Mercury in the Barometer is at the ſame Height; for the Preſſure of the Air being the ſame, its Elaſticity which depends upon it, is ſo too; but the Air is denſer by Reaſon of the Cold, and therefore its Vibrations ſlower.

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ber of Particles will move forwards and backwards in each Vibration; now since we suppose the Cause by which they put each other into Motion, (*viz.* their Elasticity,) the same, they will each receive a less Degree of Velocity; and so the Vibrations will be performed in a longer Time, whence the Succession of them will be slower and the Progress of the Sound proportionably retarded *.

WHEREAS the undulatory Motion of the Air, which constitutes Sound, is propagated in all Directions from the sounding Body, it will frequently happen, that the Air in performing its Vibrations will impinge against various Objects, which will reflect it back, and

* The Method of determining the Velocity with which Sound is propagated, is (by the help of a short Pendulum) to estimate the Time which passes between seeing the Fire of a Gun at a Distance, and hearing the Report. Its great Velocity makes it difficult to be determined exactly; accordingly Authors differ much in their Accounts. The most accurate Observers Dr. *Halley* and Dr. *Durham* have found it to be about one Thousand one Hundred and Forty two Feet, which is almost a Quarter of a Mile in a Second.

The usual Experiments to prove that the Air is necessary for the Propagation of Sound, are such as these. A small Bell being put into the Receiver of the Air-Pump may be heard at a considerable Distance before the Air is exhausted out of it, but when the Air is much rarified by exhausting, can scarcely be heard at all. When the Air is condensed, the Sound is augmented in Proportion to the Condensation. These Experiments do not only succeed in forced Rarefactions and Condensations, but in such also as are Natural; Sound being observed to be much weaker on the Tops of high Mountains, where the Air is less condensed by the Weight of the incumbent Atmosphere, than in the Valleys below.

ſo cauſe new Vibrations the contrary Way; now if the Objects are ſo ſituated, as to reflect a ſufficient Number of Vibrations back (*viz.* ſuch as proceed different Ways) to the ſame Place, the Sound will be there repeated, and is called an *Echo* *. And the greater the Diſtance of the Objects is, the longer will be the Time, before the Repetition is heard. Therefore when the Sound in its Progreſs meets with Objects at different Diſtances ſufficient to produce an Echo, the Sound will be repeated ſeveral Times ſucceſſively, according to the different Diſtances of thoſe Objects from the ſounding Body; and this makes what is called a *repeated* Echo.

If the Vibrations of the tremulous Body are propagated through a long Tube, they will be continually reverberated from the Sides of the Tube into its Axis, and by that means prevented from ſpreading, till they get out of it; whereby they will be exceedingly increaſed, and the Sound rendered much louder than it would otherwiſe be †.

* In *Woodſtock* Park in *Oxfordſhire*, there is an Echo which repeats diſtinctly ſeventeen Syllables, by Day and twenty by Night. The Reaſon why it repeats more Syllables by Night than by Day, is becauſe the Air being colder at that Time, is more denſe; and therefore the Return of the Vibrations is ſlower, which gives Time for the Repetition of more Syllables. See *Plot's* Natural Hiſtory of *Oxfordſhire*.

† This is the Caſe in the *Stentorophonic* Tube or Speaking Trumpet.

THE Difference of *Mufical* Tones depends on the different Number of Vibrations communicated to the Air in a given Time by the Tremors of the founding Body; and the quicker the Succeffion of the Vibrations is, the acuter is the Tone, and *è contra*.

A mufical Chord performs all its Vibrations, whether great or fmall in the fame Time. For if a String is ftretched between two Pins, and a Force is applied to the middle Point, to draw it out of its rectilineal Situation; it is found by Experiment, that the Diftance (if it be fmall) to which it is drawn, is as the Force applied; confequently the Velocity, with which it returns when left to it felf, will be as the Space it has to move over; it will therefore perform all its Vibrations in the fame Time: this is the Reafon, why the fame Chord

Trumpet. See *Kircher* de Re Muficâ. Lib. 9. Par. 4. *Ode* Philof. Natur. Princip. p. 293.

Upon this Principle it is, that Sound is conveyed from one Side of a *Whifpering* Gallery to the oppofite one, without being perceived by thofe who ftand in the Middle. The Form of a Whifpering Gallery is that of a Segment of a Sphere, or the like arched Figure; and the Progrefs of the Sound through it may be illuftrated in the following Manner.

Let *ABC* (*Fig.* 14.) reprefent the Segment of a Sphere, and fuppofe a low Voice uttered at *D*, the Vibrations expanding themfelves every Way, fome will impinge upon the Points *E*, *E*, &c. from thence be reflected to the Points *F*, from thence to *G*, and fo on, till they all meet in *C*, and by their Union there caufe a much ftronger Sound, than in any other Part of the Segment whatever, even at *D* the Point from whence they came.

however

however ſtruck produces the ſame Note. It is alſo found by Experiment, that when Strings of equal Diameters, but different Lengths, are equally ſtretched, the longer they are, ſo much the leſs Weights will draw them from their rectilineal Situation to the ſame Diſtance; the Forces therefore by which they return are leſs, and the Times of their Vibrations longer.

When two Chords perform their Vibrations in equal Times, the Tone produced is called an *Uniſon.* If one performs two, while the other one, 'tis an *Octave.* If one three, while the other two; 'tis a *Fifth.* If one three, while the other four; 'tis called a *Fourth* &c.

To make an Uniſon Sound, it is not neceſſary, that the Vibrations of the two Strings ſhould actually concur, but only that they ſhould be performed in equal Times; ſo that they would always concur, if they began at the ſame Inſtant. For the Ear perceives not the ſingle Vibrations diſtinctly, but only finds that Difference which proceeds from the Intervals of Time, that paſs between them*.

* Upon theſe Principles we may account for that remarkable Phænomenon in *Muſic*, that an intenſe Sound being raiſed, either with the Voice or a ſonorous Body, another ſonorous Body near it and in Uniſon with it, will thereby be made to ſound. For the Vibrations of the Air, which correſpond to the Tremors of the firſt ſounding Body, agreeing exactly in Point of Time with thoſe which are capable of being given to the other Body at Uniſon

Unison with it; when they have by their first Impulse communicated a small Degree of Motion to it, will, by conspiring with it as it moves forwards and backwards continually increase its Motion, till it becomes sensible. The contrary happens in Strings which are in Discord with each other; for in this Case, though the first Vibration of one may give Motion to the other, yet their Vibrations not being performed in equal Times the second will come *unseasonably*, *i. e.* when the other is moving the contrary Way, and obstruct its Motion. It is farther observable that in two Strings, one of which vibrates twice, while the other once; if the first be sounded, the two extreams of the other will each sound an Unison with it, while the middle Point remains at Rest. So if one vibrates thrice, while the other once, the last will be divided into three Parts, each of which will sound an Unison with it, and the two Points between those Parts will remain at Rest. For otherwise that which vibrates twice, while the other once, must necessarily interfere with it at every second Vibration; and that which vibrates, thrice while the other once, would interfere with it at every third; so that it would not be put into a sufficient Motion to produce a Sound. But when it is divided by the quiescent Points, it becomes so many Strings at Unison with the former, each of which easily receives its Vibrations from thence.

From hence likewise it is, that if we take two or three Drinking Glasses and put some Water or other Fluid into each of them and place them near to each other, taking Care to fill them, to such Heights, that (when struck) their Tones shall be in Unison; and then if we slide the Finger along the Brim of one of the Glasses pressing pretty strongly upon it, (which will cause it to sound) we shall see the Surface of the Fluids in the other Glasses begin to tremble; which shews that the Vibrations of the first Glass cause the like in the other at Unison with it, though not perhaps in a Degree sufficient to produce a Sound strong enough to be heard distinctly from the former.

Thus it is that some Persons are able to break a Drinking Glass by a Tone of their Voice at Unison with it. They first try the Tone of the Glass by striking it, then applying their Mouth near to the Brim of it, sound the same Note with their Voice; this sets the Glass a trembling; they then continually raise their Voice, sounding still the same Note; this encreases the Tremors of the Glass, which by that Means (if it is not too strong) is broke in Peices.

The Effect of *Music* upon Persons bit with the *Tarantula*, (if the Accounts we have of it from abroad may be credited) is very surprising.

ſurpriſing. A Perſon bit with the *Tarantula* after ſome Time loſes both Senſe and Motion, and dies if deſtitute of Help. The moſt effectual Remedy is *Muſic*. The Muſician tries Variety of Airs, till he hits upon one that affects the Patient, who upon that begins to move by Degrees, and keeps Time with his Fingers, Arms and Legs, afterwards with his whole Body; he then raiſes himſelf up, begins to dance, and increaſes in Activity every Moment; till after five or ſix Hours, being very much fatigued, he is put to Bed to recover Strength. The next Day the ſame Air brings him out of Bed for a new Dance. Which Exerciſe being thus continued, the Diſtemper is abated in the Space of four or five Days, the Effects of the Bite being in ſome Meaſure carried off by Sweat, and the Patient begins then to recover his Senſe and Knowledge by little and little.

The Reaſon why the Patient is thus affected by the *Muſic*, is becauſe the Nerves of his Body are ſo diſpoſed in that Diſtemper, as eaſily to be agitated by the Vibrations which are occaſioned by the principle and ſtronger Notes of what is played.

See on the Subject of this Diſſertation, Philoſoph. Tranſact. N°. 134. 243. 302. 313. 319. 337. Hiſt. de l' Acad. 1702. 1708. *Grew*'s Coſmolog. Sacr. Book I. Chap. 5. *Mead* upon Poiſons p. 59. *Keil*'s Anatomy p. 214.

DISSER-

DISSERTATION II.

Of Capillary Tubes.

BY a *Capillary Tube* is generally underftood a Glafs Pipe, the Diameter of whofe Bore is at moft but about one tenth of an Inch; though any Tube whofe Cavity does not exceed that Magnitude, may be fo called.

THE Phænomena of Capillary Tubes being fuch as contradict a known Law in *Hydroftatics*, viz. *that a Fluid rifes in a Tube to the fame Height with the Level of its Source* *; and likewife of Affinity with the Afcent of the Sap through the Stems of Plants for the Nourifhment of their Fruit, and with divers other Operations of Nature: it has been thought of no fmall Moment in Philofophy to find out and eftablifh their true Caufe; which after numerous Experiments and feveral Conjectures about it, is found to be no other than the Attraction of Cohefion, by which fmall Particles of Matter mutually run together and form larger Bodies †. I fhall lay down the fe-

* See Chap. I. §. 9. Cafe 4. in the Notes.

† See *Hauksbee*'s and *Power*'s Experiments. *Muffchenbroeck* 4°. Edit. Philofoph. Tranfact. N°. 355. Mem. de l' Acad. 1705, 1714, 1722, 1724. With others refer'd to in Quæftiones Philofoph.

veral Phænomena, as ſo many Matters of Fact, and ſubjoin to each a Solution from that Cauſe. In order to which, it may not be improper to premiſe the following Conſideration by way of *Lemma.*

Let us ſuppoſe the Veſſel ABCD *(Fig.* 15.*)* filled with a Fluid to the Height LM, and let it be conceived as divided into the equal Portions EFGH, GHIK, IKLM, *&c.* farther, let it be ſuppoſed, that each Particle of Matter in the inner Surface of the Veſſel, has a Sphere of Attraction, whoſe Semidiameter is equal to the Thickneſs of three of thoſe Portions; that is to ſay, that the Attraction of the Particle M reaches upwards as far as F, and downwards as far as S; and that of the Particle O upwards as far as H and downwards as far as U; and ſo of all the reſt quite round the Tube. From hence it will follow that every Particle of the inner Surface of the Veſſel, which lies between EF and RS conſpires in endeavouring to raiſe the Fluid towards AB the Top of the Veſſel, and that the Fluid is not affected by any other: for Inſtance, the Particle S, and all below it, will attract downwards three Strata of the Fluid (ſuch as are contained in three equal Diviſions of the Veſſel) from above, and as many upwards from below, and therefore will have no Effect at all in raiſing or depreſſing the Fluid: But the Particle Q will attract only two Strata down-

downwards, becaufe there are no more above it, and three upwards, and therefore will in fome Meafure tend to raife the Fluid; fo the Particle O will attract but one downwards and three upwards, the Particle M, none downwards, and three upwards; the Particle K two upwards, and H only one: all which may clearly be feen by their Situations in the Figure with Refpect to the Surface of the Fluid. Therefore in every Veffel, where there is a mutual Attraction between the Fluid it contains and the Particles of which it is compofed, there will be a certain Number of Particles difpofed around it in Form of a broad Periphery or Zone as reprefented by AB *Fig.* 16, half of which lies above the Surface of the Fluid and half below it, that will tend to make it rife towards the Top*. This being underftood, the following Phænomena will not be difficult.

* I have been the more particular in explaining this *Lemma*, becaufe it is not a bare *Periphery* of no Breadth, to which the Afcent of the Fluid is owing, but a *Zone* or *Cingulum* of Particles diftended equally in Breadth both ways from the Surface of the Fluid; and becaufe it is upon the Breadth of this, that fome of the following Solutions depend. As to the Thicknefs of it, that undoubtedly is equal to the Semidiameter of the Sphere of Attraction in the Particles of the Veffel; and therefore Veffels whofe Sides are of different Thickneffes (provided thofe Thickneffes be lefs than that Semidiameter) muft have different Effects upon the fame Fluid, though no one has as yet been fo accurate as to obferve it. The Reafon why a Fluid will not rife in a large Veffel, as well as in one that is Capillary, is becaufe the Attraction of its Particles does not reach far enough into the Middle of the Veffel; and therefore it only rifes about

I. Let there be two Capillary Tubes AB and CD (*Fig.* 17.) open at both Ends, and having their lower Orifices A and C immerged below the Surface of the Water contained in the Vessel FGHI: the Water will immediately rise up in each Tube above the Surface of that in the Vessel, beginning with a swift Motion, which will gradually decrease, till as much Water has entered the Tubes, as they are able to raise: and the Heights to which the Water will rise in them, will be reciprocally as their Diameters.

That the Water ought to rise in both Tubes is an immediate Consequence of the foregoing *Lemma*; because the Column of Water within the Tube is rendered lighter than an equal Column on the outside, as being attracted upwards by a Portion of the interior Surface of the Vessel; and therefore will rise till it becomes as much longer than the external ones as it is made lighter, that the *Æquilibrium* which was destroyed by the Attraction of the Tube, may be restored by the Weight of the Column. The Reason that the Velocity with which it rises, ought constantly to decrease, is because the heavier the Column is, the less is the Effect of the Attraction, which is always the same in a Tube of the same Diameter.

the Sides, standing higher there than in the middle: as may be seen in a Drinking Glass, when a Quantity of Water is put into it, somewhat less than is sufficient to fill it.

And

25.

Fig. 12. P. 28.

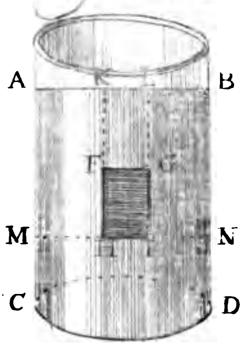
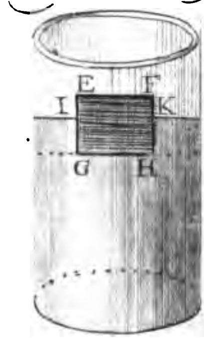
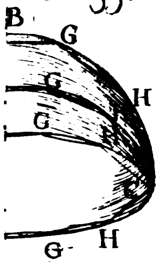


Fig. 13. P. 29.

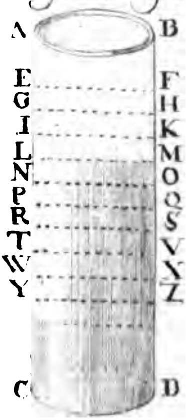


P. 53.



P. 59.

Fig. 15. P. 58.



And the Heights to which the Water rifes in them, will be reciprocally as their Diameters; for then the Quantities raifed will be directly as the Diameters*; but the Peripheries that raife them, (being always of the fame Breadth and having their Lengths equal to the Circumferences of the Tubes,) are as thofe Diameters; the Quantities of Water therefore being in the fame Ratio, are as the Peripheries, *i.e.* as the Caufes by which they are raifed.

II. If the Tubes before they are immerged in the Water, are filled to greater Heights, than thofe to which it would naturally rife in them, and then have their lower Orifices immerged in Water, the Water will fubfide till it ftands in each at the fame Height to which it would have rifen; but if they are held in a perpendicular Pofition without being immerged, the Water will not fubfide in the Tubes *quite* fo far.

The Reafon why the Water in the Tube when its lower Orifice is immerged, fubfides to the fame Height it would have rifen to, had

* The Heights to which the Water rifes, being in a reciprocal Ratio of the Diameters; and the Contents of Cylindrical Tubes being in a direct Ratio of their Heights, and of the Squares of their Diameters; the Quantities of Water raifed in this Cafe will be in a reciprocal Ratio of the Diameters, and a direct one of the Squares of the fame. Now thefe two Ratio's being compounded together, give the direct one of the Diameters themfelves; becaufe the fimple reciprocal Ratio deftroys one of thofe, which are contained in the direct one of the Squares.

the

the Tube been immerged when empty, is becauſe the Column is ſuſpended in one Caſe by the ſame Cauſe, by which it is raiſed in the other; but when the Tube full of Water is held erect, without being immerged, it will not ſubſide *quite* ſo far, becauſe the lower End of the Tube which the Water leaves behind it as it drops out, attracts it the contrary Way; ſo that the Column in this Caſe is ſuſpended not only by the inner Surface of the Tube at the Top, but alſo by its lower End; and therefore a greater Quantity of Water is ſuſpended than in the former Caſe.

III. If a Tube having its lower Orifice immerged in Water, be held obliquely, it will raiſe the Water to the ſame perpendicular Height, as when held erect.

For ſince Fluids preſs according to their perpendicular Heights, the Weight of the Column raiſed will not be proportionable to the attractive Force of the Tube, till it has arrived at the ſame perpendicular Height, to which it would have roſe, if held erect.

IV. If a Tube, when the Water is riſen into it to its wonted Height, is laid in an Horizontal Situation, the Water will move towards the Middle of the Tube, leaving the End which was immerſed a little behind.

The Solution of this Phænomenon depends on what was obſerved in the Lemma about the Breadth of the attracting Periphery, and

and its being equally ſituated on each Side the Surface of the Water; for from thence it follows, that if the Water ſhould not run from the full End of the Tube, after it is laid in an Horizontal Situation, but remain contiguous to it, that End of the Column of Water would be attracted only by ſuch a Portion of a Periphery as lies within the Surface at that End; becauſe the End of the Tube coinciding with the Surface, the other Half of the Periphery is wanting. Whereas at the other End of the Column there is a Periphery whoſe Breadth is intire, which overpowering the other, cauſes the Water to move towards the Middle of the Tube, till the Breadth of the Periphery at each End of the Tube is the ſame, after which the Water being attracted equally each way, remains at Reſt.

V. Let there be a Tube (*Fig.* 18.) conſiſting of two Parts DR and RCK, of different Diameters, it follows from what has been ſaid, that DR the ſmaller Part of the Tube is able to raiſe Water higher than the other: let then the Height to which the larger would raiſe it be TF, and that to which it would riſe in the leſſer (was it continued down to the Surface of the Fluid) be XL. If this compound Tube be filled with Water and the larger Orifice CK be immerſed in the ſame Fluid, the Surface of the Water will ſink no farther than XL, the Height to which the leſſer

leſſer Part of the Tube would have raiſed it.

BUT if the Tube be inverted as in *Fig.* 19. and the ſmaller Orifice XL be immerſed, the Water will run out till the Surface falls to TF, the Height to which the larger Part of the Tube would have raiſed it. The Size of the lower Part making no Alteration in the Height, at which the Fluid is ſuſpended in either Caſe.

IN order to account for theſe Phænomena, it muſt be conſidered, that when a Body is ſo diſpoſed, that its different Parts ſhall move with different Degrees of Velocity, the greater Proportion the Velocity of that Part to which a moving Power is applied, bears to that of the reſt, ſo much the more effectual is the Power in moving that Body; or that the ſame Power applied to different Parts, will be equivalent in Effect to different Powers applied to the ſame Part: as is the known Caſe of the *Lever*, and all the other *Mechanical* Powers.

NOW let us conceive the Tube DR (*Fig.* 18.) continued down to HI, and let it be ſuppoſed at preſent that the Fluids contained in the Tube XLHI and the compound one XLKC, are not ſuſpended by the Periphery at L, but that they preſs upon their reſpective Baſes HI and CK. Let it farther be ſuppoſed, that theſe Baſes are each of them moveable, and

and that they are raised up or let down with equal Velocities; then will the Velocity with which XL the uppermost Stratum of the Fluid XLCK moves, exceed that of the same Stratum, considered as the uppermost of the Fluid in the Tube XLHI, as much as the Tube RCK is wider than DR (by the *Lemma* Chap. 1. §. 9.) that is, as much as the Space MNKC exceeds XLIH; consequently by the foregoing Observation, the Effect of the attracting Periphery XL, as it acts upon the Fluid contained in the Vessel XLCK, exceeds its Effect, as it acts upon that in XLHI, in the same Proportion. Since therefore it is able *(ex Hypoth.)* to sustain the Weight of the Fluid XLHI by its natural Power, it is able under this *Mechanical* Advantage, to sustain the Weight of as much, as would fill the Space MNKC: but the Pressure of the Fluid XLCK is equal to that Weight, as having the same Base and an equal Height (Chap. 1. §. 9.) its Pressure therefore, or the Tendency it has to descend in the Tube, is equivalent to the Power of the attracting Periphery XL, for which Reason it ought to be suspended by it.

Again the Height *(Fig. 19)* at which the attracting Periphery in the larger Part of the Tube is able to sustain the Fluid is no greater than NF, that to which it would have raised it, had the Tube been continued down to MN. For here the Power of the at-

tracting Periphery acts under a like *Mechanical* Diſadvantage; and is thereby diminiſhed in Proportion to the Capacity of the Tube TFNM to that of HIXL; becauſe if the Baſes of theſe Tubes are ſuppoſed to be moved with equal Velocities, the Riſe or Fall of the Surface of the Fluid TFXL would be ſo much leſs than that of TFMN. And whereas the attracting Periphery TF is able by its natural Power to ſuſpend the Fluid only to the Height NF in the Tube TFMN; it is in this Caſe able to ſuſtain no greater Preſſure than what is equal to the Weight of the Fluid in the Space HIXL: but the Preſſure of the Fluid TFXL which has equal Height and the ſame Baſe with it, is equal to that Weight; and therefore is a juſt *Æquipondium* to the attracting Power.

VI. FROM hence we may clearly ſee the Reaſon, why a ſmall Quantity of Water put into a Capillary Tube, which is of a Conical Form and laid in an Horizontal Situation, will run towards the narrower End. For let AB *(Fig. 20.)* be the Tube, CD a Column of Water contained within it; when the Fluid moves, the Velocity of the End D will be to that of the End C reciprocally as the Cavity of the Tube at D, to that at C (by the *Lemma* Chap. 1. § 9.) that is, reciprocally as the Square of the Diameter at D, to the Square

of

of the Diameter at C*; but the attracting Periphery at D is to that at C, in the fimple Ratio of the Diameter at D, to the Diameter at C. Now fince the Effect of the Attraction depends as much upon the Velocity of that Part of the Fluid where it acts, as upon its natural Force, its Effect at D will be greater than at C; for though the Attraction at D be really lefs in its felf than at C, yet its Lofs of Force upon that Account, is more than compenfated by the greater Velocity of the Fluid in that Part; the Fluid will therefore move towards B.

VII. FROM hence likewife it follows, that if a Veffel as ABC (reprefented *Fig.* 21) of any Form whatever, have its upper Part drawn out into a Capillary Tube as B; and if this Veffel is filled with Water, and have its lower Orifice placed on FG the Surface of the fame Fluid; then the Water will remain fufpended in the Veffel, provided the Capillary at the Top be fmall enough, (was it continued down to the Bottom) to raife the Fluid to the Height B. Becaufe by the foregoing Propofition the lower Part of the Tube makes no Alteration in the Height at which the Capillary B is able to fuftain the Fluid.

VIII. AND if the fame Veffel be filled only to the Height DE *(Fig. 22.)* and a Drop of

* 12. *El.* 2.

Water be put into the Capillary at B, (the intermediate Part BDE being full of Air) the Water will continue ſuſpended at the Height DE.

For although the Fluid ACDE is not in Contact with the Drop of Water in the Capillary Tube, and therefore not immediately ſupported by it; yet the Preſſure of the Atmoſphere upon the Surface FG, and againſt the upper Part of the Drop in the Capillary B keeps the Fluid ADEC, and the Drop and the intermediate Air from ſeparating, juſt as in the former Caſe the Attraction of Coheſion in the Particles of the Water prevented a Separation between that in the Veſſel and that in the Capillary. Conſequently as the Water in the Capillary was able in the former Caſe to ſuſtain as much Fluid as the Veſſel could contain, it is now ſufficient to ſuſtain the Fluid ADEC *.

IX. Let there be a Capillary Siphon, as that repreſented *Fig.* 23, 24 or 25. and let EF be the Height, to which Water might be raiſed by a Periphery equal to that at A Now ſince, (as

* This happens quite otherwiſe in Vacuo, becauſe the Preſſure of the Air, which as it were connects the Drop with the Water *ADEC*, being wanting, it immediately falls for Want of a Support. Whereas the former Phænomenon equally ſucceeds in Vacuo; which ſhews that the Parts of the Fluid in the Veſſel are connected with each other, and with that in the Capillary by their own mutual Attraction of Coheſion, there being nothing elſe whereby they can be ſupported.

was

was obferved §. 2.) the lower End of a Tube when it is not immerged, caufes a longer Column to be fufpended than otherwife would be; that is, it fupports a fhort Column befides that which is fuftained by the attracting Periphery; let HI be the Height of fuch a Column as might be fufpended by the End C: then if any of thofe Tubes are filled with Water, and held as in the Figure (neither End being immerged) the Fluid will run out of the Tube at C, if CD the Difference of the Legs exceeds EF and HI added together, otherwife not.

FOR the Column AB is a Counterpoife to BD, being of the fame perpendicular Height; and therefore it is only the Weight of the Column CD which determines the Fluid to move; unlefs that Weight therefore exceeds the Force of the attracting Periphery at A (which the Fluid AB muft leave in rifing up the Tube) and alfo what may be fupported by the End C, that is, *(ex Hypoth.)* two Columns whofe Heights are EF and HI, it cannot run out at C; otherwife it will, as being deftitute of a fufficient Support.

X. IF the End A is immerged in Water (fuppofing the Tube full as before), it will run out, though CD the Difference of the Legs, only exceeds HI. For then the Attraction at A ceafes, and there is nothing to fupport the

Column

Column CD, but the Power the End C has to prevent Drops from falling off it.

XI. Again if the End C is immerged in Water, (and the other not) it will run out at A, if CD exceeds EF, otherwise not,

For in this Case, there is nothing to support the Column CD, but the attracting Periphery at A, whose Power is supposed able to raise a Column as EF, and no more.

XII. And if both Ends are immersed (the Tube being supposed full as before) the Water will run out at the lower, which ever it is. For then the Attraction of both Ends ceases; and the longer Column over-balancing the shorter, the Fluid is determined thereby to run out at the lower End.

XIII. If either of the Tubes (*Fig.* 23 or 24) are small enough to raise the Water from A to B, and if the Orifice A is immerged, the Fluid will rise to B, passing on to C, where it will run out or be suspended according to the foregoing Cases: but if the Periphery at G (*Fig.* 25.) is such as would not support the Fluid higher than AM, it will stop, when it comes at G, and only the Part ABG will be filled with it.

For that Fluid which has passed B assists by its Weight the attracting Periphery in raising the Column AB, and therefore runs down to C. But if when it comes to G, the Periphery there is not able to support more than AM

A M the Difference of the Legs A B and B G, the Fluid must necessarily stop there; since B G is no more than a Counterpoise to M B, and A M is supposed to be as much as the Periphery at G can sustain.

XIV. Tho' a Capillary Tube be shorter than the Height to which its attracting Periphery is able to raise a Fluid, *v. g.* tho' the Tube A B *(Fig. 26.)* be small enough (was it of sufficient Length) to raise Water as high as C; yet when the End A is immerged the Fluid will not run out at B, but only be suspended at that Height.

For when the Fluid is risen as high as B, it has then no more Periphery to attract it any farther; and if it was forced up a little higher, it would be attracted back again by the End *.

XV. The Ascent of different Fluids in the same Tube is various. *Musschenbroek* has found that in a Tube in which Water will rise to the Height of twenty six Lines, Oyl of Wormwood will rise but eighteen or nineteen, whereas Urine will rise thirty three or thirty four. The Reason of which is because some

* Hence we see the Absurdity of supposing that a Fluid may be made continually to flow from a lower Place to an higher by a Capillary Tube as such; for whether the Tube be of such Form, as is represented Fig. 23, 24, 25, or 26. the Fluid will always stop when it comes at the higher End; because the Attraction is then in a Direction contrary to its Motion, and the Weight of the Fluid contained in *D C* the Difference of the Legs is likewise an Impediment to it.

Fluids

Fluids are attracted more ſtrongly by Glaſs than others are. Mercury exhibits Phænomena juſt the Reverſe of the former; for the Height it riſes to in a Capillary Tube is leſs than that of the Level. This is becauſe the Particles of Mercury attract each other more forcibly than they are attracted by thoſe of Glaſs †.

† See *Jurin*'s Diſſert. Philoſoph. Tranſ. N°. 363.

According to *Muſſchenbroeck* the Length of the upper Part of a Tube, which is above the Height to which it is able to raiſe a Fluid, conduces ſomething towards the Raiſing it; and therefore in a longer Tube a Fluid riſes higher than in one of the ſame Dimenſions that is ſhorter; and that if a Tube, with ſo much Fluid contained in it, as it is able to raiſe, be laid in an Horizontal Situation, the Fluid will run to the Middle of it. But of this I have had no Experience: 'tis poſſible that ingenious Profeſſor though very accurate in making Experiments, might herein be deceived. He acknowledges, (Experiment the fifteenth,) that it ſometimes happens otherwiſe.

Other Authors beſides thoſe already referred to, that have treated on this Subject, are *Boyle* Exper. Phyſ. Mech. Exp. 9. *Sturmius* Colleg Cur. Tentam. 8. *Bernoulli* Gravit. Æth. *Hooke* Micrógr. Obſerv. 6. *Leeuwenhoek* Continuat. Arcan Nat. Epiſt. 131. *Sinclaire* Art. Gravit.

DISSER-

DISSERTATION III.

Of the Origin of Fountains.

MANY have been the Conjectures of Philofophers concerning the Origin of Fountains; and great Pains have been taken both by the Members of the Royal Society, and thofe of the Academy of Sciences at *Paris*, in order to afcertain the true Caufe of it. It was *Ariftotle*'s Opinion, and held by moft of the ancient Philofophers after him, that the Air contained in the Caverns of the Earth, being condenfed by Cold near its Surface, was thereby changed into Water; and that it made its Way through, where it could find a Paffage. But we have no Experience of any fuch Tranfmutation of Air into Water.

THOSE who imagin, that Fountains owe their Origin to Waters brought from the Sea by fubterraneous Ducts, give a tolerable Account, how they lofe their Saltnefs by Percolation as they pafs through the Earth; but they find great Difficulty in explaining by what Power the Water rifes above the Level of the Sea, near to the Tops of Mountains, where Springs generally abound; it being contrary

to the Laws of *Hydrostatics*, that a Fluid should rise in a Tube above the Level of its Source. However they have found two Ways, whereby they endeavour to extricate themselves from this Difficulty. The one is that of *Des Cartes*, who imagines that after the Water is become fresh by Percolation, it is raised out of the Caverns of the Earth in Vapour towards its Surface; where meeting with Rocks near the Tops of Mountains in the Form of Arches or Vaults, it sticks to them, and runs down their Sides, (like Water in an Alembic) till it meets with proper Receptacles, from which it supplies the Fountains. Now, this is a mere *Hypothesis* without Foundation or Probability; for in the first Place, we know of no internal Heat of the Earth to cause such an Evaporation; or if that were allowed, yet 'tis quite incredible, that there should be any Caverns so smooth, and void of Protuberances, as to answer the Ends of an Alembic, in collecting and condensing the Vapours together, in every Place where Fountains a-arise. There are others (as *Varenius* &c.) who suppose, that the Water ma[illegible]se thro' the Pores of the Earth, as thro[illegible]llary Tubes by Attraction; but here [illegible]hew, that they are quite unacquain[illegible] what relates to the Motion of a Fl[illegible]n su[illegible] Tubes. For when a Capill[illegible]pen[illegible]

to a Cavity at its upper End, or grows larger and larger, so as to cease to be Capillary at that End; the Water will not ascend through that Tube into the Cavity, or beyond where the Tube is Capillary; because there the Force of Attraction is exerted the contrary Way: Nay, if the Cavity is continually supplied with Water, it will be attracted into the Capillary Tube, and run down it, as through a Funnel, if the lower End is immerged in the same Fluid, as in this Case it is supposed to be *.

IT has been a generally received Opinion, and much espoused by *Marriotte* (a diligent Observer of Nature,) that the Rise of Springs is owing to the Rains and melted Snow. According to him, the Rain Water which falls upon the Hills and Mountains, penetrating the Surface, meets with Clay or Rocks contiguous to each other, along which it runs, without being able to penetrate them, till being got to the Bottom of the Mountain, or to a considerable Distance from the Top, it breaks out of the Ground and forms Springs.

IN order to examine this Opinion, Mr. *Perrault*, *De la Hire* and *D. Sideleau* endeavoured to make an Estimate of the Quantity of Rain and Snow, that falls in the Space of a Year, to see whether it would be sufficient to afford a Quantity of Water, equal to that

* See the latter Part of the foregoing Differtation.

which is annually difcharged into the Sea by the Rivers. The Refult of whofe Inquiries was, that the Quantity of Rain and Snow which fell in a Year into a Cylindrical Veffel, would fill it (if fecured from evaporating) to the Height of about nineteen Inches. Which Quantity D. *Sideleau* * fhewed, was not fufficient to fupply the Rivers; for that thofe of *England*, *Ireland* and *Spain* difcharge a greater Quantity of Water annually, than the Rain, according to that Experiment, is able to fupply. Befides which, another Obfervation was made by them at the fame Time, *viz.* that the Quantity of Water raifed in Vapour one Year with another, amounted to about Thirty two Inches, which is thirteen more than falls in Rain: a plain Indication, that the Water of Fountains is not fupplied by Rains and melted Snow.

THUS, the true Caufe of the Origin of Fountains remained undifcovered, till Dr. *Halley* in making his Celeftial Obfervations upon the Tops of the Mountains at St. *Helena*, about eight Hundred Yards above the Level of the Sea, found that the Quantity of Vapour which fell there (even when the Sky was clear) was fo great that it very much impeded his Obfervations, by covering his Glaffes with Water every half Quarter of an Hour; and upon that

* Memoirs of the Royal Academy of Sciences for the Year 1693.

attempted

attempted to determine by Experiment the Quantity of Vapour, exhaled from the Surface of the Sea, as far as it arifes from Heat; in order to try, whether that might be a fufficient Supply for the Water continually difcharged by Fountains. The Procefs of his Experiment was as follows. He took a Veffel of Water falted to the fame Degree with that of Sea Water, in which he placed a Thermometer, and by means of a Pan of Coals, brought the Water to the fame Degree of Heat, which is obferved to be that of the Air in our hotteft Summer: This done, he affixed the Veffel of Water with the Thermometer in it, to one End of a Pair of Scales, and exactly counterpoifed it with Weights on the other. Then at the End of two Hours he found by the Alteration made in the Weight of the Veffel, that about a 60*th* Part of an Inch of the Depth of the Water, was gone off in Vapour; and therefore in twelve Hours, one tenth of an Inch would have gone off. Now this accurate Obferver allows the Mediterranean Sea to be 40 Degrees long and 4 broad (the broader Parts compenfating for the narrower) fo that its whole Surface is 160 fquare Degrees, which according to the Experiment muft yield at leaft 5280 Millions of Tons. In which Account no Regard is had to the Wind, and the Agitation of the

Surface

Surface of the Sea; both which undoubtedly promote the Evaporation.

It remained now to compare this Quantity of Water, with that which is daily conveyed into the same Sea by the Rivers. The only way to do which, was to compare them with some known River; and accordingly he takes his Computation from the River *Thames*, and to avoid all Objections, he makes such Allowances as are probably more than the Truth.

The Mediterranean receives the following considerable Rivers *viz.* the *Iberus*, the *Rhone*, the *Tibur*, the *Po*, the *Danube*, the *Neister*, the *Borysthenes*, the *Tanais* and the *Nile*. Each of these he supposes to bring down ten Times as much Water as the *Thames*, whereby he allows for smaller Rivers, which fall into the same Sea. The *Thames* then he finds by Mensuration to discharge about 20300000 Tons of Water a Day. If therefore the abovesaid nine Rivers yield ten Times as much Water as the Thames doth, it will follow, that all of them together yeild but 1827 Millions of Tons in a Day; which is but little more than one Third of what is proved to be raised in Vapour out of the Mediterranean in the same Time. We have therefore from hence a Source abundantly sufficient for the Supply of Fountains.

Now having found, that the Vapour exhaled from the Sea, is a sufficient Supply for the

the Fountains; he proceeds in the next Place to consider the Manner in which they are raised, and how they are condensed into Water again, and conveyed to the Sources of Springs.

In order to this he considers, that if an Atom of Water was expanded into a Shell or Bubble, so as to be ten Times as big in Diameter as when it was Water, that Atom would become specifically lighter than Air; and therefore would rise so long as the Warmth which first separated it from the Surface of the Water should continue to distend it to the same Degree; and consequently that Vapours may be raised from the Surface of the Sea in that Manner, till they arrive at a certain Height in the Atmosphere, at which they find the Air of equal specific Gravity with themseves. Here they will float, till being condensed by Cold, they become specifically heavier than the Air, and fall down in Dew or being driven by the Winds against the Sides of the Mountains, (many of which far surpass the usual Height to which the Vapours would of themselves ascend) are compelled by the Stream of the Air to mount up with it to the Tops of them: where being condensed into Water they presently precipitate, and gleeting down by the Crannies of the Stone, Part of them enters into the Caverns of the Hills; which being once filled, all the over-

overplus of Water that comes thither, runs over by the loweſt Place, and breaking out by the Sides of the Hills, forms ſingle Springs. Many of theſe running down by the Valleys between the Ridges of the Hills, and coming to unite, form little Rivulets or Brooks: many of theſe again meeting in one common Valley, and gaining the plain Ground, being grown leſs rapid, become a River; and many of theſe being united in one common Channel, make ſuch Streams as the *Rhine* and the *Danube*; which latter, he obſerves, one would hardly think to be a Collection of Water condenſed out of Vapour, unleſs we conſider how vaſt a Tract of Ground that River drains, and that it is the Sum of all thoſe Springs, which break out on the South Side of the *Carpathian* Mountains, and on the North Side of the immenſe Ridge of the *Alps*, which is one continued Chain of Mountains from *Switzerland* to the *Black-Sea*.

Thus one Part of the Vapours, which are blown on to the Land, is returned by the Rivers into the Sea, from whence it came; another Part falls into the Sea before it reaches the Land; and this is the Reaſon, why the Rivers do not return ſo much Water into the *Mediterranean* as is raiſed in Vapour. A third Part falls on the Low-Lands, and is the *Pabulum* of Plants, where yet it does not reſt, but is again exhaled in Vapour by the Action

Action of the Sun, and is either carried by the Winds to the Sea, to fall in Rain or Dew there, or else to the Mountains to be there turned into Springs.

HOWEVER it is not to be supposed that all Fountains are owing to one and the same Cause, but that some proceed from Rain and melted Snow, which subsiding through the Surface of the Earth, makes it Way into certain Cavities and thence issues out in the Form of Springs; because the Waters of several are found to increase and diminish in Proportion to the Rain which falls: that others again, especially such as are salt, and spring near the Sea Shore, owe their Origin to Sea Water percolated through the Earth, and some to both these Causes: though without doubt most of all, and especially such as spring near the Tops of high Mountains, receive their Waters from Vapours, as before explained *.

* There is a certain Species of Springs which ebb and flow alternately, and some that cease to flow for a Time, and from thence are called reciprocating or intermitting ones. Their Reciprocations may be accounted for in the following Manner.

Let *ABC* represent one Side of an Hill in which there is a Cavity *DEF*, and from this a subterraneous Duct *IKL*. Now as this Cavity fills with Water (suppose from Vapours percolating through the Surface of the Hill, or in any other Manner whatever) its Surface will rise in the Duct as it does in the Cavity, till it arrives at *M*, the Level with the upper Part of the Duct; at which Time it will run over at *K*, filling *KLA* the other Part of the Duct. Now if the Column *KL* is longer than *KI* it will overpoise the other, and so cause the Water to run out at *A*, till its Surface in the Cavity sinks as far as *I*, (provided the Duct is large enough to convey the Water away faster

than it enters the Cavity) at which Time the Fountain at *A* will cease to play, till the Surface of the Water in the Cavity rises again to *M*, and runs over at *K* as before. The Reason why the Water continues running (when the Duct is once full) till its Surface sinks to *I*, is because the Air pressing against it as it runs out at *G* and also upon its Surface in the Cavity, keeps the Duct full, as long as the Water in the Cavity is high enough to feed its Orifice at *I*.

See more on this Subject in Philosop. Transact. N°. 119. 189. 192. 384. 424. History de l' Acad. 1693. 1703. 1713. *Gulielmini* della Natura de Fiumi. *Dale's* History of *Harwich*. *Marriotte's* Hydrostatics. *Nieuwentyt* Contempt. 19. *Varenius* Geograph. Cap 16. *Regnault* Vol. 2. Conversat. 6. *Hales's* Statical Essays Vol. 1. Experiment 19. *Michelottus* in Append. ad *J. Bernoullii* de Effervesc.

DISSERTATION IV.

Of the Barometer.

IN treating of the Properties of the Air (Chap. III.) I have already taken Notice of the Construction of the common Barometer; and proved, that the Ascent and Suspension of the Mercury therein, is owing to the Pressure * of the Air. I proceed now to a more particular Inquiry into the Original, and Use of this Instrument; and the different Forms under which it has appeared, since the Time of its Inventor *Torricelli*.

In

* To say the Ascent and Suspension of the Mercury is owing to the Pressure and Elasticity of the Air, as is commonly done, is inaccurate. The Variation, indeed, in the Height of the Mercury, may be ascribed to the Elasticity of the Air, but no otherwise, than as to its *remote* Cause; *viz.* as it occasions an Alteration in the Quantity of Air, impending over the Place where the Variation happens; which alters its Weight, and so the Mercury is proportionably raised or depressed. To illustrate this, let it be supposed; that the Air is every where in *Æquilibrio*, quite round the Globe, and at perfect Rest; and then, that its Elasticity, in some one Place near the Surface of the Earth, is augmented by the Heat of the Sun, all the rest of it remaining as before. The Consequence of this will be, that the superior Part of the Atmosphere, over this Place, will be raised higher by the Expansion of the inferior Air; and therefore, being unconfined, will spread itself, every way, over the neighbouring Columns, which we suppose to retain their former State. The Quantity of Matter therefore in those Columns of Air, in whose lower Parts its Elasticity was increased, will be diminished, and that of the neighbouring

In the Beginning of the laſt Century, it was a prevailing Opinion among Philoſophers, that the Univerſe was full of Matter; and that Nature (as they expreſſed it) abhorred a *Vacuum:* Accordingly they imagined, that if a Fluid was ſucked up a Pipe with a ſufficient Force, it would riſe to any Height whatever; ſince Nature would not ſuffer any Part of the Pipe to remain empty. *Galilæo*, who flouriſhed about that Time, found upon Trial, that the common Pump would not raiſe Water, unleſs the Sucker reached within three and thirty Feet of its Surface in the Well *: From hence

ones augmented. A Barometer therefore placed in thoſe Regions, where the Air was rarified, will ſubſide; while one in the neighbouring Countries will aſcend; and they will continue at different Heights, till the denſer Air, ruſhing in upon the rarified, reſtores the *Æquilibrium*. Thus, we ſee, the Variation of the Air's Elaſticity is not the *immediate* Cauſe of the Variation in the Barometer; it firſt affects the Weight of the Air, by altering the Quantity incumbent over any Place, and that affects the Barometer. But, if we may have Recourſe to *remote* Cauſes, we may, if we pleaſe, go one Step farther; and ſay, the Aſcent and Suſpenſion of the Mercury is owing to the Heat of the Sun; for by the foregoing Inſtance, a Variation in the Heat of the Sun may ſometimes be the occaſion of a Variation in the Height of the Mercury.

Neither is the Suſpenſion of the Mercury, in a Tube, that is kept within Doors, to be aſcribed to the Elaſticity of the Air; for that exerts no Force, but as the internal Air is preſſed by the external, which endeavours to get in, where-ever it can find a Way.

* It is a common Notion, that a ſucking Pump will not raiſe Water above thirty-three Feet, whereas it will raiſe it to any Height whatever, if the Sucker reaches within thirty-three Feet of the Surface of the Water; as will be evident to any one that conſiders the Structure of the Pump: For all the Water, which has once

hence he judiciously inferred, that a Column of Water thirty-three Feet high was a Counterpoise to a Column of Air of an equal Base, whose Height extended to the Top of the Atmosphere; and that, for this Reason, the Water would not follow the Sucker any farther. *Torricelli*, observing this, took the Hint; and considered, that, if a Column of Water, of about thirty-three Feet, was equal in Weight to a Column of Air, of the same Base *; a Co-

once passed through the Valve in the Sucker, is supported by that, as the Sucker is drawn up, and rests upon a Valve placed in the Pump below, as it is let down; so that it can be no Impediment to the rising of the Water below the Sucker, whatever the Length of the Column, which it forms, may be. The placing one Pump above another, where Water is to be raised from great Depths, is rather for Strength and Conveniency, than out of Necessity.

* Perhaps it may be enquired here, how it comes to pass, that the Column of Air, which presses upon the stagnant Mercury in the Bason, is always supposed to have an equal Base with the suspended Column in the Tube; whereas, in Reality, its Base is equal to the Surface of the stagnant Mercury. The Reason is, that, as the Base of the Column of Air increases, in the same Proportion the Velocity, wherewith it descends, decreases, when it forces down the Surface of the Mercury in the Bason; consequently its Moment, or Pressure upon the Surface of the stagnant Mercury (so far as it relates to the suspending of it in the Tube) is no greater, than it would have been, had its Base been equal to that of the suspended Column; and therefore, in considering it as suspending a Fluid in a Tube, it is properly enough said to be a Column of such a Base.

Neither is this Supposition inconsistent with the ninth Proposition of the first Chapter, where it is demonstrated, that the Pressure of a Fluid is in Proportion to its perpendicular Height, and the Quantity of Surface, against which it presses. For, as the Surface of the Mercury may be considered as a Base on which the

Column

Column of Mercury, no longer than about twenty-nine Inches and a half, would be ſo too; ſuch a Column of Mercury being as heavy, as thirty-three Feet of Water. Accordingly he tried the Experiment in a Glaſs Tube (in the Manner laid down, Chap. III, §. 4.) and found it to ſucceed *. The Apparatus he made Uſe of,

Column of Air reſts, ſo the Baſe of the Column of Air may be conſider'd as a Surface againſt which the Mercury preſſes. Theſe two being equal, 'tis clear, that only the Relation of the Heights of the Columns are to be conſidered, and not that of their Baſes.

* Notwithſtanding this clear Proof of the Preſſure of the Atmoſphere, the Aſſerters of a *Plenum* would by no Means be prevailed upon to allow it to be ſuch; but tried all Ways to account for this *Phænomenon* from ſome other Cauſe. The moſt chimerical Solution, and which at the ſame Time gave the adverſe Party the greateſt Difficulty to overthrow, was that of *Linus*. He contended, that in the upper Part of the Tube, there is a Film, or Rope of Mercury, extended thro' the ſeeming Vacuity, and that the reſt was ſuſpended by it, and kept from falling into the Baſon; and that this Film is able to ſupport about twenty-nine Inches of Mercury. He confirms his *Hypotheſis* by the following Experiment. Take, ſays he, a ſmall Tube, open at both Ends, ſuppoſe about twenty Inches long; fill this Tube with Mercury, ſtopping the lower Orifice with your Thumb: Then cloſing the upper with your Finger, and immerging the lower in ſtagnant Mercury, you ſhall perceive, upon the Removal of your Thumb, a manifeſt Suction of your Finger into the Tube; and the Tube and Mercury will both ſtick ſo cloſe to it, that you may carry them about the Room. Therefore, ſays he, the internal Cylinder of Mercury in the Tube is not held up by the preponderant Air without; for if ſo, whence comes ſo ſtrong a Suction, and ſo firm an Adheſion of the Tube to your Finger?

Or if you fill the ſame Tube almoſt full of Mercury, leaving a little Space of Air within, and then immerge it in the ſtagnant Mercury, you will find, that, notwithſtanding its Surface is at ſome Diſtance from your Finger, there will be a conſiderable Suction,

of, is now the common Barometer or Weather Glass *.

The Mercury standing at a less Height, the nearer it is carried to the Top of the Atmosphere,

tion of it, as before. From hence he infers, that the Finger supports the Mercury, by Means of the abovementioned Film, and that the Pressure of the Atmosphere is not concerned.

But, when it was found, that the Mercury would not stand so high in the Tube, on the Top of a Mountain, as below; and would quite fall, when the circumambient Air was extracted from it by the Pump, all Objections vanished; and *Linus*'s funicular *Hypothesis* (as it was called) though it seemed to solve all other *Phænomena*, relating to the Suspension of the Mercury, was with Justice rejected.

Kircher, when this new Doctrine of a *Vacuum* was first advanced at *Rome*, contended, that the Authors of it were establishing Principles not only repugnant to those of Nature, but such as would be prejudicial to the Orthodox Faith; as endeavouring to evince by this subtile Experiment, that there might be in Nature *locatum sine loco, accidentia sine subjecto*, and therefore made the Experiment with Water, in the following Manner. He caused a small Bell to be fixed in the upper Part of the Tube, imagining, that, if there should be a *Vacuum*, the Bell would not be made to sound: But in making the Experiment, some Air got into the Tube (for he tells us, that but ten Feet of Water remained in the Tube, after it was inverted) the Bell therefore was heard to sound; and so the Notion of a *Vacuum*, till more accurate Experiments evinced the contrary, was exploded with Contempt.

* *Huygens* observed, that, if a Tube seventy-five Inches long, was filled with Mercury well purged of its Air, the whole Quantity of Mercury would remain suspended; whereas, according to the *Torricellian* Experiment, the Mercury ought to have subsided to the Height of about twenty-nine Inches.

The Cause of this *Phænomenon* seems to be, that, by the great Weight of so long a Column of Mercury, it was pressed into so close Contact with the Glass in pouring in, that by the mutual Attraction of Cohesion between the Mercury and the Glass, the whole Column was sustained, after the Tube was inverted.

(Chap.

(Chap. III. §. 7.) renders it uſeful in determining the Height of Mountains; and finding out the different Elevation of one Place above another. Accordingly, Dr. *Halley* has given us a Table for that Purpoſe, in the Philoſophical Tranſactions N°. 181, ſhewing how many Feet the Deſcent of the Mercury each Inch anſwers to, as it is conveyed to the Top of a Mountain, or other elevated Place. And Dr. *Nettleton* has done the like in the Philoſophical Tranſactions N°. 388, ſhewing what Number of Feet anſwers to each tenth Part of an Inch, from twenty-ſix to thirty-one Inches of Mercury.

But the principal Uſe of it is, to eſtimate the Gravity of the Air at different Times, in Order to foreſee the Alterations of the Weather, which are conſequent thereon. To this End, Dr. *Halley* in the ſame Tranſaction has alſo laid down the more remarkable *Phænomena*, relating to the different Heights of the Mercury at different Times, together with the Solution of each; which are ſo juſt, and ſo agreeable to true Philoſophy, that I doubt not but the Reader will excuſe me for giving his Account in his own Words, rather than to render it imperfect, by endeavouring to vary from it, or abridge it.

" 1. In calm Weather, when the Air is in-" clined to Rain, the Mercury is commonly " low.

" In

"2. In ferene, good, fettled Weather, the
" Mercury is generally high.

"3. Upon very great Winds, though they
" be not accompanied with Rain, the Mercury
" finks loweft of all, with Relation to the
" Point of the Compafs the Wind blows upon.

"4. *Cæteris paribus*, the greateft Heights
" of the Mercury are found upon eafterly and
" north-eafterly Winds.

"5. In calm frofty Weather, the Mercury
" generally ftands high.

"6. After very great Storms of Wind,
" when the Mercury has been low, it gene-
" rally rifes again very faft.

"7. The more northerly Places have greater
" Alterations of the Barometer, than the more
" foutherly.

"8. Within the Tropics, and near them,
" thofe Accounts we have had from others,
" and my own Obfervations at St. *Helena*,
" make very little or no Variation of the
" Height of the Mercury in all Weathers.

"Hence I conceive that the principal Caufe
" of the Rife and Fall of the Mercury, is from
" the variable Winds, which are found in the
" temperate Zone, and whofe great Uncon-
" ftancy, here in *England*, is moft notorious.

"A fecond Caufe is the uncertain Exhalation
" and Precipitation of the Vapours lodging in
" the Air, whereby it comes to be at one Time,
" much more crouded than at another, and

" con-

" consequently heavier, but this latter in a great " Measure depends upon the former. Now, " from these Principles, I shall endeavour to " explicate the several *Phænomena* of the Ba- " rometer, taking them in the same Order I " laid them down. Thus:

" 1. The Mercury's being low, inclines it " to rain, because the Air being light, the " Vapours are no longer supported thereby, " being become specifically heavier than the " Medium wherein they floated, so that they " descend towards the Earth, and in their Fall, " meeting with other aqueous Particles, they " incorporate together, and form little Drops " of Rain; but the Mercury's being at one " Time lower than at another, is the Effect of " two contrary Winds blowing from the Place " where the Barometer stands; whereby the " Air of that Place is carry'd both Ways from " it, and, consequently, the incumbent Cylin- " der of Air is diminished, and accordingly the " Mercury sinks: As for Instance, if in the " *German* Ocean it should blow a Gale of " westerly Wind, and at the same Time an " easterly Wind in the *Irish* Sea; or if in " *France* it should blow a northerly Wind, " and in *Scotland* a southerly; it must be " granted, that that Part of the Atmosphere " impendant over *England*, would thereby be " exhausted and attenuated, and the Mercury " would subside, and the Vapours, which be-

" fore

"fore floated in thofe Parts of the Air of "equal Gravity with themfelves, would fink "to the Earth.

"2. The greater Height of the Barometer "is occafioned by two contrary Winds blow- "ing towards the Place of Obfervation, where- "by the Air of other Places is brought thither "and accumulated; fo that the incumbent "Cylinder of Air, being encreafed both in "Height and Weight, the Mercury preffed "thereby muft needs ftand high, as long as "the Winds continue fo to blow; and then "the Air being fpecifically heavier, the Va- "pours are better kept fufpended, fo that they "have no Inclination to precipitate and fall "down in Drops, which is the Reafon of the "ferene good Weather which attends the "greater Heights of the Mercury.

"3. The Mercury finks the loweft of all by "the very rapid Motion of the Air in Storms "of Wind. For the Tract or Region of the "Earth's Surface, wherein the Winds rage, "not extending all round the Globe, that "ftagnant Air which is left behind, as like- "wife that on the Sides, cannot come in fo "faft as to fupply the Evacuation made by fo "fwift a Current, fo that the Air muft necef- "farily be attenuated, when and where the "faid Winds continue to blow, and that more "or lefs, according to their Violence; add "to which, that the horizontal Motion of the

" Air being so quick as it is, may, in all Pro-
" bability, take off some Part of the perpendi-
" cular Pressure thereof; and the great Agi-
" tation of its Particles is the Reason why the
" Vapours are dissipated, and do not condense
" into Drops, so as to form Rain, otherwise
" the natural Consequence of the Air's Rare-
" faction †.

" 4. The Mercury stands the highest upon
" the easterly and north-easterly Wind, be-
" cause in the great *Atlantic* Ocean, on this
" Side the thirty-fifth Degree of north Lati-
" tude, the Winds are almost always wester-
" ly or south-westerly; so that whenever
" here the Winds come up at east and north-
" east, 'tis sure to be checked by a contrary
" Gale as soon as it reaches the Ocean; where-
" fore, according to what is made out in our
" second Remark, the Air must needs be heap-

† The Reason the Doctor assigns for the sinking of the Mercury the lowest of all in violent Storms of Wind, seems not sufficient. Perhaps it may be better accounted for thus; the Cause why the Wind blows at all, is in order to restore the *Æquilibrium* of the Atmosphere, when lost (as may be inferred from what was said in the first Note of this, and will be more largely explained in the following Dissertation;) it therefore always blows towards that Point, where the Air is most rarefied and lightest. Now the Air in its Progress to that Point, must certainly move faster and faster; for the Cause which gave it Motion at first, continues to act upon it all the Way. Consequently, in whatever Place the Wind blows with great Rapidity, that Place is at, or near the Point, where the Air is most rarefied, and lightest; which is a sufficient Reason for the Mercury's standing low at that Place.

" ed

" ed over this Iſland, and conſequently the
" Mercury muſt ſtand high, as often as theſe
" Winds blow. This holds true in this Coun-
" try, but is not a general Rule for others,
" where the Winds are under different Cir-
" cumſtances; and I have ſometimes ſeen the
" Mercury here, as low as twenty-nine Inches
" upon an eaſterly Wind, but then it blew ex-
" ceeding hard, and ſo comes to be accounted
" for, by what was obſerved upon the third
" Remark.

" 5. In calm froſty Weather the Mercury
" generally ſtands high, becauſe (as I con-
" ceive) it ſeldom freezes, but when the Winds
" come out of the northern, and north-eaſtern
" Quarters; or, at leaſt, unleſs thoſe Winds
" blow at no great Diſtance off: For the
" north Parts of *Germany*, *Denmark*, *Swe-*
" *den*, *Norway*, and all that Tract from
" whence north-eaſtern Winds come, are
" ſubject to almoſt continual Froſt all the
" Winter; and thereby the lower Air is very
" much condenſed, and in that State is brought
" thitherwards by thoſe Winds, and being ac-
" cumulated by the Oppoſition of the weſter-
" ly Wind blowing in the *Ocean*, the Mer-
" cury muſt needs be preſſed to a more than
" ordinary Height: and, as a concurring
" Cauſe, the ſhrinking of the lower Parts of
" the Air into leſſer Room by Cold, muſt

" needs cause a Descent of the upper Parts of " the Atmosphere, to reduce the Cavity made " by this Contraction to an *Æquilibrium*.

" 6. After great Storms, when the Mer- " cury has been very low, it generally rises " again very fast: I once observed it to rise " one Inch and a half in less than six Hours, " after a long continued Storm of south-west " Wind. The Reason is, because the Air be- " ing very much rarefied, by the great Eva- " cuations which such continued Storms make " thereof, the neighbouring Air runs in the " more swiftly, to bring it to an *Æquilibri-* " *um*; as we see Water runs the faster for " having a greater Declivity.

" 7. The Variations are greater in the more " northerly Places, as at *Stockholm*, greater " than that at *Paris* (compar'd by M. *Pas-* " *chal*;) because the more northerly Parts " have usually greater Storms of Wind than " the more southerly, whereby the Mercury " should sink lower in that Extream; and " then the northerly Winds bringing the more " dense and ponderous Air from the Neigh- " bourhood of the Pole, and that again being " checked by a southerly Wind at no great " Distance, and so heaped, must of Necessity " make the Mercury in such Case stand higher " in the other Extream.

" 8. Lastly,

" 8. Laftly, this Remark, That there is " little or no Variation near the *Equinoctial*, " does above all others, confirm the *Hypothe-* " *fis* of the variable Winds being the Caufe of " thefe Variations of the Height of the Mer- " cury; for in the Places above-named, there " is always an eafy Gale of Wind blowing " nearly upon the fame Point, *viz.* eaft-north- " eaft, at *Barbadoes*, and eaft-fouth-eaft at " St. *Helena* *, fo that there being no contra- " ry Currents of Air to exhauft or accumulate " it, the Atmofphere continues much in the " fame State: However, upon Hurricanes " (the moft violent of Storms) the Mercury " has been obferved very low, but this is but " once in two or three Years, and it foon re- " covers its fettled State about 29 ½ Inches."

Monfieur *Leibnitz* accounted for the Defcent of the Mercury before Rain, upon another Principle †, *viz.* as a Body fpecifically lighter than a Fluid, while it is fufpended by it, adds more Weight to that Fluid, than when, by being reduced in its Bulk, it becomes fpecifically heavier, and defcends; fo the Vapour, after it is reduced into the Form of Clouds, and defcends, adds lefs Weight to the Air, than before; and therefore the Mercury

* See the Caufe of this affigned in the following Differtation.

† Memoir. de l'Acad. 1711.

cury falls. To which it is anſwered, 1*ſt*. That when a Body deſcends in a Fluid, its Motion, in a very little Time, becomes uniform, (or nearly ſo) a farther Acceleration of it being prevented by the Reſiſtance of the Fluid; and then, by the third Law of Nature, it preſſes the Fluid downwards, with a Force equal to that whereby it tends to be farther accelerated, that is, with a Force equal to its whole Weight. 2*dly*. The Mercury, by its Deſcent, foretells Rain a much longer Time before it comes, than the Vapour, after it is condenſed into Clouds, can be ſuppoſed to take up in falling. 3*dly*. Suppoſing that as many Vapours, as fall in Rain, during the Space of a whole Year, were at once to be condenſed into Clouds, and even quite ceaſe to gravitate upon the Air, its Gravity would ſcarce be diminiſhed thereby, ſo much as is equivalent to the Deſcent of two Inches of Mercury in the Barometer. Farther, in many Places between the Tropics, the Rains fall at certain Seaſons, in very great Quantities *, and yet the Barometer ſhews there very little or no Alteration in the Weight of the Air.

The following are Mr. *Patrick*'s Obſervations on the riſing and falling of the Mercury. They are very juſt, and are to be accounted

* See Diſſertation the VIth.

for

for on the fame Principles with thofe of Dr. *Halley*.

" 1. The rifing of the Mercury prefages in " general fair Weather; and its falling, foul " Weather; as Rain, Snow, high Winds and " Storms.

" 2. In very hot Weather, the falling of " the Mercury forefhews Thunder.

" 3. In Winter the rifing prefages Froft; " and in frofty Weather, if the Mercury falls " three or four Divifions, there will *certain-* " *ly* follow a Thaw. But in a continued " Froft, if the Mercury rifes, it will *certain-* " *ly* fnow.

" 4. When foul Weather happens foon after " the Falling of the Mercury, expect but little " of it. And, on the contrary, expect but " little fair Weather, when it proves fair " fhortly after the Mercury has rifen.

" 5. In foul Weather, when the Mercury " rifes much and high, and fo continues for " two or three Days before the foul Weather " is quite over, then expect a Continuance of " fair Weather to follow.

" 6. In fair Weather, when the Mercury " falls much and low, and thus continues for " two or three Days before the Rain comes; " then expect a great deal of wet, and pro- " bably high Winds.

" 7. The

" 7. The unſettled Motion of the Mercury " denotes uncertain and changeable Weather.

" 8. You are not ſo ſtrictly to obſerve the " Words engraven on the Plates (though, for " the moſt Part, it will agree with them) as " the Mercury's *Riſing* and *Falling*: For if " it ſtands at *Much Rain*, and then riſes up " to *Changeable*, it preſages fair Weather, al- " though not to continue ſo long, as it would " have done, if the Mercury were higher; " And ſo on the contrary, if the Mercury " ſtood at *Fair*, and falls to *Changeable*, it " preſages foul Weather; though not ſo much " of it, as if it had ſunk down lower."

From theſe Obſervations, it appears, That it is not ſo much the Height of the Mercury in the Tube, that indicates the Weather, as the Motion of it up and down; wherefore, in Order to paſs a right Judgment of what Weather is to be expected, we ought to know, whether the Mercury is actually *Riſing* or *Falling*, to which End, the following Rules are of Uſe.

1. If the Surface of the Mercury is convex, ſtanding higher in the Middle of the Tube than at the Sides, it is generally a Sign that the Mercury is then riſing.

2. If the Surface is concave, or hollow in the Middle, it is ſinking. And,

3. If it is plain, the Mercury is stationary, or rather, if it is a little convex; for Mercury being put into a Glass Tube, especially a small one, will naturally have its Surface a little convex; because the Particles of Mercury attract each other more forcibly than they are attracted by Glass. Further,

4. If the Glass is small, shake the Tube; and if the Air is growing heavier, the Mercury will rise about half the tenth of an Inch higher, than it stood before; if it is growing lighter, it will sink as much. This proceeds from the Mercury's sticking to the Sides of the Tube, which prevents the free Motion of it, till it is disengaged by the Shock. And therefore, when an Observation is to be made with such a Tube, it ought always to be shaken first, for sometimes the Mercury will not vary of its own Accord, till the Weather, it ought to have indicated, is present.

The Usefulness of knowing, whether the Mercury is actually rising or falling; and the Advantage that would arise from perceiving the most minute Variations in estimating the Heights of Places, have given Occasion to the Invention of several Kinds of Barometers different from the *Torricellian*, though founded on the same Principle; wherein the Scale of Variation, which in that is not above three Inches,

ſhould be conſiderably larger. Of which I am now to give ſome Account.

1. The firſt is that of *Des Cartes*, which was made in the Form expreſſed *Fig.* 28. where AB is a Tube hermetically * ſealed at A, and having its lower Orifice immerged in ſtagnant Mercury EF, and filled with the ſame Fluid to G, from thence to H with Water, and empty from thence to the Top. Now, when the Mercury riſes in this Tube, ſuppoſe from G to L, the Water will be raiſed in the ſmall Tube, perhaps from H to M, *viz.* as many Times further, as the Tube CA is ſmaller than CD; by which Means the Variations become much more ſenſible, than they are in the common Barometer. The Inconvenience of this was, that the Air, included in the Water, getting looſe by Degrees, filled the void Space at the Top, and ſo ſpoiled the Machine.

2. He then contrived it thus, ABC (*Fig.* 29.) is a bent Tube hermetically ſealed at A, filled with Water from F to D (tinged with *Aqua Regia* to prevent its freezing,) from D to E with Mercury, and empty from thence to the Top. Then, upon the Mercury's riſing, ſuppoſe from

* A Tube is ſaid to be *hermetically* ſealed, when the End is ſo cloſed, that nothing can poſſibly evaporate through it. And, becauſe this is beſt done, when it is cloſed up with its own Subſtance; or when its Bore does not reach quite through it, it is then ſaid to be *hermetically* ſealed.

E to

E to M, and falling as much at D, the Surface of the Water at F would fink fo many Times farther than the Surface of the Mercury at D, as the Tube CG was fmaller than GH. The Water here is liable to evaporate, though that may, in fome Meafure, be prevented, by pouring a few Drops of Oil of fweet Almonds upon it. Others have contrived

3. The Horizontal or Rectangular Barometer (*Fig.* 30.) hermetically fealed at A, and filled with Mercury from D to E; then as the upper Surface of it rifes in the Tube, fuppofe from E to F, the lower will be driven from D to G, as many Times farther, as this Part of the Tube is lefs than that at E. But it often happens, that fome Parts of the Mercury break off from the reft in the Leg BC, and are left behind. This Inconvenience is remedied in

4. The Diagonal Barometer ABC (*Fig.* 31.) wherein the Mercury, inftead of rifing from B to D (fuppofe that Space to correfpond to the Scale of Variation in a ftrait Tube) will rife from B to A; for it will always ftand at the fame perpendicular Height, whatever be the Inclination of the Tube; becaufe Fluids prefs only according to their perpendicular Altitude *. But the Tube AB muft not be too much inclined, left the Mercury break in it, as in the former.

* Chapter I. §. 9.

5. AB (*Fig.* 32.) is Dr. *Rook*'s Wheel-Barometer, wherein ABD is a Tube filled with Mercury from *a* to E; *a* is an Iron Ball, ſwimming on the Surface of the Mercury; this as it ſubſides with the Surface of the Mercury, draws the little Wheel *m n* round, to whoſe Circumference it is fixed by Means of the String *a c* *: This Wheel carries the Index PQ, which points to the graduated Edge of the Circle KL, and by its Motion ſhews the moſt minute Variations of the Mercury. When the Ball *a* is raiſed by the Mercury on which it ſwims, the Index is drawn the contrary Way by a leſſer Ball *b*, which hangs on the other Side the Wheel. The Friction in this Machine, unleſs it be made with great Accuracy indeed, renders it uſeleſs.

6. The pendent Barometer is another Contrivance to render the Variations more ſenſible. It conſiſts of a ſmall conical Tube, (repreſented *Fig.* 33.) hermetically ſealed at A, and filled with Mercury from C to D, and empty from thence to A. Now, ſuppoſing the Gravity of the Air encreaſed, it will raiſe the Mercury higher in the Tube, and ſo force it into a narrower Part; by which Means the Column becoming longer, its perpendicular Preſſure upon the Air below will be proportionably encreaſed.

* The Tube is ſmaller at *a* than at E, that the greateſt Variation may be at that Surface of the Mercury on which the Ball reſts.

On

On the contrary, when the Air becomes lighter, the Mercury descends into a larger Part of the Tube, and by that Means has the Length of its Column proportionably contracted. The Inconvenience that attends this Barometer, is that the Tube must be very small, otherwise the Mercury will fall out; or the Air will be apt to get into it, and divide the Column in several Places; and when the Tube is very small, the Friction of the Mercury against the Sides of it, will hinder it from rising and falling freely.

7. Dr. *Hook*, observing how unfit the common Barometer was to be used on Board of Ship, by Reason its Position ought to be steady, contrived the following one, called, from its Use, a *Marine* Barometer, consisting of two Parts, the one A B (*Fig.* 34.) the common Spirit Thermometer, the other C D, a Tube filled with Air from C to E, and from thence to the End D with tinged Water. This End is immerged in the same Fluid contained in the Vessel G F; and having its Surface exposed to the Pressure of the external Air. Now, the last of these Machines will be affected both by the Warmth of the external Air, and also by its Pressure: The former dilating the Air included in C E, and by that Means driving the Water downwards; the latter pressing it up higher in the Tube: Whereas the other, *viz.* AB, is affected by the Warmth of the Air alone. Consequently,

ſequently, were theſe Inſtruments graduated in ſuch a Manner, that, if the Gravity of the external Air ſhould always remain the ſame it was, when the Inſtruments were made, their Variations (then only depending on its Warmth) ſhould exactly correſpond with each other; that is, when the Spirit in the Tube A B, ſhould aſcend to 1, the Water in C D, ſhould deſcend to 1, *&c.* Then, whenever their Variations ſhould be obſerved to differ from each other, the Difference could only be aſcribed to ſome Alteration in the Preſſure of the Air upon the Surface of the Water in the Veſſel G F. In Proportion therefore as this Difference is greater, or leſs, ſo is the Alteration in the Gravity of the Air, from what it was when the Inſtruments were adjuſted. For Inſtance, when the Water ſtands above the Diviſion, which correſponds to that, which the Spirit points to in the other Machine, it is an Indication, that the Preſſure of the Air is greater at that Time, than when the Inſtruments were graduated, and *vice verſâ*.

This Machine is not only more uſeful at Sea, than the common one, as not requiring a ſteady Poſition; but may have its Scale of Variation conſiderably enlarged, by making the Bore of the Tube C D very ſmall, in Proportion to the Capacity of its Head C.

But

But it is obſerved, that in long keeping this Inſtrument, the included Air loſes ſomewhat of its Elaſticity; whereby, in Proceſs of Time, the Water ſtands higher than it ought, and therefore indicates the Gravity of the Air to be greater than what it is.

In the Philoſophical Tranſactions N°. 427. I have given an Account of a Barometer, wherein the Scale of Variation may be encreaſed *ad Infinitum*. The Deſcription of it is as follows: A B C D (*Fig.* 35.) is a cylindrical Veſſel, filled with a Fluid to the Height W, in which is immerged the Barometer S V, conſiſting of the following Parts: The Principal of which is a Glaſs Tube T P (repreſented ſeparately at *t p*) whoſe upper End T is hermetically ſealed: This End does not appear to the Eye, being received into the lower End of a Tin Pipe G H, which in its other End G receives a cylindrical Rod, or Tube ST, and thereby fixes it to the Tube TP. This Rod ST may be taken off, in Order to put in its ſtead a larger, or leſſer, as Occaſion requires. S is a Star at the Top of the Rod ST, and ſerves as an Index, by pointing to the graduated Scale L A, which is fixed to the Cover of the Veſſel A B C D. MN is a large cylindrical Tube made of Tin (repreſented ſeparately at *m n*) which receives in its Cavity the ſmaller Part of the Tube TP, and is well cemented to it

at

at both Ends, that none of the Fluid may get in.

The Tube TP, with this Apparatus, being filled with Mercury, and plunged into the Bason V, which hangs by two, or more Wires, upon the lower End of the Tube MN, must be so poised, as to float in the Liquor contained in the Vessel A B C D, and then the whole Machine will rise, when the Atmosphere becomes lighter, and *vice versa*.

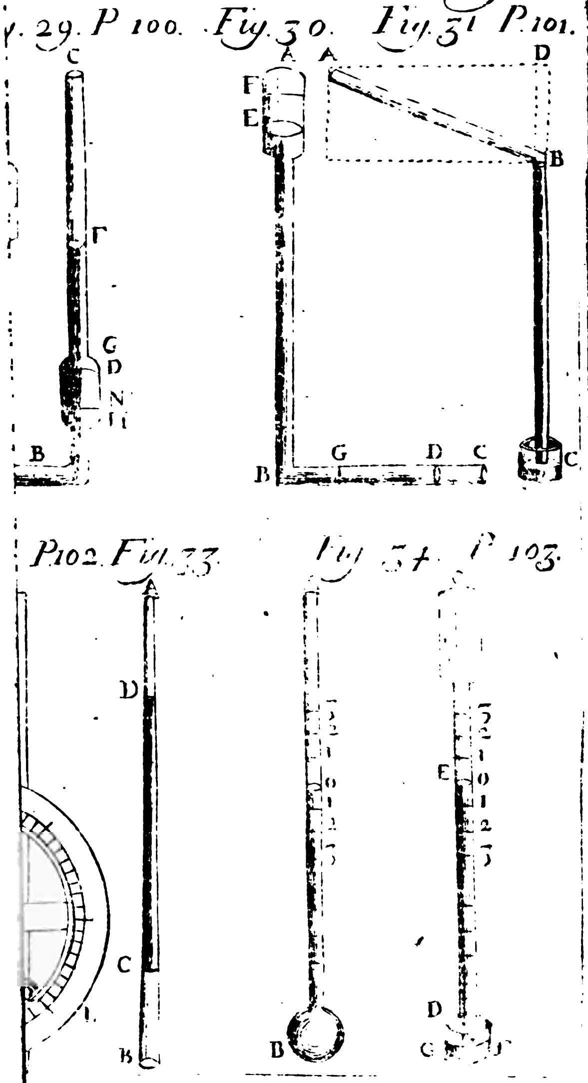
I shall here add a Computation, in Order to shew the Possibility of the Variation being infinite, upon a given finite Variation of the Weight of the Atmosphere, and withal, the Reason why it may be so. And for the Sake of those who would see a Mathematical Proof of it, I shall give the Demonstration in a Note below.*

Let

* Let the specific Gravity of Quicksilver be to that of Water, or to the Liquor the Barometer floats in, as s to 1; and if it be proposed, that the Variations in this compound Barometer shall be to the contemporary Variations of the common Barometer in the given Ratio of n to 1, this Effect will be obtained, by making the Diameter of the Rod ST to the Diameter of the Cavity of the Tube HI, as $\sqrt{\frac{n+s}{ns}}$ to 1, which may be thus demonstrated.

Let us suppose, that the Variation in the Height of the Quicksilver in the common Barometer, which we call v, is such, that a cubic Inch of Quicksilver shall rise into the *Vacuum* XT; in Order to which, a cubic Inch of Quicksilver must rise from the Vessel V; that is, the Surface P must subside so far, that a cubic Inch of Water (if that be the Fluid made Use of) shall enter the Vessel V; by which Means the Barometer with the Parts annexed will be heavier by a cubic Inch of the Fluid. Now

Part II. Plate V. Pag. 106.



Let it be supposed, that the Fluid made use of is Water, and that the given Variation in the Weight of the Atmosphere is such, that, by

Now this additional Weight of a cubic Inch of Fluid, will make the whole Barometer subside (according to the Laws of Hydrostatics) till a cubic Inch of the Rod HS, immediately extant above the Surface at W, shall come under it; but the Length of such a Magnitude of HS will exceed the Length of an equal Magnitude of Quicksilver in the larger Tube X, as many Times as the Square of the Diameter at X exceeds the Square of the Diameter at H (the Lengths of equal Cylinders being reciprocal to their Bases.) That is, the perpendicular Descent of the compound Barometer will be to v, the perpendicular Ascent of the Mercury in the common Barometer, as d to 1 (supposing this the Ratio of their Bases, and consequently will be equal to dv.

But, by this Descent, the Distance PW, between the Surface of the stagnant Quicksilver and the Top of the Fluid, will be augmented by a Column, whose Height is dv, the Descent of the compound Barometer; and consequently the Weight of the whole Column of the Fluid pressing on the lower Surface of the Quicksilver (to which the Height X is partly owing) will be encreased by a Column of that Length; and this Increase would produce a second Ascent of the Mercury at X equal to itself, namely, dv, were the Fluid as heavy as Quicksilver; but since it is supposed to be lighter in the Ratio of s to 1, the Ascent of the Quicksilver, on this Account, will only be $\frac{dv}{s}$.

But now, as in the former Case, when the Ascent of the Mercury was v, the Descent of the compound Barometer was shewn to be dv; so here, the Ascent of the Mercury being $\frac{dv}{s}$, the Descent of the common Barometer will be $\frac{ddv}{s}$, and the next Descent $\frac{dddv}{ss}$, and the next $\frac{d^4v}{s^3}$, and so on to Infinity. Therefore the whole Descent of the compound Barometer, is to the Ascent of the Mercury in the common Barometer, that is, n is to 1 as $d+\frac{dd}{s}+\frac{ddd}{ss}+\frac{d^4}{s^3}+$ &c. to 1, or as $\frac{ds}{s-d}$ to 1; because the Terms of the Series being in geometrical Proportion, the Sum

by pressing upon the Surface of it at W, the Surface of the Mercury at X may be raised an Inch higher (measuring from its Surface at P) than before; and that the Breadth of the Cavity of the Tube at X, and of the Bason at P are such, that by this Ascent of the Mercury, there may be a cubic Inch of it in the Cavity X more than before, and consequently in the Bason a cubic Inch less. Now, upon this Supposition, there will be a cubic Inch of Water in the Bason more than there was before; because the Water will succeed the Mercury to fill up its Place. Upon this Account, the whole Machine will be render'd heavier, than it was before, by the Weight of a cubic Inch of Water, and therefore will sink, according to the Laws of Hydrostatics (Chap II. §. 5.) till a cubic Inch of that Part of the Rod WS, which

Sum of them all is $\frac{ds}{s-d}$. Hence we have $n = \frac{ds}{s-d}$, and therefore $ns = ds + dn$; that is, $1 : d :: n+s : ns :: \frac{n+s}{ns} : 1$; and therefore, by extracting the square Roots of each Term in the Proportion, $1 : \sqrt{d}$ (that is, the Diameter of ST to the Diameter of HI) as $\sqrt{\frac{n+s}{ns}}$ to 1. *Q. E. D.*

Example 1. Putting $s=14$ and $n=1$, the Variation in each Barometer will be equal, by taking the Diameter of ST to the Diameter of HI, as $\sqrt{\frac{15}{14}}$ to 1, that is, as 30 to 29 nearly.

Example 2. If n be put infinite, the Diameter of ST will be to the Diameter of HI, as $\sqrt{\frac{1}{s}}$ to 1, or 1 to $\sqrt{14}$; that is, as 1 to $3\frac{1}{4}$ nearly.

Was

was above the Surface of the Water at W, comes under it. Then, if we ſuppoſe this Rod ſo ſmall, that a cubic Inch of it ſhall be fourteen Inches in Length, the whole Machine will ſink fourteen Inches lower into the Fluid than before, and conſequently the Surface of the Mercury in the Baſon will be preſſed more than it was before, by a Column of Water fourteen Inches high. But the Preſſure of fourteen Inches of Water is equivalent to one of Mercury (becauſe Water is about fourteen Times lighter than Mercury) this additional Preſſure therefore will make the Mercury aſcend at X, as much as the ſuppoſed Variation in the Weight of the Air did at firſt. This Aſcent will give Room for a ſecond cubic Inch of Water to enter the Baſon; the Machine will therefore be again rendered heavier by the Weight of ſo much Water, and accordingly will ſubſide fourteen Inches farther. This will occaſion another additional Preſſure of Water, which will raiſe another Inch of Mercury, and make the Machine ſink fourteen Inches more, and ſo on, without ever approaching nearer to an *Æquilibrium* with the external Air: and therefore a Scale, anſwering to the Variation of this Barometer, ought ſtrictly and properly to be of an infinite Length; becauſe after this Barometer has ſunk or riſen thouſands of Miles (if that were poſſible) it would ſtill have the ſame Tendency to ſink or riſe on, as when it firſt ſet out.

Now, was the Rod WS so small, that a cubic Inch of it should be more than fourteen Inches long (the other Parts remaining as was supposed above) the Variation in this Barometer would be more than infinite, or negative with Respect to those of the common Barometer. The Meaning of which is, that whereas in the common Barometer, the suspended Column of Mercury, by its rising or falling, approaches nearer to an *Æquilibrium* with the external Air, this Barometer would continually recede from an *Æquilibrium* with it; so that the farther it should move up or down, instead of acquiring by that Means a less Tendency to move on, as the Mercury in the common Barometer does, it would acquire a greater.

On the contrary, when a cubic Inch of the Rod is less than fourteen Inches in Length, the Variation will be finite; and may be made to bear any Proportion to those of the common Barometer whatever, as demonstrated in the foregoing Note.

While I am writing this, another Method occurs to me of making a Barometer, wherein the Scale of Variation shall bear any Proportion to that of the common one. It is this; Let there be a compound Tube, as ABC (*Fig.* 36.) hermetically sealed at A, and open at C, empty from A to D, filled with Mercury from thence to B, and from thence to E with Water:

ter: Then, if the Tube FC be a little more than five Times less in Diameter than the Tube FA, the Variation in the lower Surface of the Water at E will be infinite; if it be above so many Times less, it will be more than infinite, otherwise it will be finite. See the Demonstration in the Note *.

That

* Let v denote a given Variation in the common Barometer, x the correspondent Variation at E sought. Let the Ratio of m to 1, express that of the specific Gravity of Mercury, to that of Water; and d to 1, that of the Diameter of the Tube FA to FC. Then the Variation at E, the lower Surface of the Water, being supposed x, the Variation of it at B, the upper Surface of it will be $\frac{x}{dd}$, and consequently GE, the Difference of the Legs EK and KB, will vary $x+\frac{x}{dd}$. Again, the Variation of the Surface of the Mercury at B will be the same with that of the Water in the same Place, *viz.* $\frac{x}{dd}$; and, if the Tube is of the same Diameter at D, as at B, the Variation of the Surface at D will also be the same, that is, $\frac{x}{dd}$: The Sum of both Variations, or the Variation of HD the Difference of the Legs, will therefore be $\frac{2x}{dd}$. Now the Pressure of the Mercury and Water together upon the Air at E, is owing to the Lengths of HD and GE; and since one of these will always shorten, when the other lengthens, the Variation in their Pressure will depend on the Variation of the Difference of their Weights, that is, of the Difference between the Weight of $x+\frac{x}{dd}$ and of $\frac{2x}{dd}$. But the Weight of $x+\frac{x}{dd}$ (being the Weight of a Column of Water) compared to that of a Column of Mercury of the same Length, is only $\frac{x+\frac{x}{dd}}{m}$. The Difference therefore between

That the Variation in this Barometer may be infinite, will appear from the following Computation.

Let the Proportion between the Bores of the Tube AF and FC be ſuch, that when HD, the Difference of the Legs wherein the Mercury is contained, is augmented one Inch, GE the Difference of the Legs, wherein the Water is contained, ſhall be diminiſhed fourteen; then, as much as the Preſſure of the Mercury is augmented, that of the Water will be diminiſhed, and ſo the Preſſure of both taken together will remain as it was. And conſequently, after it has began to riſe, it will always have the ſame Tendency to riſe on, without ever coming to an *Æquilibrium* with the Air.

How far this Barometer will ſucceed in Practice, muſt be left to Experience to determine.

tween $\frac{x+\frac{x}{dd}}{m}$ and $\frac{2x}{dd}$ will always be equal to the Variation in the common Barometer, and therefore $\frac{2x}{dd}-\frac{x+\frac{x}{dd}}{m}=v$, and by the common Method of Reduction, $x=\frac{vmdd}{2m-dd-1}$: That is, $x : v :: mdd : 2m-dd-1$. Now, if we put $m=14$, and $d=5$, 2, $-dd$ and -1 will be as much as $2m$, and therefore $2m-dd-1$ will be equal to nothing; and ſo x being by the Proportion as many Times more than v, as mdd is than nothing, 'tis infinite. And if m be put $=14$, and $d=5$, mdd will be equal to 350, and $2m-dd-1=2$; and therefore the Variations, in this Caſe, will be to thoſe in the common Barometer, as 175 to one.

Pro-

Probably, if the Bore of the Tube FC be made very ſmall, *viz.* about the twentieth Part of an Inch Diameter, the Air will not aſcend through the Water, as it is apt to do through the Mercury in the pendent Barometer; and the Smallneſs of the Bore will not prevent the Water, from moving, near ſo much as it does the Mercury in that Barometer.

There is an Improvement of another Kind in the common Barometer, whereby it is rendered *portable.* The Tube containing the Mercury, inſtead of having its lower End immerged in a Veſſel of that Fluid, has it tied up in a leathern Bag, not quite full of Mercury. And though the external Air cannot get into the Bag to ſuſpend the Mercury in the Tube, by preſſing on its Surface, as in the common one; yet it has the ſame Effect by preſſing on the Outſide of the Bag, which, being pliant, yields to the Preſſure, and keeps the Mercury ſuſpended in the Tube at its proper Height. This Bag is generally incloſed in a little Box, through the Bottom of which paſſes a Screw; with this Screw the Bag may be compreſſed, ſo as to force the Mercury up to the Top of the Tube; which keeps it ſteady, and hinders it from breaking the Tube by daſhing againſt the Top when it is carried about, as it otherwiſe would be apt to do.

See

See more on the Subject of this Dissertation, *Weidleri* Institutiones Mathemat. p. 608. *Melchior Verdries* Phys. Pars specialis, Cap. IV. §. 15. Mr. *Paschal*'s Traitè de l'Equilibre des Liqueurs. *Sinclair*'s Ars magna Gravitatis & Levitatis. *Mariotte*'s Second Essay de la Nature de l'Air. Philosoph. Burgund. Tom. II. p. 850. *Saul*'s Treatise on the Barometer. *Regnault*'s Philosoph. Conversat. 22. *Clare*'s Motion of Fluids, p. 141. Mem. de l'Acad. 1705, 1711. Philosophical Transactions N°. 9, 10, 11, 55, 86, 91, 165, 181, 185, 208, 229, 236, 237, 240, 243, 269, 351, 366, 385, 388, 405, 406, 427. *Cotes*'s Hydrostatical and Pneumatical Lecture. Lect. 7. With several other Authors referred to in Mr. *Johnson*'s Quæstiones Philosophicæ, Cap. VI. Quæst. 36, 37.

DIS-

DISSERTATION V.

Of the Origin of the Winds.

THE Wind is no other, than the Motion of the Air, upon the Surface of the Globe. Some of the Ancients took it to be Air, rushing out of the Bowels and Cavities of the Earth: And others thought it an Exhalation from its Surface. But these are *Hypotheses* too chimerical to stand in Need of a particular Confutation. Some of the Moderns, who held a *Plenum*, have accounted for it thus. They imagined, that the Air being confined above, as it must be, if we suppose a *Plenum*, would, when more than ordinarily rarefied, or stocked with Vapours, drive away the neighbouring Air, in order to make room for itself; and by this Means occasion a Wind. Others, observing a constant and perpetual easterly Wind to blow at the *Equator*, ascribed its Origin to the diurnal Rotation of the Earth, about its Axis from West to East; which they thought would occasion the Air upon its Surface, to seem to move the contrary Way, being in some Measure left behind. But, whereas there are Winds, in some Places near the *Equator*,

that blow on other Points of the Compaſs (as we ſhall ſee hereafter) this *Hypotheſis* is inſufficient. Beſides, the Air preſſing upon the Surface of the Earth by its Gravity, like other Bodies; and having nothing to hinder it from moving freely along with it, muſt neceſſarily, in Time, acquire an equal Degree of Velocity, and ſo keep Pace with it, all the Way round.

The principal Cauſe of the Wind, or, in other Words, of the Air's moving from Place to Place, upon the Surface of the Earth, is the Atmoſphere's being heated over one Part more than over another. For, in this Caſe, the warmer Air, being rendered ſpecifically lighter than the reſt, riſes up into the ſuperior Parts of the Atmoſphere, and there diffuſes itſelf every Way; while the neigbouring inferior Air ruſhes in from all Parts at the Bottom, to reſtore the *Æquilibrium*.

Upon this Principle it is, that moſt of the Winds may be accounted for.

To begin with thoſe which blow under the *Equator*.

1. Under the *Equator*, the Wind is always obſerved to blow from the Eaſt Point *.

For,

* For the Reader's Eaſe (who perhaps is not furniſhed with the Philoſophical Tranſactions) I ſhall here inſert by Way of Note, from Dr. *Halley*'s Account, ſo much of the Hiſtory of the Winds, as may be neceſſary for the underſtanding this Theory.

" The

For, ſuppoſing the Sun to continue vertical over ſome one Place, the Air will be moſt rarefied there; and conſequently, the neighbouring

" The univerſal Ocean, ſays he, may moſt properly be divided " into three Parts, *viz.* 1. The *Atlantic* and *Æthiopic* Seas. " 2. The *Indian* Ocean. 3. The great *South* Sea, or the *Pacific* " Ocean.

" I. In the *Atlantic* and *Æthiopic* Seas, between the *Tropics*, " there is a general eaſterly Wind all the Year long, without " any conſiderable Variation; excepting, that it is ſubject to be " deflected therefrom, ſome few Points of the Compaſs, towards " the North, or South, according to the Poſition of the Place.

" 1. Near the Coaſt of *Africa*, as ſoon as you have paſſed the " *Canary* Iſles, you are ſure to meet a freſh Gale of North-eaſt " Wind, about the Latitude of twenty-eight Degrees North; " which ſeldom comes to the Eaſtwards of the Eaſt-north-eaſt, " or paſſes the North-north-eaſt. This Wind accompanies thoſe " bound to the Southward, to the Latitude of ten Degrees North, " and about an hundred Leagues from the *Guinea* Coaſt; where, " till the fourth Degree of North Latitude, they fall into *Calms* " and *Tornadoes*, or ſudden Storms.

" 2. Thoſe bound to the *Caribbe* Iſles, find, as they approach " the *American* Side, that the aforeſaid North-eaſt Wind be-" comes ſtill more and more eaſterly, ſo as ſometimes to be Eaſt, " ſometimes Eaſt by South, but yet moſt commonly to the North-" ward of the Eaſt, a Point or two, ſeldom more. 'Tis likewiſe " obſerved, that the Strength of theſe does gradually decreaſe, " as you ſail to the Weſtward.

" 3. That the Limits of the *Trade* and *variable* Winds in " this Ocean, are farther extended on the *American* Side, than " the *African*; for, whereas you meet not with this certain " Wind, till after you have paſs'd the Latitude of twenty eight " Degrees on this Side; on the contrary Side it commonly holds " to thirty, thirty-one, or thirty-two Degrees of Latitude; and this " is verified likewiſe to the Southward of the *Æquinoctial*; for " near the *Cape of Good Hope*, the Limits of the *Trade* Winds are " three or four Degrees nearer the *Line*, than on the Coaſt " of *Braxil*.

ing Air will rush in from every Quarter with equal Force. But, as the Sun is continually shifting to the Westward, the Part, where the Air

" 4. That from the Latitude of four Degrees North, to " the aforesaid Limits on the South Side of the *Equator*, the " Winds are generally and perpetually between the South " and East, and most commonly between the South-east and " East; observing always this Rule, that on the *African* Side, " they are more southerly, on the *Brasilian* more easterly, so as " to become almost due East, the little Deflection they have be- " ing still to the Southward. In this Part of the Ocean, it " has been my Fortune to pass a full Year, in an Employ- " ment that obliged me to regard more than ordinarily the " Weather, and I found the Winds constantly about the " South-east, the most usual Point South-east by East: When " it was easterly, it generally blew hard, and was gloomy, " dark, and sometimes rainy Weather: If it came to the South- " wards, it was generally serene, and a small Gale next to a " Calm; but this not very common. But I never saw it to the " Westwards of the South, or Northwards of the East.

" 5. That the Season of the Year has some small Effect on " these *Trade* Winds; for that when the Sun is considerably to " the Northward of the *Equator*, the South-east Winds, espe- " cially in the Streight of this Ocean (if I may so call it) " between *Brasil*, and the Coast of *Guinea*, do vary a Point or " two to the Southward, and the North-east become more " easterly; and, on the contrary, when the Sun is towards the " *Tropic* of *Capricorn*, the South-easterly Winds become more " easterly, and the North-easterly Winds, on this Side the " *Line*, veer more to the Northward.

" 6. That as there is no general Rule, that admits not of some " Exception, so there is in this Ocean a Tract of Sea, wherein " the southerly and South-west Winds are perpetual, *viz.* all " along the Coast of *Guinea*, for above five hundred Leagues to- " gether, from *Sierra Leona*, to the Isle of St. *Thomas*: For " the South-east *Trade* Wind having pass'd the *Line*, and ap- " proaching the Coast of *Guinea* within eighty or an hundred " Leagues, inclines towards the Shore, and becomes South-south- " east; and by Degrees, as you come nearer, it veers about to " South, South-south-west, and in with the Land South-west, and " sometimes

Air is most rarefied, is carried the same Way: and therefore the Tendency of all the lower Air, taken together, is greater that Way, than any other.

" sometimes West-south-west. These are the Winds, which are " observed on this Coast when it blows true; but there are fre- " quent Calms, violent sudden Gusts, called *Tornadoes*, from all " Points of the Compass, and sometimes unwholsome foggy " easterly Winds, called *Hermitæ*, by the Natives, which too " often infest the Navigation of these Parts

" 7. That to the Northwards of the *Line*, between four and " ten Degrees of Latitude, and between the Meridians of *Cape* " *Verde*, and of the eastermost Islands that bear that Name, there " is a Tract of Sea, wherein it were improper to say there is " any *Trade* Wind, or yet a *variable*; for it seems condemned to " perpetual Calms, attended with terrible Thunder and Light- " ning, and Rains so frequent, that our Navigators from thence " call this Part of the Sea, the *Rains*: The little Winds that " are, being only some certain Gusts, of very little Conti- " nuance, and less Extent; so that sometimes each Hour you " shall have a different Gale, which dies away into a Calm " before another succeeds: And in a Fleet of Ships in Sight of " one another, each shall have the Wind from a several Point of " the Compass: With these weak Breezes, Ships are obliged to " make the best of their Way to the Southward, through the " aforesaid six Degrees; wherein it is reported some have been " detained whole Months for want of Wind. ——

" II. In the *Indian* Ocean, the Winds are partly general, " as in the *Æthiopic* Ocean; partly periodical, that is, half " the Year they blow one Way, and the other half, near upon " the opposite Points; and these Points and Times of shifting, " are different in different Parts of this Ocean.

" 1. Between the Latitudes of ten Degrees and thirty Degrees " South, between *Madagascar* and *Hollandia nova*, the *Gene-* " *ral* Trade-Winds about the South-east and by East, are found " to blow all the Year long, to all Intents and Purposes, " after the same Manner, as in the same Latitudes in the " *Æthiopic* Ocean, as it is described in the fourth Remark " aforegoing.

" 2. That

other. Thus the Tendency of the Air towards the West, becomes general, and its Parts impelling one another, and continuing to move

" 2. That the aforesaid South-east Winds extend to within " two Degrees of the *Equator*, during the Months of *June*, " *July*, and *August*, &c. to *November*, at which Time, between " the South Latitude of three and ten Degrees, being near the " *Meridian* of the North End of *Madagascar*, and between two " and twelve South Latitude, being near *Sumatra* and *Java*; " the contrary Winds from the North-west, or between the " North and West, set in, and blow for half a Year, *viz.* " from the Beginning of *December* till *May*: And this *Monsoon* is observed as far as the *Malucca* Isles.

" 3. That to the Northward of three Degrees South Latitude, over the whole *Arabian* and *Indian* Sea, and Gulf of " *Bengal*, from *Sumatra* to the Coast of *Africa*, there is another " *Monsoon*, blowing from *October* to *April*, upon the North-" east Points: But in the other half Year, from *April* to *October*, " upon the opposite Points of South-west and West-south-west, " and that with rather more Force than the other, accompanied " with dark, rainy Weather; whereas the North-east blows clear. " 'Tis likewise to be noted, that the Winds are not so constant, " either in Strength or Point, in the Gulf of *Bengal*, as they " are in the *Indian* Sea, where a certain steady Gale scarce ever " fails. 'Tis also remarkable, that the South-west Winds, in " these Seas, are generally more southerly on the *African* Side, " and more westerly on the *Indian*.

" 4. There is a Tract of Sea to the Southwards of the " *Equator*, subject to the same Changes of the Winds, *viz.* " near the *African* Coast, between it and the Island *Madagascar*, " or St. *Laurence*, and from thence Northwards, as far as the " *Line*; wherein from *April* to *October*, there is found a con-" stant fresh South-south-west Wind, which, as you go more " northerly, becomes still more and more westerly, so as to fall " in with the West-south-west Winds, mentioned before in those " Months of the Year to be certain to the Northward of the " *Equator*. What Winds blow in those Seas, for the other " half Year, I have not yet been able to obtain to my full Satis-" faction: The Account which has been given me, is only this,

" That

move till the next Return of the Sun, so much of its Motion as was lost by his Absence, is again restored, and therefore the easterly Wind becomes *Perpetual*.

Some

" That the Winds are much easterly hereabouts, and as often to " the North of the true East, as to the Southward thereof.

" 5. That to the Eastward of *Sumatra* and *Malacca*, to the " Northwards of the *Line*; and along the Coast of *Cambodia* " and *China*, the *Monsoons* blow North and South; that is to " say, the North-east Winds are much northerly, and the " South-west much southerly. This Constitution reaches to the " Eastward of the *Philippine* Isles, and as far northerly as " *Japan*; the northern *Monsoon* setting in, in these Seas, in " *October* or *November*; and the southern in *May*, blowing all " the Summer Months. Here it is to be noted, that the Points of " the Compass from whence the Winds come, in these Parts of " the World, are not so fixed, as those lately described; for the " southerly will frequently pass a Point or two to the Eastwards " of the South, and the northerly as much as to the Westwards " of the North, which seems occasioned by the great Quantity " of Land which is interspersed in these Seas.

" 6. That in the same Meridians, but to the Southwards of " the *Equator*, being that Tract lying between *Sumatra* and " *Java* to the West, and New *Guinea* to the East, the same " northerly and southerly *Monsoons* are observed; but with this " Difference, that the Inclination of the northerly is towards " the North-west, and of the southerly towards the South-east: " But the *Plagæ Venti* are not more constant here than in the " former, *viz.* *variable* five or six Points. Besides, the Times of " the Change of these Winds are not the same, as in the *Chinese* " Seas, but about a Month, or six Weeks later.

" 7. That the contrary Winds do not shift all at once, but in " some Places the Time of the Change is attended with *Calms*, " in others with *variable* Winds; and it is particularly remark- " able, that the End of the westerly *Monsoon*, in the Seas of " *China*, are very subject to be tempestuous. The Violence of " these Storms is such, that they seem to be of the Nature of the " *West-India* Hurricanes, and render the Navigation of these Parts " very unsafe about that Time of the Year. These Tempests are

" by

Some are inclined to think, that the continual shifting of the Sun to the Westward, should produce a westerly Wind under the *Equator*, by causing the Current of the Air from the West to exceed and over-balance that;

" by our Seamen; usually term'd the *Breaking up of the Monsoons.*

" III. The Third Ocean, called *Mare Pacificum*, whose Extent is equal to that of the other two (it being from the West Coast of *America* to the *Philippine* Islands, not less than an hundred and fifty Degrees of Longitude) is that which is least known to our own, or the neighbouring Nations: That Navigation that there is on it, is by the *Spaniards*; who go yearly from the Coast of *New-Spain* to the *Manilha*'s: But that but by one beaten Track; so that I cannot be so particular here, as in the other Two. What the *Spanish* Authors say of the Winds, they find in their Courses; and what is confirmed by the old Accounts of *Drake* and *Candish*, and since by *Schooten*, who sailed the whole Breadth of this Sea, in the southern Latitude of fifteen or sixteen Degrees, is, that there is a great Conformity between the Winds of this Sea, and those of the *Atlantic* and *Ethiopic*; that is to say, that to the Northward of the *Equator*, the predominant Wind is between the East and North-east; and to the Southwards thereof, there is a constant, steady Gale, between the East and South-east, and that on both Sides the *Line*, with so much Constancy, that they scarce ever need to attend the Sails; and strength, that it is rare to fail of crossing this vast Ocean in ten Weeks Time; which is about an hundred and thirty Miles a Day; Besides, 'tis said, that Storms and Tempests are never known in these Parts, wherefore some have thought it might be as short a Voyage to *Japan* and *China*, to go by the Streights of *Magellan*, as by the Cape of *Good Hope*.

" The Limits of these *General* Winds are much the same as in the *Atlantic* Sea, *viz.* about the thirtieth Degree of Latitude on both Sides. Besides, a farther Analogy between the Winds of this Ocean, and the *Ethiopic*, appears in that, that upon the Coasts of *Peru*, they are always much southerly, like as they are found near the Shores of *Angola*.

which

which opposes it from the East. For, whereas the eastern Air retains its Heat some time after the Sun is removed from it, it must always be rarefied to a greater Degree, and also to a greater Distance from the Place to which the Sun is vertical, than the western Air is; and therefore the western Air, being more ponderous, should be an Over-balance for the eastern, and drive its Current before it.

But it is to be observed, that we are not to consider the Point to which the Sun is vertical, but the Point of greatest Rarefaction (which, upon Account of the Sun's Motion to the Westward, lies always to the Eastward); and then see, which Side of the Column of Air, incumbent over that Point, sustains the greater Pressure from the neighbouring Columns. Now, although the Air is rarefied even farther to the East of this Point, than to the West, yet still, if we suppose the Point to keep its place, the Air over it will sustain an equal Degree of Pressure on each Side. For, since no Column can be assigned on the western Side, but one also on the eastern, may be found under an equal Degree of Rarefaction, and therefore of the same specific Gravity: And since Fluids of equal Heights, and the same specific Gravities (whatever be the Breadth of their Columns) press equally against equal Objects, (Chap. I. §. 9.) 'tis very evident, that the Column of Air, over the

Point of greateſt Rarefaction, is preſſed equally both Ways; and ſo, upon this Suppoſition, each Wind will blow towards that Point with equal Force. But, if we ſuppoſe the Point of greateſt Rarefaction to ſhift towards the Weſt, we ſhall find, that this *Æquilibrium* will by that Means be deſtroyed, and the Motion of the Air (upon the whole) determined that Way. For let us conſider any Portion of the weſtern Air approaching towards the Point of greateſt Rarefaction, if that Point ſhifts, in the mean Time, towards the Weſt, then will that Portion reach the ſaid Point ſooner than it otherwiſe would have done; thereby loſing a Part of its Motion, by which Means the weſterly Current will be diminiſh'd. Again, if, while the Eaſt Wind blows towards the Point of greateſt Rarefaction, that Point moves on before it, then will the eaſtern Air have a greater Quantity of Motion, than it otherwiſe would have had; that, which ſhould have been an Impediment to it, being, upon this Suppoſition, in ſome Meaſure withdrawn; and ſo the Eaſt Wind will be augmented. Thus, the Weſt Wind having its Force leſſen'd by the Motion of the Sun; and the Eaſt one being increaſed, the latter at length as it were abſorbs the former, and carries it away in its own Direction.

2. On each Side of the *Equator*, to about the thirtieth Degree of Latitude, the Wind is found

found to vary from the East Point, so as to become North-east on the northern Side, and South-east on the southern.

The Reason of which is, that, as the *equatorial* Parts are hotter than any other, both the northern and southern Air, ought to have a Tendency that Way; the northern Current therefore, meeting in this Passage with the eastern, produces a North-east Wind on that Side; as the southern Current joining with the same, on the other Side the *Equator*, forms a South-east Wind there.

These two Propositions are to be understood of open Seas, and of such Parts of them as are distant from the Land; for near the Shores, where the neighbouring Air is much rarefied, by the Reflection of the Sun's Heat from the Land, it frequently happens otherwise particularly.

3. On the *Guinea* Coast, the Wind always sets in upon the Land, blowing westerly instead of easterly. This is because the Deserts of *Africa* lying near the *Equator*, and being a very sandy Soil, reflect a great Degree of Heat into the Air above them. This therefore being rendered lighter, than that which is over the Sea, the Wind continually rushes in upon the Land to restore the *Æquilibrium*.

4. That Part of the Ocean, which is called the *Rains*, is attended with perpetual Calms, the Wind scarce blowing sensibly either one

Way or other. (See its Situation described in Note, Page 119, Remark 7th). For this Tract being placed between the westerly Wind blowing from thence towards the Coast of *Guinea*; and the easterly Wind blowing from the same Place to the Westward thereof, the Air stands in *Æquilibrio* between both, and its Gravity is so much diminished thereby, that it is not able to support the Vapour it contains, but lets it fall in continual Rain, from whence this Part of the Ocean has its Name.

5. There is a Species of Winds, observable in some Places within the *Tropics*, called by the Sailors *Monsoons*, which, during six Months of the Year, blow one Way; and the remaining six, the contrary.

The Occasion of them in general is this: When the Sun approaches the northern *Tropic*, there are several Countries, as *Arabia*, *Persia*, *India*, &c. which become hotter, and reflect more Heat than the Seas beyond the *Equator*, which the Sun has left; the Winds therefore, instead of blowing from thence to the Parts under the *Equator*, blow the contrary Way; and when the Sun leaves those Countries, and draws near the other *Tropic*, the Winds turn about, and blow on the opposite Point of the Compass.

The Regularity of these Winds making them more than ordinarily useful in Navigation, they are from thence called *Trade*

Winds.

Winds. As to other Circumstances relating to them, and the particular Times and Places of the *Monsoons*, &c. See the Historical Account in the foregoing Note; all which might easily be solved upon the same Principle, had we *Data* to go upon, and were all the several Circumstances of Situation, Heat, Cold, *&c.* sufficiently known *.

From the Solution of the general *Trade* Winds, we may see the Reason, why in the *Atlantic* Ocean, a little on this Side the thirtieth Degree of North Latitude, or thereabouts, as was observed in the foregoing Dissertation, there is generally a West, or South-west Wind. For, as the inferior Air, within the Limits of those Winds, is constantly rushing towards the *Equator*, from the North-east Point, or thereabouts, the superior Air moves the contrary Way; and therefore after it has reached these Limits, and meets with Air, that has little or no Tendency to

* Some have thought, that the Regularity of the general *Trade* Winds is partly owing to the diurnal Motion of the Moon from East to West. For, as the Sun renders the Air specifically lighter by its Heat, so does the Moon by attracting it, in the same Manner, as it does the Sea, in raising the Tides. But it is to be observed, that as the Moon acts with the greatest Force upon the superior Parts of the Air, and as those superior Parts are unconfined, and therefore more at Liberty to rush in, in Order to restore the *Æquilibrium*; it will from hence follow, that the rushing in of the superior Parts of the Atmosphere will principally contribute towards restoring the *Æquilibrium*; and so the Motion, produced below, will be very inconsiderable.

any one Point more than to another, by Reason of the Sun's Distance, it will determine it to move in the same Direction with itself.

In our own Climate we frequently experience, in calm Weather, gentle Breezes blowing from the Sea to the Land, in the Heat of the Day; which *Phænomenon* is very agreeable to the Principle laid down above: For the inferior Air over the Land being rarefied by the Beams of the Sun, reflected from its Surface, more than that which impends over the Water which reflects fewer, the latter is constantly moving on to the Shore, in order to restore the *Æquilibrium*, when not disturbed by stronger Winds from another Quarter *.

From what has been observed, nothing is more easy than to see, why the northern and southern Parts of the World, beyond the Limits of the *Trade* Winds, are subject to such

* In Confirmation of this, we have an easy, and very pertinent Experiment, related by Mr. *Clare*, in his *Motion of Fluids*. " Take, *says he*, a large Dish, fill it with cold Water; into the " Middle of this put a Water-Plate, filled with warm Water. " The first will represent the Ocean; and the other an Island, " rarefying the Air above it. Blow out a Wax Candle, and if " the Place be still, on applying it successively to every Side of " the Dish, the fuliginous Particles of the Smoak, being visible " and very light, will be seen to move towards the Plate, " and rising over it, point out the Course of the Air from Sea " to Land. Again, if the ambient Water be warmed, and the " Plate filled with cold Water, let the smoaking Wick of the " Candle be held over the Plate, and the contrary will happen.

Variety

Variety of Winds. For the Air, upon Account of its Distance from the *Equator*, being undetermined to move towards any fixed Point, is continually shifting from Place to Place, in Order to restore the *Æquilibrium*, wherever it is destroyed; whether by the Heat of the Sun, the rising of Vapours, or Exhalations, the melting of Snow upon the Mountains, and a great Variety of other Circumstances, more than can be easily enumerated.

We are told by Historians, of certain Caves that emit Wind; if so, it is when the included Air is rarefied by Heat, and therefore rushes out for want of Room; or, when the Pressure of the external Air, incumbent upon the Mouth of the Cave, is diminished, and so permits the internal Air to dilate itself, and issue out.

For more on this Subject, see *Nieuwentyt*'s Religious Philosopher. *Regnault*'s Philosophical Conversations. *Clare*'s Motion of Fluids. *Martin*'s Philosophical Grammar. And the Authors referred to in Mr. *Johnson*'s Quæstiones Philosoph. Cap. IV. Quæst. 1. 2.

DIS-

DISSERTATION VI.

Of the Formation, and Ascent of Vapours, and their Resolution into Rain, Snow, *and* Hail.

THAT Vapours are raised from off the Surface of Water by the Action of the Sun's Heat, is agreed on by all: But the Manner in which this is done, has ever been a Controversy among Philosophers; neither is it at this Time sufficiently explain'd by any one.

If we consult a *Cartesian* upon this Matter, he immediately tells us, that, by the Action of the Sun upon the Water, small Particles of the Water, are formed into hollow Spheres, filled with *Materia Subtilis*, and by that Means becoming lighter than an equal Bulk of Air, are easily buoyed up in it. But, as this *Materia Subtilis* is only a Fiction, the Solution is not to be regarded.

Dr. *Nieuwentyt*, and several other Philosophers, who maintain, that Fire is a particular Substance, distinct from other Matter, account for the Formation and Ascent of Vapours thus: They say, that the Rays of the Sun, or Particles of Fire separated from them, adhering to Particles of the Water, make up

up little Bodies, lighter than an equal Bulk of Air; which therefore, by the Laws of Hydrostatics, will ascend in it, till they come to an Height where the Air is of the same specific Gravity with themselves. And, that Rain is produced by the Separation of the Particles of the Fire from those of the Water; which last, being then left without Support, can no longer be sustained by the Air, but falls down in Drops of Rain *.

This Opinion is liable to the following Difficulties; *First*, Fire has never been yet proved to be a distinct Element, or a particular Substance †; and the Change of Weight in Bodies in chymical Preparations, heretofore thought to arise from the Adhesion of Particles of Fire, is found to proceed from the Adhesion of Particles of Air §.

Secondly, Should the above-mentioned Supposition be allowed, the fiery Particles, which are joined to the watery ones to buoy them up, must be considerably large, or else a very great Number must fix upon a single Particle of Water; and then a Person, being on the Top of an Hill in a Cloud, would be sensible of the Heat, and find the Rain produced from that Vapour, much colder than the Vapour it-

* See *Nieuwentyt*'s Religious Philosopher, Contempl. 19.

† See the Authors referred to in Mr. *Johnson*'s Quæstiones Philosoph. Cap. I. Quæst. 30.

§ By Dr. *Hales*, in his *vegetable Statics*.

ſelf: whereas the contrary is evident to our Senſes; the Tops of Hills, though in the Clouds, being much colder than the Rain which falls below.

Beſides, the Manner in which the Particles of Water ſhould be ſeparated from thoſe of the Fire, ſo as to fall in Rain, is not eaſily to be conceived.

The moſt generally received Opinion is, That by the Action of the Sun, on the Surface of the Water, the aqueous Particles become formed into Bubbles, filled with a *Flatus*, or *warm Air*, which renders them ſpecifically lighter than common Air, and makes them riſe therein, till they meet with ſuch as is of the ſame ſpecific Gravity with themſelves *. But I aſk,

Firſt, How comes the Air in the Bubbles to be ſpecifically lighter than that without, ſince the Sun's Rays, which act upon the Water from whence they are raiſed, are equally denſe over all its Surface?

Secondly, If it could be poſſible for rarer Air to be ſeparated from the denſer ambient Air, to form the Bubbles (as Bubbles of ſoaped Water are blown up by warm Air from the Lungs, whilſt the ambient Air is colder and denſer) what would hinder the external Air from reducing that, which is incloſed in the Bubbles, immediately to the ſame Degree of

* Philoſophical Tranſactions, N°. 192.

Cold-

Coldneſs, and ſpecific Gravity with itſelf; (Cold being readily communicated through ſuch thin Shells of Water). By which means, the Bubbles would become ſpecifically heavier than the circumambient Air, and would no longer be ſupported therein; but fall down, almoſt as ſoon as they were formed?

Thirdly, If we ſhould grant all the reſt of the Suppoſition, yet the following Difficulty will ſtill remain. If Clouds are made up of Bubbles of Water filled with Air, why do not thoſe Bubbles always expand, when the ambient Air is rarefied, and preſſes leſs upon them than it did before; and why are they not condenſed, when the ambient Air is condenſed by the Accumulation of the ſuperior Air? But if this Condenſation and Rarefaction ſhould happen to them, the Clouds would always continue at the ſame Height, contrary to Obſervation; and we ſhould never have any Rain.

The two laſt Opinions are more largely examined by Dr. *Deſaguliers* in the Philoſophical Tranſactions N°. 407. After which he endeavours to eſtabliſh one of his own.

He obſerves, with Sir *Iſaac Newton*, that, when by Heat or Fermentation the Particles of a Body are ſeparated from their Contact, their repulſive Force grows ſtronger, and the Particles exert that Force at greater Diſtances; ſo that the ſame Body ſhall be expanded into a very large Space, by becoming fluid; and

may ſometimes take up more than a Million of Times the Room it did in a ſolid and incompreſſible State. "Thus, ſays he, if the "Particles of Water are turned into Vapour, "by repelling each other ſtrongly, and repel "Air more than they repel each other; Aggregates of ſuch Particles, made up of Va- "pour and Vacuity, may riſe in Air of different "Denſities, according to their own Denſity "depending on their Degree of Heat." He obſerves farther, that Heat acts more powerfully on Water, than on common Air; for that the ſame Degree of Heat, which rarefies Air two Thirds, will rarefy Water near fourteen thouſand Times, changing it into Steam or Vapour as it boils it. And in Winter, that ſmall Degree of Heat, which in Reſpect of our Bodies appears cold, will raiſe a Steam or Vapour from Water, at the ſame Time that it condenſes Air. Laſtly, he obſerves, That the Denſity and Rarity of this Vapour depends chiefly on its Degree of Heat, and but little on the Preſſure of the circumambient Air. From all which he infers, That the Vapour being more rarefied near the Surface of the Earth, than the Air is there by the ſame Degree of Heat, muſt neceſſarily be buoyed up into the Atmoſphere; and ſince it does not expand itſelf much, though the Preſſure of the incumbent Air grows leſs, at length it finds a Place where the Atmoſphere is of the ſame ſpecific

Gravity

Gravity with itself, and there floats, till by ſome Accident or other, it is converted again into Drops of Water, and falls down in Rain.

And to ſhew that Air is not neceſſary for the Formation of Steam or Vapour, he gives us this Experiment.

" ABCD (*Fig.* 37.) is a pretty large Veſ-
" ſel of Water, which muſt be ſet upon the
" Fire to boil. In this Veſſel muſt be ſuſpend-
" ed the Glaſs Bell E, made heavy enough
" to ſink in Water; but put in, in ſuch a Man-
" ner, that it be filled with Water when up-
" right, without any Bubbles of Air at its
" Crown within, the Crown being all under
" Water. As the Water boils, the Bell will
" by Degrees be emptied of its Water, being
" preſſed down by the Steam, which riſes
" above the Water in the Bell; but, as that
" Steam has the Appearance of Air, in Order
" to know whether it be Air or not, take the
" Veſſel off the Fire, and draw up the Bell
" by a String faſtened to its Knob at Top,
" till only the Mouth remains under Water;
" then as the Steam condenſes by the cold
" Air on the Outſide of the Bell, the Water
" will riſe up into the Bell at F, quite to the
" Top, without any Bubble above it; which
" ſhews, that the Steam, which kept out the
" Water, was not Air."

This

This Hypothesis, howevever plausible it may appear, is attended with Difficulties, as well as the other. For,

First, If the repulsive Power of the Particles of Water is sufficiently augmented by Heat as such (and that by a very small Degree of it) to make them recede from each other, and become specifically lighter than common Air; how comes it to pass, that all the Particles of Water, as soon as, or before it boils, have not their repulsive Forces thus augmented, since they are all under a much greater Degree of Heat, than is necessary to raise Vapour?

Secondly, Allowing that they may rise off from the Surface of the Water, and float in the circumambient Air, as being lighter than it, why do not their repulsive Forces, as they rise up into the Air, and the superincumbent Pressure becomes less, drive them to greater Distances from each other, and so cause them to continue lighter than the Air about them at all Heights?

Thirdly, If the Pressure of the Air about them, when near the Surface of the Earth, is not sufficient to bring them into so close Contact, as to form Drops of Water there, what Force will they find sufficient for it, when they are carried aloft into a Region of Air, where the Pressure is not near so great? The Doctor hints, that they are formed into Rain, "when "by the great Diminution of the specific Gra-
" vity

" vity of the Air about a Cloud, it has a " great Way to fall, in which Case, he says, " the Resistance of the Air, which increases " as the Square of the Velocity of the de- " scending Cloud, causes the floating Particles " of Water to come within the Power of each " other's Attraction, and form such big Drops " as, being specifically heavier than any Air, " must fall in Rain." But as the inferior Air, from the Cloud to a considerable Depth below it, cannot be supposed to leave it all at once, the Vapours must necessarily fall all the Way through a resisting *Medium*; so that the little Velocity the Cloud can acquire, while it is in the Form of Vapour, will never produce a Resistance from the Air greater than the Pressure which the Vapours sustained below

Lastly, The Experiment, brought to make Way for this Hypothesis, shows clearly, that Vapour formed without the Resistance of Air, is immediately condensed into Water again, as soon as it is suffered to cool: Which is widely different from what happens to Vapours buoyed up into the colder Regions of the Air, and floating there, till they fall in Rain.

These are all, or the principal *Hypotheses*, that have been framed for the Solution of this common *Phænomenon*: Which as they seem inadequate to the Effects produced, and therefore unsatisfactory, I thought it not amiss to sug-

suggest to the Reader the chief Difficulties, with which I conceive them to be attended. But as it is easier to pull down, than to build up, to overturn a weak *Hypothesis*, than to raise and support a strong and sufficient one; so, I must own, in this Case, I can think of no Way of accounting for the Rise of Vapours, according to the received Principles of Philosophy, or say wherein their Nature consists. Upon this Account it is impossible I should give a Philosophical Account of their Resolution into Rain. It must suffice just to mention the Causes, which are observed to produce that Change.

The first is, That Part of the Air beneath them is carried away by contrary Winds blowing from the same Place*; for then the Remainder being too light to buoy them up, the upper ones, in all Probability, precipitate down upon the lower ones, unite with them, and forms Drops of Rain †. Another is, great Quan-

* Perhaps it may be thought, that when the Air, which impends over any Place, is carried away from thence by contrary Winds, the Vapours which float in it should be carried away too; and so the few that remain should be easily supported; or, if they fall, should not produce much Rain.

Now, in Answer to this, it must be considered, that as the inferior Air is carried away from any Place by contrary Winds, the Height of the Atmosphere is diminished thereby; wherefore, in all Probability, the superior Air rushes in by a contrary Current to fill up the Vacuity at the Top, which bringing in its Contents of Vapour affords a continual Supply for the Rain that falls.

† A remarkable Instance we have of this, in that Part of the *Atlantic* Ocean, which the Sailors call the *Rains*, (see Dissert. V.) from

Quantities of them, being driven by the Winds against the Sides of Mountains, are thereby made to coalesce, and so constitute Rain*. It

from the frequent Rains that fall there: the Occasion of which is, that the Atmosphere is diminished by continual contrary Winds blowing from thence, so that it is not able to sustain the Vapour it receives.

* Upon this depends the Difference of the Seasons of the Year at *Malabar* and *Coromandel* in the *East-Indies*, and at *Jamaica* in the West. See Dr. *Gordon*'s Discourse on the Causes of the Change of Weather, Philosophical Transactions, N°. 17.—— " The Rivers of *Indus* and *Ganges*, says he, where they enter the " Ocean, contain between them a large *Chersonesus*, which is di- " vided in the Middle by a Ridge of Hills, which they call the " *Gate*, which run along from East to West, and quite through to " Cape *Comori*. On the one Side is *Malabar*, and on the other *Co-* " *romandel*. On the *Malabar* Side, between that Ridge of Moun- " tains and the Sea, it is after their Appellation Summer from *Sep-* " *tember* till *April*; in which Time it is always a clear Sky, with- " out once, or very little Raining. On the other Side the Hills, on " the Coast of *Coromandel*, it is at the same Time their Winter, " every Day and Night yielding Abundance of Rain, and from " *April* to *September* it is, on the *Malabar* Side their Winter, and " on the other Side their Summer: So that in little more than twen- " ty Leagues Journey in some Places, as where they cross the Hills " to St. *Thomas*, on the one Side of the Hill you ascend with a " fair Summer; on the other you descend with a stormy Winter. " The like is said to be at Cape *Razalgate* in *Arabia*. And Dr. " *Tropham* relates the same of *Jamaica*, intimating that there is " a Ridge of Hills which runs from East to West, through the " midst of the Island, and that the Plantations on the South Side " of the Hills have, from *November* to *April*, a continual Sum- " mer, whilst those on the North Side have as constant a " Winter, and the contrary from *April* to *November*.

" From these and such like Accounts it seems evident, that a " bare lessening of the Atmosphere's Gravity will not occasion " Rain; but that there is also needful either a sudden Change of " Winds, or a Ridge of Hills to meet the Current of the Air and " Vapours, whereby the Particles of the Vapours are driven toge- " ther, and so fall down in Drops of Rain. And hence it is,

T " that

It is generally thought, That if the Vapours in their Descent pass through a Region of Air sufficiently cold, they are there frozen into Icicles,

" that whilst the Wind blows from the North-east, *viz.* from *No-* " *vember* to *April* (see the Account of the Monsoons in the fore-going Dissertation) " there are continual Rains in the northerly " Plantations of *Jamaica*, and on the Side of *Coromandel* in " the *East-Indies*, because the Winds beat against that Side of " the Hills; and so there is fair Weather on the other Side of " these Hills, in *Malabar* and in the southern Plantations of *Ja-* " *maica*, there being no Winds to drive the Vapours together. " But in the southerly Monsoons, *viz.* from *April* to *November*, " *Malabar* and the southerly Plantations of *Jamaica* have Floods " of Rain, the Wind beating against that Side of the Hills, whilst " in *Coromandel*, and the other Side of *Jamaica*, there is fair " and clear Weather.

" This serves also to clear the *Singularity* of *Seasons* in *Peru*, be-" yond any other Parts of the Earth, and seems to be assigned by *Aco-* " *sta* as the Cause of it. *Peru* runs along from the *Line* Southwards " about a thousand Leagues. It is said to be divided into three Parts, " long and narrow, which they call *Lanos*, *Sierras*, and *Andes*; the " *Lanos*, or Plains, run along the South-Sea Coast; the *Sierras* are " all Hills with some Vallies; and the *Andes* steep and craggy Moun-" tains. The *Lanos* have some ten Leagues in Breadth, in some " Parts less, and in some more; the *Sierras* contain some twenty " Leagues in Breadth, the *Andes* as much, sometimes more, some-" times less; they run in Length from North to South, and in " Breadth from East to West. This Part of the World is said to " have these remarkable Things: 1. All along the Coast, in the " *Lanos*, it blows continually with one only Wind, which is South " and South-west, contrary to that which usually blows under the " torrid Zone. 2. It never rains, thunders, snows, or hails in " all this Coast, or *Lanos*, though there falls sometimes a small " Dew. 3. Upon the *Andes* it rains almost continually, though " it be sometimes more clear than other. 4. In the *Sierras*, " which lie between both Extreams, it rains from *September* to " *April* but in the other Seasons it is more clear, which is when " the Sun is farthest off, and the contrary when it is nearest. " Now the Reason of all seems to be this; The eastern Breezes, " which blow constantly under the *Line*, being stopp'd in their

" Course

Icicles, and from Snow. But this Opinion seems to be false; because it frequently snows when the Barometer is high, at which Time the Vapours cannot begin to fall. It is therefore more probable, that they are first frozen into Icicles, and by that means shooting forth into several Points every Way from the Center (agreeably to the Nature of Freezing) lose their Form; and so becoming specifically heavier than Air fall down, and in their Passage, several being congealed together, form Fleeces of Snow*.

Hail is evidently no other, than Drops of Rain congealed into Ice. This happens, when in their Passage through the inferior Air, they

" Course by the *Sierras* and *Andes*, and yet the same Breezes being to be found in the South Sea beyond *Peru*, as appears by the easy Voyages from *Peru* to the *Philippines*, a Current of Wind blows from the South on the Plains of *Peru*, to supply the eastern Breeze on the South-Seas, and there being but one constant Gale on these Plains, and no contrary Winds or Hills for it to beat upon, this seems to be the Reason why the Vapours are never or very seldom driven into Rain. And the *Andes* being as high perhaps in many Places as the Vapours ascend in the highest Degree of the Atmosphere's Gravity, this may probably be the Reason, why the eastern Breeze, beating constantly against these Hills, occasions Rain upon them at all Seasons of the Year. And the *Sierras* being it seems lower than the *Andes*, therefore from *September* to *April*, when the Sun is nearest, and so the Atmosphere's Gravity less, and the Vapours lower, they are driven against the *Sierras* into Rain."

The like is to be said of other Countries. They are all, *cæteris paribus*, more or less rainy, as they are more or less mountainous. *Ægypt*, which is quite without Mountains, has seldom or never any Rain; but is plentifully watered by the *Nile*, which yearly rises above its Banks, and overflows the whole Country.

* See a Discourse on the Nature of Snow, Philosophical Transactions N°. 92.

meet with *nitrous* Particles, which are known to contribute greatly to Freezing. Their Magnitude is owing to a fresh Accession of Matter, as they pass along. Hence we see the Reason why Hail is so frequent in Summer, because at that Time greater Quantities of Nitre are exhaled from the Earth, and float up and down in the Air.

If the Vapours, after they are exhaled from off the Waters, do not rise very high in the Atmosphere, but hover near the Surface of the Earth, they then constitute what we call a Fog. And, if they ascend higher, they still appear in the same Form to those, who, being upon the Tops or Sides of Mountains, are at an equal Height with them; though to those, who are below, they appear as Clouds.

If they fall to the Earth, without uniting into Drops large enough to be called Rain, they are then said to fall in Dew.

See farther on this Subject, *Derham*'s Physſ. Theolog. Book I. Chap. 3, and Book II. Chap 5. Spectacle de la Nature, Dialog, 21, and 23. *Nieuwentyt* Contempl. 19. *Clare*'s Motion of Fluids. *Regnault*, Vol. III. Conversat. 7. *Musschenbroek* Epitom. Physſ. Mathemat. Par. II. Cap. 24. *Melchior Verdries* Physic. Pars special. Cap. V. §. 8. And the Authors referred to in Mr. *Johnson*'s Questiones Philosoph. Cap. IV, Quæst. 7.

DIS-

DISSERTATION VII.

Of the Causes of Thunder and Lightning, and of the Aurora Borealis.

THOSE Philosophers, who maintain that Vapours are buoyed up into the Air, by Particles of Fire adhering to them (as explained in the foregoing Dissertation) account for the *Phænomena* of Thunder and Lightning, in the following Manner. They suppose, that from the Particles of Sulphur, Nitre, and other combustible Matter, which are exhaled from the Earth, and carried into the higher Regions of the Atmosphere, together with the ascending Vapours, is formed an inflammable Substance; which, when a sufficient Quantity of fiery Particles is separated from the Vapour by the Collision of two Clouds, or otherwise, takes Fire, and shoots out into a Train of Light, greater or less, according to the Strength and Quantity of the Materials.

This Opinion is false for the Reasons mentioned in the foregoing Dissertation, which plainly show, that it is impossible the Vapours should be attended with such fiery Particles, as is here supposed.

Neither

Neither have we Occasion to fly to such an *Hypothesis*; for, as Vapours exhaled from the Surface of the Water are carried up into the Atmosphere: in like Manner, the *Effluvia* of solid Bodies are continually ascending thither: Now we find by Experiment, that there are several inflammable Bodies, which, being mixed together in due Proportion, will kindle into Flame by Fermentation alone,* without the Help of any fiery Particles †. When therefore

* See the Theory of Fermentation explained in the following Dissertation.

† Monsieur *Lemery* having covered up in the Earth about fifty Pounds of a Mixture composed of equal Parts of Sulphur, and Filings of Iron tempered with Water; after eight or nine Hours Time, the Earth, where it was laid, vomited up Flames. Hist. de l'Acad. 1700, p. 574.

From this Experiment we see the true Cause of the Fire of *Ætna* and *Vesuvius*, and other burning Mountains. They probably are Mountains of Sulphur, and some other Matter proper to ferment with it, and take Fire. From like Causes proceeds the Heat of Bath-waters, and other hot Springs.

Mix a small Quantity of Gun-Powder with Oyl of Cloves, pour gently upon this Mixture, two or three times as much Spirit of Nitre, and you will observe a bright Inflammation suddenly arising from it. A Mixture of the two Fluids alone will take Fire; the Powder is added only to augment the Inflammation.

Take two Pounds of Nitre, or refined Salt Petre well dried and reduced to the finest Powder, with a Pound of Oyl of common Vitriol: From this Mixture may be drawn by Distillation a Spirit of Nitre capable of inflaming Oyl of Turpentine. Mem. de l'Acad. 1726, p. 97, &c. Put into a Glass an Ounce of this Spirit of Nitre, with an Ounce of Oyl of Vitriol; pour upon it an equal Quantity of Oyl of Turpentine, and a very fine Flame will arise suddenly with a great Explosion. When the Liquors are fresh the Effect is much greater. If we mix a Dram of the Spirit of Nitre and three of Oyl of Turpentine, with only one of the Spirit of

fore there happens to be a proper Mixture of the *Effluvia* of such Bodies floating in the Air, they ferment, kindle, and flashing like Gunpowder, occasion those Explosions, and Streams of Fire, which we call Thunder and Lightning.

As to the particular Species of *Effluvia*, which compose this Mixture, that cannot exactly be determined; they are thought to be chiefly sulphureous and nitrous: *sulphureous*, because of the sulphureous Smell which Lightning generally leaves behind it, and of that sultry Heat in the Air which is commonly the Fore-runner of it: *nitrous*, because we don't know of any Body so liable to a sudden and violent Explosion, as Nitre is *.

The

of Vitriol, the Mixture will take Fire immediately. If the same Experiment be made with the Balm of *Mecca*, a sudden Flame will arise, with a Noise like that of the Report of a Gun.

There are two celebrated Experiments of this Kind, though they do not come up exactly to the present Purpose, because they will not succeed, unless the Ingredients be first heated, the one of *Aurum fulminans*, and the other of *Pulvis fulminans*. The first is a Mixture of Salt of Tartar, and Gold dissolved by *Aqua Regia*. A small Quantity of this, if put into a Brass Spoon, and heated with the Flame of a Candle, causes a sudden Noise resembling that of Thunder; and goes off with great Violence. The other is a Mixture of three Parts of Nitre, two of Salt of Tartar, and one of Sulphur. This Mixture when set upon the Fire, and warmed to a certain Degree, is dissipated all on a sudden with great Thundering, like the *Aurum fulminans*.

See an Account of Exhalations taking Fire of their own Accord in Cole-Pits. *Power*'s Experimental Philosophy, p. 62. and 181.

* Dr. *Lister* is of Opinion, That the Matter both of Thunder and Lightning, and also of Earthquakes, is the *Effluvia* of the *Pyrites*,

The Effects of Thunder and Lightning are owing to the sudden and violent Agitation the Air is put into thereby, together with the Force of the Explosion *; and not to Thunderbolts falling from the Clouds; as supposed by the Vulgar †.

Pyrites; as he does, that the Matter of *Vulcano*'s is the *Pyrites* itself. This is a Mineral that emits copious Exhalations, and is exceedingly apt to take Fire upon the Admission of Moisture. See the Doctor's Defence of his Notion in the Philosophical Transactions, N° 157. He thinks this may be the Reason why *England* is so little troubled with Earthquakes, and *Italy*, and almost all Places round the *Mediterranean* Sea, so very much, *viz.* because the *Pyrites* are rarely found in *England*; and where they are, they lie very thin, in Comparison of what they do in those Countries; as the vast Quantity of Sulphur, emitted from the burning Mountains there, seems to shew.

* Lightning is said to have dissolved Silver, without burning the Purse it was in; and to have melted the Sword, without touching the Scabbard, and the like. The Occasion of this may possibly be, that the Matter of the Exhalation may be so subtile and penetrating, that, as we see it happens with *Aqua Fortis*, or *volatile Salts*, it may pass through soft Bodies without altering their Texture, while it spends its whole Force on hard ones, in which it finds the greater Resistance.

† Some are inclined to think, that Thunderbolts are artificial, and that they were applied by the Ancients to some Use. What confirms them in their Opinion, is, that they are found more frequently where Sepulchres have been, than in other Places. And, as it was the Custom of the Ancients to have their Arms buried with their Ashes, they think they might be of some Use in War. Some are of Opinion, they were used in Sacrifices. See *Melchior Verdrie*'s Physic. Pars special. Cap. V. §. 9. *Wedelius* Exercit. Med. Philol. Cont. II. Dec. I. p. 103. *Schminckius* Profess. Marpurg. Differtat. de Urnis Sepulchralibus, & Armis Lapideis, A. 1714. *Herman Nunningius* Sepulchret. Westphal. Mimigard. Gentil. p. 44. *Jo. Henr. Cobarsen* Offileg. Histor. Physic. p 44.

The

The Diſtance the Thunder is from us, may nearly be eſtimated by the Interval of Time between our ſeeing the Lightning, and hearing the Thunder. For, as the Motion of Light is ſo very quick, that the Time it takes up, in coming to us from the Cloud, is not perceptible; and as that of Sound is about a thouſand Feet in a Second; allowing a thouſand Feet for every Second, that paſſes between our ſeeing the one, and hearing the other; we have the Diſtance of the Cloud, pretty nearly, from whence the Thunder comes.

We ſometimes ſee Flaſhes of Lightning, though the Sky be clear and free from Clouds; in this Caſe they proceed from Clouds, that lie immediately below our Horizon.

Of Affinity with the *Phænomena* of Lightning are thoſe of the *Aurora Borealis*, or *Northern Lights*, which, of late Years, have very frequently appeared in our Climate *. Theſe Lights differ ſo much from each other, that to give a Deſcription of one alone, would not be

* *Phænomena* of this Kind are reported to have been very frequent in *Groenland*, *Iceland*, and *Norway*, Countries near the Pole. The only ones, that are upon Record, as having appeared in *England*, before that of *March* the 6th, $17\frac{15}{16}$, are thoſe of *January* the 30th, 1560, *October* the 7th, 1564, *November* 14th and 15th, 1574, and a ſmall one ſeen near *London* on the 9th of *Auguſt*, 1708. On *November* the 16th, 1707, a ſmall one appeared in *Ireland*. Since that of *March* the 6th, $17\frac{15}{16}$, they have been, and ſtill continue very common.

sufficient to acquaint the Reader with all the Circumstances wherewith they are attended. I shall therefore collect together such *Phænomena*, as have been most generally observed, and reduce them to the ten following Propositions, adding in the Notes, by Way of *Specimen*, a full Account of that most remarkable *Aurora*, which was seen *March* the 6th 17$\frac{15}{16}$, as it was laid before the *Royal Society* by Dr. *Halley*, at their Request *.

The

* "On *Tuesday* the 6th of *March*, in the Year 1716 (the Afternoon having been very serene and calm, and somewhat warmer than ordinary) about the Time it began to grow dark (much about seven of the Clock) not only in *London*, but in all Parts of *England*, where the Beginning of this wonderful Sight was seen; out of what seemed a dusky Cloud, in the North-east Parts of the Horizon, and scarce ten Degrees high, the Edges whereof were tinged with a reddish Yellow, like as if the Moon had been hid behind it, there arose very long luminous Rays, or Streaks perpendicular to the Horizon, some of which seemed nearly to ascend to the *Zenith*. Presently after, that reddish Cloud was swiftly propagated along the northern Horizon into the North-west, and still farther westerly; and immediately sent forth its Rays from all Parts, now here, now there, they observing no Rule or Order in their rising. Many of those Rays, seeming to concur near the *Zenith*, formed there a *Corona*, or Image, which drew the Attention of all Spectators. Some likened it to that Representation of Glory, wherewith our Painters in Churches surround the Holy Name of God. Others to those radiating *Stars*, wherewith the Breasts of *Knights* of the Order of the *Garter* are adorned. Many compared it to the *Concave* of the great *Cupola* of St. *Paul's* Church, distinguished with Streaks alternately light and obscure, and having in the Middle a Space less bright than the rest, resembling the Lanthorn. Whilst others, to express as well the Motion as Figure thereof, would have it to be like the Flame in an Oven, reverberated and rolling against the arched "Roof

The most general *Phænomena* of an *Aurora-Borealis* are these that follow.

1. In the northern Parts of the *Horizon*, there is commonly the Appearance of a very black

" Roof thereof: Some thought it liker to that tremulous Light, " which is cast against the Cieling by the Beams of the Sun, re- " flected from the Surface of the Water in a Bason, that's a little " shaken. But all agree, that this *Spectrum* lasted only a few " Minutes, and exhibited itself variously tinged with Colours, Yel- " low, Red, and a dusky Green: Nor did it keep in in the same " Place; for when first it began, it appeared a little to the " Northwards of the *Zenith*, but by Degrees declining towards " the South, the long *Striæ* of Light, which arose from all Parts " of the northern Semicircle of the *Horizon*, seemed to meet to- " gether, not much above the Head of *Castor*, or the northern " *Twin*, and there soon disappeared.

" After the first *Impetus* of the ascending Vapour was over, the " *Corona* appeared no more; but still, without any Order, as to " Time or Place, or Size, luminous *Radii*, like the former, con- " tinued to arise perpendicularly, now oftener, and again sel- " domer; now here, now there; now larger, now shorter. " Nor did they proceed as at first, out of a Cloud, but oftener " would emerge at once out of the pure Sky, which was more " than ordinary serene and still. Nor were they all of the same " Form. Most of them seemed to end in a Point upwards, like " erect Cones; others like truncate Cones, or Cylinders, so much " resembling the long Tails of Comets, that at first Sight, they " might well be taken for such. Some of those Rays would con- " tinue visible for several Minutes; when others, and those the " much greater Part, just shewed themselves, and died away. " Some seemed to have little Motion, and to stand, as it were, " fixed among the Stars, whilst others, with a very perceptible " Translation, moved from East to West under the Pole, contrary " to the Motion of the Heavens; by which Means they would " sometimes seem to run together, and at other Times to fly one " another.

" After this Sight had continued about an Hour and a half, " those Beams began to rise much fewer in Number, and not near " so high; and by Degrees, that diffused Light, which had illus-

black Cloud; but it is evident that it is no real Cloud, because the smallest Stars are visible through it. This apparent Cloud is extended sometimes farther towards the West, than

" trated the northern Parts of the Hemisphere, seemed to subside, " and settling on the Horizon, formed the Resemblance of a very " bright *Crepusculum*. That this was the State of this *Phænomenon*, " in the first Hours, is abundantly confirmed by the unanimous " Consent of several. For, by the Letters we have received from " almost all the extreme Parts of the Kingdom, there is found very " little Difference from what appeared at *London* and *Oxford*; un- " less that in the North of *England*, and in *Scotland*, the Light " seemed somewhat stronger and brighter.

" Hitherto I have related the Observations of others; as to " myself, I had no Notice of this Matter, till about nine of the " Clock: I immediately perceived toward the South and South- " west Quarter, that though the Sky was clear, yet it was tinged " with a strange Sort of Light; so that the smaller Stars were " scarce to be seen, and much as it is when the Moon of four " Days old appears after Twilight. I perceived at the same Time " a very thin Vapour to pass before us, which arose from the pre- " cise East-Part of the Horizon, ascending obliquely, so as to " leave the *Zenith* about fifteen or twenty Degrees to the North- " ward. But the Swiftness, wherewith it proceeded, was scarce to " be believed, seeming not inferior to that of Lightning; and " exhibiting, as it passed on, a Sort of momentaneous *Nubecula*, " which discover'd itself by a very diluted and feint Whiteness; " and was no sooner formed, but before the Eye could well take " it, it was gone, and left no Signs behind it. Nor was this a " single Appearance; but for several Minutes, about six or seven " Times in a Minute, the same was again and again repeated; " these Waves of Vapour regularly succeeding one another, and " at Intervals very nearly equal; all of them in their Ascent pro- " ducing a like transient *Nubecula*.

" By this Particular we were first assured; that the Vapour we " saw, became conspicuous by its own proper Light, without the " Help of the Sun's Beams; for these *Nubeculæ* did not discover " themselves in any other Part of their Passage, but only between " the South-east and South, where being opposite to the Sun, " they

than to the East; sometimes farther towards the East, then to the West; and generally takes up a Quarter of the Horizon, more or less.

2. The

" they were deepest immersed in the Cone of the Earth's Shadow; " nor were they visible before or after. Whereas the contrary " must have happened, had they borrowed their Light from the " Sun.

" On the western Side of the northern Horizon, *viz.* between " West and North-west, not much past ten of the Clock, I ob- " served the Representation of a very bright Twilight, contiguous " to the Horizon, out of which arose very long Beams of Light, " not exactly erect towards the *Vertex*, but something declining " towards the South; which, ascending by a quick and undulating " Motion to a considerable Height, vanished in a little Time; " whilst others, though at uncertain Intervals, supplied their " Place. But at the same Time, through all the rest of the nor- " thern Horizon, *viz.* from the North-west to the true East, " there did not appear any Sign of Light to arise from, or join " to, the *Horizon*; but what appeared to be an exceeding black " and dismal Cloud, seemed to hang over all that Part of it. Yet " was it no Cloud, but only the serene Sky, more than ordinary " pure and limpid, so that the bright Stars shone clearly in it, " and particularly *Canuda Cygni*, then very low in the North; " the great Blackness manifestly proceeding from the Neighbour- " hood of the Light, which was collected above it. For the " Light had now put on a Form quite different from all that we " have been describing, and had fashioned itself into the Shape of " two *Laminæ*, or Streaks, lying in a Position parallel to the *Hori-* " *zon*, whose Edges were but ill terminated. They extended them- " selves from the North by East to the North-east, and were each " about a Degree broad; the undermost about eight or nine Degrees " high, and the other about four or five Degrees over it; these " kept their Places for a long Time, and made the Sky so light, " that I believe a Man might easily have read an ordinary Print " by the Help thereof.

" Whilst I was viewing this surprizing Light, and expecting " what was farther to come, the northern End of the upper *La-* " *mina* by degrees bent downwards, and at length closed with " the End of the other that was under it, so as to shut up on the " North-Side an intermediate Space, which still continued open

" to

2. The upper Edge of this Cloud (which is sometimes within less than six Degrees of the Horizon, and sometimes forty or fifty above it)

" to the East. Not long after this, in the said included Space, I " saw a great Number of small Columns, or whitish Streaks, to " appear suddenly erect to the Horizon, and reaching from the " one *Lamina* to the other; which instantly disappearing, were " too quick for the Eye, so that I could not judge, whether they " arose from the under, or fell from the upper; by their sudden " Alterations, they made such an Appearance, as might well " enough be taken to resemble the Conflict of Men in Battle.

" And much about the same Time, there began on a sudden to " appear, low under the Pole, and very near due North, three or " four lucid *Areas*, like Clouds, discovering themselves in the " pure but very black Sky, by their yellowish Light. These, as " they broke out at once, so after they had continued a few Mi- " nutes, disappeared as quick, as if a Curtain had been drawn " over them: Nor were they of any determined Figure, but " both in Shape and Size might properly be compared to small " Clouds illuminated by the full Moon, but brighter.

" Not long after this, from above the foresaid two *Laminæ*, " there arose a very great *Pyramidal* Figure, like a Spear, sharp " at the Top, whose Sides were inclined to each other, with an " Angle of about four or five Degrees, and which seemed to reach " up to the *Zenith*, or beyond it. This was carried with an " equable, and not very slow Motion, from the North-east where " it arose, into the North-west, where it disappeared, still keep- " ing in a perpendicular Situation, or very near it; and passing " successively over all the Stars of the *little Bear*, did not efface " the smaller ones in the Tail, which are of the fifth Magni- " tude; such was the extream Rarity, and Perspicuity of the " Matter whereof it consisted.

" This single Beam was very remarkable for its Height above " all those, that, for a great while before, had preceeded it, or " that followed it.

" It being now past eleven of the Clock, and nothing new of- " fering itself to our View, but repeated *Phases* of the same Spec- " tacle; I observed, that the two *Laminæ*, or Streaks, parallel " to the *Horizon*, had now wholly disappeared; and the whole " Spectacle reduced itself to the Resemblance of a very bright

" Cre-

it) is generally terminated with a very lucid Arch, from one to four or five Degrees broad, whose Center is below the Horizon. Sometimes

" *Crepusculum* setting on the Northern *Horizon*, so as to be " brightest and highest under the Pole itself; from whence it " spread both Ways into the North-east and North-west. Un- " der this, in the Middle thereof, there appeared a very black " Space, as it were the Segment of a lesser Circle of the Sphere " cut off by the *Horizon*. It seemed to the Eye like a dark Cloud; " but was not so; for by the Telescope the small Stars appeared " through it more clearly than usual, considering how low they " were: And upon this as a *Basis*, our *Lumen Auroriforme* rest- " ed, which was no other than a Segment of a Ring, or *Zone* " of the Sphere, intercepted between two parallel lesser Circles, " cut off likewise by the *Horizon*; or the Segment of a very " broad *Iris*, but of one uniform Colour, *viz.* a Flame-Colour " inclining to Yellow, the Center thereof being about forty De- " grees below the *Horizon*. And above this there were seen " some Rudiments of a much larger Segment, with an Interval " of dark Sky between, but this was so exceeding feint and " uncertain, that I could make no proper Estimate thereof.

" I attended this *Phænomenon* till near three in the Morning, and " the Rising of the Moon: But for above two Hours together it " had no Manner of Change in its Appearance, nor Diminution, " nor Increase of Light; only sometimes, for very short Intervals, " as if new Fewel had been cast on a Fire, the Light seemed to " undulate and sparkle not unlike the rising of a vaporous Smoak " out of a great Blaze when agitated. But one Thing I assured " myself of, that the *Iris*-like Figure did by no means owe its " Origin to the Sun's Beams: For that about three in the Morn- " ing, the Sun being in the Middle between the North and East, " our *Aurora* had not followed him, but ended in that very Point " where he then was: Whereas in the true North, which the " Sun had long passed, the Light remained unchanged, and in " its full Lustre.

Appearances of this Kind have been taken Notice of both by *Pliny*, *Seneca*, and *Aristotle*; the last of which calls the vibrating Light near the *Zenith*, Δαλοί; the more steady perpendicular Streams, Δοκοί; and the *Aurora* itself, from the apparent dark Cloud just below it, Χάσμα. That *Aurora* which was observed by

times there are two or more of these Arches, one above another. In some, the Cloud is not terminated by an Arch, but by a long bright Streak

by Monsieur *Gassendi* in *Provence*, on the 21st of *September*, in the Year 1621, was very remarkable, at that Time.

He tells us, That about the End of Twilight in the Evening, when the Sky was very clear, and there was no Moon, there appeared in the North a Sort of a rising Morn, which ascending by Degrees, became intermingled with certain Streaks, as it were, or Rays perpendicular to the Horizon: And that at the same Time there appeared some small *passing* whitish Clouds between the South and the Place of the Sun's setting in Winter; and that in the Place where the Sun sets in Summer, a bright Redness seemed to arise in the Form of a *Pyramid*, which moved towards the setting of the Sun at the *Equinox*; and which was distinguished into three several *Pyramids*, which in a little Time were confounded one with another, and at last disappeared. When this Redness ceased, the northern Whiteness was risen forty Degrees and more; that is, about the Altitude of the Pole-Star, forming itself into an Arch, and taking up near sixty Degrees of the *Horizon*. After this, certain *Chevrons*, or Columns of Rays, some whiter, and some a little darker, began more plainly to be distinguished, of about two Degrees in Breadth, and perpendicularly posited; so that all that Part appeared as it were fluted. The whole Circumference immediately appeared scalloped; and then some of those Columns which were in the Middle, and that were the whitest, began as it were to leave their Places with great Impetuosity, and in less than a Quarter of a Minute, raised themselves almost to the Top, putting on the Form of *Pyramids*, which they would retain four or five Minutes. It was about nine of the Clock, when the Arch of Whiteness began to decrease or sink; at which Time certain very white Streams of Smoke began to issue out from the Columns which were remaining under the *Pyramids*, and darting upwards with very great Rapidity through the *Pyramids*, like Javelins, vanished immediately when they came to the Tops of them. This Spectacle lasted about an Hour; after which the Whiteness sunk down to about six Degrees of the *Horizon*. *Vide Abregé de Gassendi*, Tom. V. P. 245.

This

Streak of Light, lying parallel to the *Horizon*. The Limb of this luminous Arch, or parallel Streak is not always even and regular, but sinks lower in some Parts, than in others.

3. Out of this Arch proceed Streams of Light, generally perpendicular to the *Horizon*, but sometimes a little inclined to it. Most of them seem to end in a Point, like *Pyramids* or *Cones*; and often very much resemble the Tails of *Comets*. Sometimes there is no luminous Arch, nor Streak of Light; and then the Streams seem to issue out from behind the dark Cloud, being distinct from each other at their *Bases*.

4. The upper Ends of the Streams incessantly appear and vanish again, as quick as if a Curtain were drawn backwards and forwards before them; which sometimes causes such a seeming trembling in the Air, that you would

This *Phænomenon* appeared not only to *Gassendi* in *Provence*, but was seen at Places very distant from thence, as at *Tolose*, *Montauban*, *Bourdeaux*, *Grenoble*, *Dijon*, *Paris*, and *Roan*, &c. and at most other Places in *France*, and the neighbouring Countries, that lie to the Northwards of *Provence*, unless where it was intercepted by Clouds; but no where in such as lie at any great Distance to the Southwards of it.

Monsieur *Gassendi* is thought to have given the Name of *Aurora Borealis* to this *Phænomenon*; but this is observed by Monsieur *Mairan*, to be a Mistake. See Mr. *Mairan*'s Physical and Historical Treatise of the *Aurora Borealis*, in the *Memoires de l'Academie Royale des Sciences*, *Année* 1731. or an Abstract of it in Philosoph. Transact. N°. 431.

imagine the upper Part of the Heavens to be, as it were, in Convulſions *.

5. They ſometimes ſeem to meet in the *Zenith*, or more commonly to the Southward of it, about ten Degrees, more or leſs (ſometimes they deviate a little to the South-eaſt of the *Meridian*, and ſometimes to the South-weſt); and there curling round, in ſome Meaſure, imitate Flame confined under an Arch; and being frequently tinged with various Orders of Colours, exhibit a moſt beautiful Appearance, reſembling a Canopy finely painted †, of about ten or twenty Degrees in Breadth.

In many *Aurora*'s, there are Streams aſcending from thoſe Parts of the Heavens, which lie ſeveral Degrees to the Southwards of the Canopy; and in ſome, they appear to ariſe, though very rarely, almoſt as large, and numerous from the ſouthern, as from the northern Parts of the Horizon.

6. The Height of the *Aurora Borealis* is very great; for that of *March* the ſixth $17\frac{1}{1}\frac{5}{6}$ was viſible from the Weſt Side of *Ireland*, to the Confines of *Ruſſia* and *Poland* on the Eaſt, and perhaps farther; extending at leaſt over thirty Degrees of Longitude, and in La-

* See their Motions well deſcribed in the Account we have of an *Aurora* in the Philoſoph. Tranſact. N°. 395, Art. 2.

† See the various Colours of the Canopy, as well as thoſe of an *Aurora* itſelf, accurately deſcribed by Pr. *Greenwood*, in Philoſoph. Tranſact. N°. 418, Art. 1.

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titude from about the fiftieth Degree over almost all the North of *Europe*, and at all Places, exhibiting, nearly at the same Time, the same Appearances.

7. These Appearances have always been very frequent (as far as the Accounts we have of them inform us) in Countries that lie in, or near the *frigid Zone*, but very rare in those of our Latitude; they are now become very frequent with us, but always seem to proceed from the North; and are as yet unknown to the Inhabitants of the more southern Parts of our *Hemisphere*. Whether they are seen to those, who inhabit in, or near the other *frigid Zone*, is to us unknown.

8. In some, there are Trains of Light running horizontally, sometimes from the Middle to the Extremes, and sometimes from one Extreme to the other. And from these Trains often arise Streams perpendicular to the *Horizon*, and accompanying them as they pass along.

9. When all the Streaming is over, the *Aurora Borealis* commonly degenerate nto a bright Twilight in the North, and there gradually dies away.

10. It is probable, that these *Phænomena* often happen in cloudy Nights, though we are not sensible of them; for 'tis observable, that in such Nights, there is frequently more Light

than what usually proceeds from the Stars alone.

The most obvious Solution of the *Aurora Borealis*, or at least what would appear so, to such as have only attended to the Circumstances of some particular ones, and which has accordingly been affixed by several * to their Accounts of the *Aurora's* they have seen, is that it is a thin Nitro-sulphureous Vapour, raised in our Atmosphere considerably higher than the Clouds; that this Vapour by Fermentation takes Fire, and the Explosion of one Portion of it kindling the next, the Flashes succeed one another, till the whole Quantity of Vapour within their Reach is set on Fire.

* Professor *Cotes*, at the End of his Description of a *Phænomenon* of this Kind, inserted in the Philosophical Transactions N°. 365, observes, that supposing a Fund of Vapours or Exhalations at a considerable Height above us to be diffused every Way into a large and spacious Plane, parallel to the *Horizon*, that Fund of mixed Matter by Fermentation will emit Streams; and that if the Wind be still, they will ascend perpendicularly upwards; otherwise they will be inclined towards that Point of the Horizon which is opposite to that from which the Wind blows; and that they will appear, by the Rules of Perspective, in the first Case, to converge to the Spectator's *Zenith*, in the other, to some Point not far from it; and that if this Fund of Vapours be situated more to the North than the South, it will produce Streams of Light attended with such Circumstances, as then appeared: But he does not say, why the Vapours should be situated rather to the North than the South, or proceed to account for all the *Phænomena* of the *Aurora Borealis* in general from thence.

Some

Some have thought, that Vapours rarefied exceedingly by subterraneous Fire, and tinged with sulphureous Steams, might from thence be disposed to shine in the Night, and rising up to the Top of the Atmosphere, or even beyond its Limits (as we find the Vapours of Gun-powder, when heated in *Vacuo*, will shine in the Dark, and ascend to the Top of the Receiver, though exhausted) might produce those Undulations in the Air, which constitute this *Phænomenon*.

But these *Hypotheses* have been rejected, as insufficient: it having been thought impossible to account for all the Circumstances of the *Aurora* by them *.

* In the Commentaries of the Academy of Sciences at *Petersburgh*, I find a late Solution of the *Aurora Borealis* from Exhalations fermenting and taking Fire in the Atmosphere, which the Author *Chr. Maier* says, occasion the Appearance of the lucid Arch in the North, and thinks that the Streams, which seem to issue from thence, are no other than the Light of that Arch reflected to us from the under Side of some thin Clouds, that lie above it. As to its appearing in the North rather than in the South, he supposes that may be owing to the Cold of those Regions condensing the Exhalations, and thereby rendering them more liable to ferment than they are in the southern; but acknowledges ingenuously, that he has no Reason to suppose this, but its being necessary to his Solution. At the End he tells, That it was known in ancient Times as well as lately: But omits taking Notice, that it appears much oftener of late Years in our Climate than it used to do; and so avoids accounting for that Particular. *Vide Commentar. Academ. Scientiar. Imperial. Petropolitan.* Tom. I. p. 351.

Dr.

Dr. *Halley* therefore has Recourſe to the *magnetic Effluvia* of the Earth, which he ſuppoſes to perform the ſame Kind of Circulation with Regard to it, as the *Effluvia* of any particular *Terrella* * do with reſpect to that, *viz.* that they enter the Earth near the South Pole, and pervading its Pores, paſs out again at the ſame Diſtance from the northern: And thinks, they may ſometimes, by the Concourſe of ſeveral Cauſes very rarely coincident, and to us as yet unknown, be capable of producing a ſmall Degree of Light, either from the greater Denſity of the Matter, or perhaps from the greater Velocity of its Motion; after the ſame Manner, as we ſee the *Effluvia* of *Electric* Bodies emit Light in the Dark.

Monſieur de *Mairan* has given us a Phyſical and Hiſtorical Treatiſe of the *Aurora Borealis*, and endeavours to prove that it is owing to the *Zodiacal Light*, or the Atmoſphere of the Sun, ſpread on each Side of it along the *Zodiac* in the form of a *Pyramid*. This, he ſays, is ſometimes extended to ſuch a Length as to reach beyond the Orbit of our Earth, and then mixing itſelf with our Atmoſphere, and being of an *Heterogeneous* Nature, produces

* A round Magnet, ſo called fr m tl.e Reſem.blance it bears to the Earth.

the

the ſeveral Appearances, which are obſerved in the *Aurora Borealis* *.

I have juſt mentioned theſe two Solutions, becauſe they come from two very ingenious Philoſophers; though I doubt not but the Reader will agree with me, that they are much too fine ſpun to hold, and that they are no other than the ingenious *Reveries* of Perſons determined to frame an Hypotheſis at any Rate. I queſtion not, but we may find Materials for the *Aurora Borealis*, without going ſo far for them, as theſe Gentlemen have done; and in particular that we have no Occaſion to have Recourſe either to the *magnetic Effluvia* of the Earth, or the *Zodiacal* Light, the Nature of both which we are wholly unacquainted with. The Materials employed in the firſt Solution (I mean ſuch *Effluvia* as are continually exhaled from the Surface and Bowels of the Earth) if rightly conſidered, will afford a very eaſy and natural one, as I ſhall endeavour to ſhew in the following Manner.

Firſt, we are aſſured by Experiment, that there are ſome Steams (as inflammable ſulphureous ones) which are capable of ſo great a Degree of Expanſion, that they will render themſelves lighter than the Air they float in, though it be

* See his Account at large, referred to at the End of Note, Page 155.

be as rare, as it can be made by Art; for they will rise to the Top of the Receiver, when exhausted *, that is when as much Air, as is possible, is drawn out †: Such Steams therefore or Exhalations, rising out of the Earth from Mines, Vulcano's, &c. must necessarily be buoyed up towards the Top of the Atmosphere; at least, till they come to a Region, where the Air is as rare and expanded, as it can be made by the *Air Pump*, here below. This Height according to Dr. *Halley*'s Computation §, (which he found upon the Degree of the Air's Elasticity) is about forty or fifty Miles: but probably it is much greater; for the Air, which is higher than Vapours and other *Heterogeneous* Matter that is not elastic, rise to, being much purer than any we can make Experiments upon, may be indued with a much greater Degree of Elasticity, and so the Atmosphere may be considerably higher, than what he upon that Principle, computes it to be.

Secondly, These *Effluvia* being raised to the Top of the Atmosphere, or near it, and floating there, will necessarily be carried towards the poplar Parts thereof, for the following Reasons. 1. Because the superior Current

* See Philosoph. Transact. N°. 347 and 360.

† It is impossible to extract all the Air out of a Vessel, because it is by the Spring of the Air remaining in the Vessel, that the Valves of the Pump are opened at each Stroke.

§ Philosoph. Transact. N°. 181.

of

of the Air, to a great Distance from the *Equator*, is that Way *. 2. We know from Experiment, that whatever swims upon a Fluid which revolves about an *Axis*, is thereby carried towards that *Axis*. This is exactly the Case of these *Effluvia*, for they swim near the Top of the Atmosphere which continually revolves about the *Axis* of the Earth; they must therefore necessarily be carried towards the polar Parts thereof.

Thirdly, These *Effluvia* being collected together at, or near the Poles, and of an inflammable Nature, may easily be supposed to ferment, when they meet with other *heterogeneous* ones proper to produce such an Effect, and emit Streams of Fire; which Streams will naturally rise up into such Parts of the Atmosphere as are still lighter than that wherein the *Effluvia* rest, that is, directly upwards from the Center of the Earth. But, according to the Rules of *Perspective*, those Streams, though they really diverge, as *Radii* from a Center, will appear to a Spectator on the Surface of the Earth to converge towards a Point: Which Point will seem to be directly over his Head, if the Streams ascend in right Lines from the Center of the Earth: but if they deviate all one Way from that Direction, the Point will be on

* As explained in Dissertation V.

that Side the *Zenith* towards which they incline*.

To illustrate this; suppose several Strings hung down from the Ceiling of a Room, and a Candle placed upon a Table below them, the Shadows of them all will converge towards the Point, that is over the Candle. And, if they are made to incline, suppose all one Way, the Point of Convergency will remove from over the Candle, towards that Side of the Room to which the upper Ends of the Strings incline: Now if a Person had viewed them from the Place where the Candle was, and referred their Places to the Ceiling, they would have seemed to him to have converged towards the Point, where their Shadows did.

And if the Streams spread themselves as they arise, but not too much, they will nevertheless appear tapering towards the upper Ends, like *Cones* or *Pyramids*; just as the Sides of a long Walk seem to a Person that views them

* This may be made to appear in the following Manner; Let ADB (*Fig.* 38.) represent the *Concave* of the Heavens, AB the *Horizon*, C the Place of the Spectator, TV a luminous Substance sending forth the parallel Streams EG, LM, NO, *&c.* These Streams will all seem to converge towards the Point D, if that Point be taken such, that the Line DC drawn from thence to the Spectator's Eye, be parallel to the Streams. For Instance, the Stream EG will seem to rise from *e* to *g*, LM from *l* to *m*, and FH from *f* to *h*, and so of the rest. And NO will appear wholly in D the Place to which the rest seem to converge. And if the Streams are as large, or somewhat larger at the upper Ends, than at the lower, they will still appear less there, those Ends being farthest from the Spectator's Eye.

from

from one End of it, or from a distant Place, to approach each other at that which is farthest from him.

This being premised, we may now account for the several *Phænomena* of the *Aurora Borealis* before laid down. As,

1. The Blackness of the Sky, which generally appears in the northern Parts of the *Horizon*, like a dark Cloud, is occasioned by the Brightness of the luminous Matter of the *Aurora* just above it. That the Sky is clear here, is evident (as was observed before) because the smallest Stars are seen through it.

2. The lucid Arch immediately above, is the luminous Matter of the *Aurora* itself, which sometimes exhibits the Appearance of a Curve, sometimes of a strait Line, according to its Form and Situation in the Atmosphere, though generally that of a Curve: For, by the Rules of *Perspective*, when a strait Line is distended horizontally, and above the Spectator's Eye, it ought to appear bent into a Curve, whose Center is below the *Horizon* *. Sometimes it appears on one Side the North Point, more

* Thus, when a Person stands fronting a Row of Houses, and looks over the Tops of them, if they are all of an equal Height, that House which is nearest him, will seem to cut the Heavens in a Point that will be higher than where it is cut by any of the rest; and the Points where the Heavens will seem to be cut by the Tops of those, which are on the Right and Left Hand of the Spectator, will descend lower and lower, as the Houses are farther off; so that the Points, taken all together, will represent a Curve.

than on the other; ſometimes regular, ſometimes irregular, as the various Circumſtances of the Air's Motion at the Top of the Atmoſphere, and of the Situation of the flaming Matter may be.

3. The Streams of Light, iſſuing out of the lucid Arch, are Streams of Fire emitted upwards from the Matter of the *Aurora*, and ſeem, for the Reaſons already laid down, to converge towards the *Zenith* of the Spectator. Why they incline a little ſometimes from the Perpendicular, will be explained in the fifth Remark, where we account for the Situation of the Canopy. When no luminous Arch appears, it is probable, that it is intercepted by the *Horizon*, or by the Vapours which float in great Quantities therein.

4. The trembling obſerved in the upper Part of the Heavens, is owing to the Quickneſs wherewith the Flaſhes ſucceed one another, and alſo to the irregular Motions and Agitations of the ſuperior Parts of the Atmoſphere.

5. So long as the luminous Matter of the *Aurora* is all of it towards the North of us, the Streams cannot ſeem to meet in a Point at the Top, as will appear to any one that conſiders the Figure referred to in the Note (p. 164), but after it has advanced forwards, or become kindled over our Heads, then they appear to meet, and form the Canopy already deſcribed; and when it has paſſed further ſtill, they ſeem

to

to arise from all Parts; though they are much fainter on the southern than on the northern Side, so long as the main Body of the *Aurora* remains on the northern Side of the Canopy, which it rarely, if ever, passes. The Reason why the Center of the Canopy is generally a few Degrees to the South of the Spectator's *Zenith* *, is because the luminous Streams, which issue forth from the extreme Parts of the Substance of the *Aurora*, will naturally diverge a little from the middle ones; and, as those which appear to us, proceed chiefly from the southern Side (that being nearest to us) the Point of Convergency will necessarily be placed to the South of our *Zenith*, according to what was said above about the Inclination of the Strings hanging from the Cieling of a Room. If the Center of the Canopy is sometimes to the Eastward, and sometimes to the Westward of the Meridian, that depends upon the Motion of that Part of the Air, which is above the Substance of the *Aurora*, and through which the Streams pass, as they rise. This also it is that makes the Streams seem to arise sometimes a little obliquely.

* According to this Theory, the Center of the Canopy will always be near the Spectator's Zenith, where-ever he is; which I believe is the Case, for I have met with no Account where it is otherwise; and so every Spectator sees a different Canopy, just as when several Persons are viewing a Rainbow, no two Persons see the same Rainbow at the same Time.

6. The

6. The great Height of the *Aurora* is owing to the exceeding Lightneſs of the *Effluvia*, which compoſe the Subſtance of it (as explained above) and the darting of the Streams upwards, into Regions perhaps quite above the Atmoſphere, occaſions it to exhibit at very diſtant Places the ſame Appearances at the ſame Time.

7. That the *Aurora* appears near the *Pole*, and never at or near the *Equator*, is becauſe of the Tendency the Matter of it has towards the Poles, as explained above. And that it appears in Places more diſtant from the *Pole*, than it formerly did, is becauſe the *Effluvia*, which are now raiſed from the Earth, are prevented from approaching ſo near the polar Parts of the Atmoſphere, as they uſed to do; thoſe Parts being already ſtocked with others, which were formerly raiſed, and are now grown *effete* by frequent Fermentations and Exploſions.

8. The *horizontal* Trains of Light are the Subſtance of the *Aurora* juſt taking Fire, which runs from one Part to another, as in a Train of Gunpowder kindled in any one Part; and ſends up Streams perpendicularly from Places, where it meets with a greater Quantity of Matter than ordinary.

9. When the Matter of the *Aurora* is ſo far ſpent, as to emit no more Streams, it appears only as a bright ſteady Light in the North, which

which gradually dies away, for Want of fresh Fewel to support it.

10. As the Vapours, of which Clouds are formed, never rise so high, as where the Matter of the *Aurora Borealis* floats; it is not at all inconsistent with the foregoing Theory, if it is sometimes intercepted from our Sight, by the Interposition of Clouds below.

See farther on this Subject, *Aristotel.* Meteor. Lib. I. Cap. 4, 5. *Plinii* Histor. Natural. Cap. 26, 27. *Senec.* Quæst. Natural. Lib. I. *Lycost.* Prodigiorum ac Ostentorum Chronicon, passim. *Julius Obsequens* de Prodigiis, Cap. 13, 43, 88. *Gassendi* Animadvers. in *Diog. Laert.* Lib. X. p. 1157. *Cornelius Gemma* de divinis Naturæ Characterismis. *Nicephori* Histor. Ecclesiast. Lib. XII. Cap. 37. *Isid. Hispal.* Histor. Goth. Tom. I. p. 65. Bibliothec. Orientalis Clementino-Vaticana, Tom. I. p. 407. *Gregor. Tur.* passim. Mem. de Lit. de l'Acad. des Inscriptions & belles Lettres, Tom. IV. p. 431. Miscellan. *Berolin.* Tom. I. p. 137. Theatr. Comet. *Stanisl. Lubienietz,* p. 264, 348. Mem. pour servir â l'Histor. de *France,* Tom. I. p. 168. Mem. de l'Acad. Royal. de Sciences, for almost each Year since 1716. Philosoph. Transf. N°. 305, 310, 320, 347, 348, 349, 351, 352, 363, 365, 368, 376, 385, 395, 398, 399, 402, 410, 418, 431; and the Authors referred to by Mr. *Johnson,* in his Quæst. Philosoph. Cap. IV. §. 3,

DIS-

DISSERTATION VIII.

Of Fermentation.

HAving had Occaſion to mention ſome of the Effects of Fermentation, it may not be amiſs, before I put an End to theſe Diſſertations, to add a ſhort Account of the Nature of it, and to ſhew how thoſe Effects are produced by it.

Fermentation is a mutual Commotion of the conſtituent Particles of Bodies, one among another; and ariſes from an Inequality in their Attractions of Coheſion. Authors diſtinguiſh it into two Kinds; the one is that which happens when a Solid is diſſolved by a Fluid; the other is, when two Fluids, being mixed together, ferment with each other.

Thoſe Authors, who have treated of the firſt of theſe, tell us, That to cauſe a Fermentation between a Solid and a Fluid, ſeveral Circumſtances are neceſſary. Particularly Dr. *Friend* *, and *Keil* † are of Opinion,

1. That the Particles of the Solid muſt attract thoſe of the Fluid with a greater Force, than the Particles of the Fluid attract one another.

* See his Chemical Lectures.
† See his Letter to Dr. *Cockburn*, *De Legibus Attractionis*.

2. That

2. That the Pores of the Solid must not be too small to admit the Particles of the Fluid into them.

3. That the Body be of so loose a Contexture, that the Force of Impact, with which the Particles of the Fluid rush into its Pores, may be sufficient to disunite its Parts.

4. That the Elasticity of the Particles tends very much to promote, and augment the Fermentation.

Pr. *Boerhaave* makes also four Conditions requisite *.

1. That there be a due Proportion between the Size of the Particles of the Fluid, and the Pores of the Body to be dissolved.

2. That the Figure of the Particles of the Fluid have a determinate Relation to that of the Pores of the Solid.

3. That the Particles of the Fluid be sufficiently solid, that their Moment, or Force of Action may not be too weak.

4. The last Qualification, he mentions, is a fit Disposition of the Particles of the Fluid, when received into the Pores of the Solid, to make some stay there, and not immediately to pass through; but to act every Way upon the Solid, as they move towards the external Surface thereof.

* Pr. *Boerhaave's* Chemistry, by *Shaw*, p. 344.

But we have no Occasion to have Recourse to so many Suppositions; if the Particles of the Solid attract those of the Fluid with a greater Degree of Force than either those of the Fluid, or those of the Solid attract one another *, it is sufficient; and there will follow a Dissolution of the Body, as may clearly be demonstrated from the Laws of *Mechanics*, whatever the other Circumstances relating to the Figure or Magnitude of Pores, &c. may be †.

When

* This may be thought an impossible Supposition, for the Force of Attraction of Cohesion being as the Surfaces of the attracting Particles, whatever Size or Form the Particles of the Solid and Fluid are of, there cannot be a greater Quantity of Surface between every two Particles, one of which is a Particle of the Solid, and the other a Particle of the Fluid, than there is between every two Particles, which are either both of the Solid, or both of the Fluid; and therefore the Particles of the Solid cannot attract those of the Fluid with greater Force than either those of the Solid or those of the Fluid attract one another. But it is to be considered, that we are not so well acquainted with the Nature of the Attraction of Cohesion, as to determine *exactly* in what Manner, and by what Laws it acts. The Experiments made Use of for this Purpose, only shew that so long as we try them with the same Kind of Bodies, the Attraction is larger where the Contact is so. See Part I. Chap. III. But we have no Method of determining, whether the Difference of Attraction, which *various* Bodies exert upon one another, arises solely from a Difference in their Surfaces, or not.

† *Dem.* Thus, let *f, f, f,* &c. (*Fig.* 39.) represent a Series of the Particles of a Fluid, and *s, s, s,* &c. a Series of those of a solid Body, contiguous to one another: and let the pricked Lines *ff, ff,* &c. represent the Forces of Attraction between the Fluid Particles one among another, and *ss, ss,* &c, those of the solid ones among themselves; and let the black Lines *fs, sf,* &c. express those which are between the fluid and solid Particles. Now, the latter Forces being by the Supposition stronger than the former, the fluid Particles will recede

from

When a Solid is put into a Fluid, if their Particles have the above-mentioned Relation to each other, those of the Solid, being attracted with greater Force towards the Fluid, than they are the contrary Way, they will fall off from the Fluid, and enter in between the Particles of the Fluid; and for the like Reason, those of the Fluid will open to themselves a Way in between those of the Solid, and will separate them from each other. Neither will their Respective Motions cease, unless their

from each other, and suffer those of the Solid to enter in between them; and for the same Reason the solid Particles will give Way to those of the Fluid. By which means, the Distances represented by the pricked Lines becoming greater, the Attractions, which they express, will be diminished; so that the fluid Particles will enter quite in between the solid ones, and the solid ones between the fluid ones; and both of them together will constitute such a Series, as is represented in *Figure* 40, in the middle Row *s*, *f*, *s*, *f*, &c. where the solid and fluid Particles lie mixed interchangeably one with another in a right Line. Now let it be supposed, that this Series is contiguous to one which consists wholly of Fluid above it, as is expressed in the Figure, and to another below, consisting of solid Particles only. Every solid Particle in this Series will be attracted upwards with greater Force, than it is downwards; and every fluid one with greater Force downwards than it is upwards, as appears by bare Inspection of the Figure, where the black Lines, as in the former, express the stronger Attractions, and the pricked ones the weaker. And, if we suppose the Number of Particles in the Solid and in the Fluid to be nearly equal, those of the Fluid will not stop, till they have quite passed through the Solid; for they will always find a Series wholly consisting of solid Particles before them, whilst that which they leave behind, will be a Mixture of both. In like manner, the solid ones will pass quite through the fluid ones; for they will always meet with more fluid ones before them, than they leave behind within the Sphere of their own Attraction.

Quantities be very unequal, till they are diffused uniformly one among another, as we may very eaſily conceive; for till then, there will always be ſome Particles attracted with a greater Degree of Force one Way than they are another.

And if more of the Solid be added to this Fluid, the Particles of the Fluid will alſo enter into that Solid, till each is ſurrounded on all Sides with ſolid Particles, as far as its attractive Force reaches. After which the Fluid is ſaid to be *ſaturated*, and will diſſolve no more.

Again, if more of the Fluid be poured upon that Solid, the ſolid Particles will diffuſe themſelves farther into the Fluid, till each of them is encompaſſed with Particles of the Fluid, as far as its attractive Force extends; and then they will ſpread themſelves no farther.

But in either Caſe, if another Solid, or Fluid, the attractive Force of whoſe Particles differ from thoſe of the former, be added, a freſh Fermentation will begin, provided the attractive Forces between the Particles of the former Mixture, and of thoſe which are now added, have ſuch Relation to each other, as is neceſſary to produce it.

Upon this Principle it ſhould ſeem, that a Fluid ſhould always be capable of diſſolving more than an equal Quantity of a Solid; and that a Solid ſhould be capable of entring in

and

and diffusing itself through more than an equal Quantity of Fluid. The Reason why it is frequently not so, is, because it commonly happens, that the Fluid and the Solid are not of equal specific Gravities. When the Solid is heaviest, so many of its Particles will not ascend and enter into the Fluid, as would otherwise have done; and on the contrary, when the Fluid is heaviest, the Weight of its Particles will be an Impediment to their rising into, and dissolving the Solid.

We have no Occasion to distinguish Fermentation into two Kinds, with regard to its Causes; for, according to the foregoing Theory, whenever two Fluids, or a Solid and a Fluid, are put together, if the Particles of the one attract those of the other, with greater Force than either those of the one, or those of the other attract themselves, a Fermentation will *equally* ensue, the Cause being the same in both Cases.

When two Fluids, or a Solid and a Fluid, ferment with each other, if the Agitation and *intestine* Motion of their Particles be very great, or continues a long Time, and if the Substance of them be of the inflammable Kind, they will, by continually rubbing one against another, be sufficiently heated to take Fire, and burst out into Flame; as was said of the several Compositions mentioned in the foregoing Dissertation.

See

See the Authors, who have explained and defended the old Solution, referred to by Mr. *Johnson* in his Quæstiones Philosoph. Cap. III. Quæst. 5, 6, 7.

The End of the Second Part.

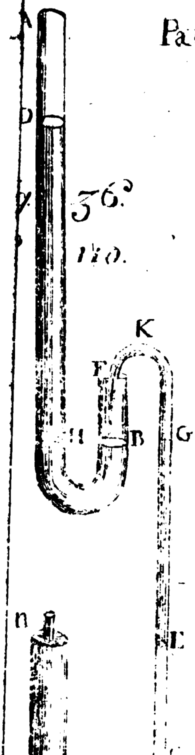


Fig. 36. P. 170.

Fig. 37. P. 135.

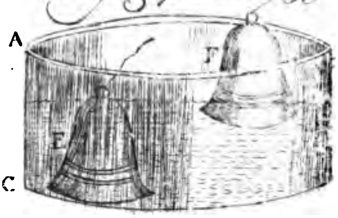


Fig. 39. P. 172.

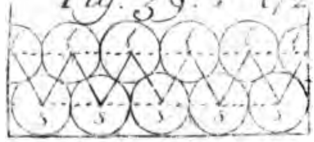


Fig. 40. P. 173.

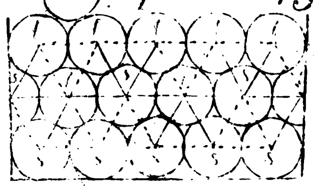
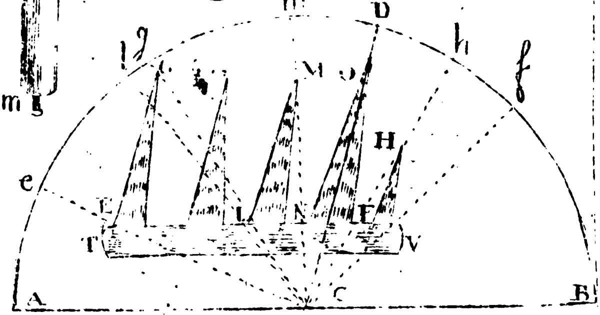


Fig. 38. P. 184.



A COMPENDIOUS SYSTEM OF Natural Philosophy.

With NOTES

Containing the

MATHEMATICAL DEMONSTRATIONS,

AND

Some Occasional REMARKS.

PART III.

OPTICS.

To which is annexed a DISSERTATION on the Subject of the Horizontal MOON.

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M.DCC.XXXVII.

A COMPENDIOUS SYSTEM OF Natural Philosophy.

PART III.

OPTICS.

CHAP. I.

Of the Nature and Propagation of Light.

IN treating of the Nature of Fluids, I have explained such *Phænomena* as result from small Particles of Matter collected together, and acted upon according to the Laws of *Mechanism*; the Order of my Design now brings me to shew how according to the same Laws such *Phænomena*, as result from the Emission of infinitely small Particles from luminous Bodies, are produced; which *Phænomena*, being the Means, whereby the Images of external Objects are represented to our Minds, by the Intervention of our Organs of

Sight, are for that Reason called *Optical*, and the Doctrine, by which they are explained, the Science of *Optics* *.

Every visible Body emits or reflects inconceivably small Particles of Matter from each Point of its Surface, which issue from it continually, (not unlike Sparks from a Coal) in strait Lines and in all Directions. These Particles entring the Eye, and striking upon the *Retina* (a Nerve expanded on the back Part of the Eye to receive their Impulses) excite in our Minds the Idea of Light. And as they differ in Substance, Density, Velocity, or Magnitude †, they produce in us the Ideas of different Colours; as will be explained in its proper Place.

That

* Optics is generally divided into two Parts, *viz.* *Dioptrics*, under which is comprehended every Thing that relates to the Appearances of Bodies seen through transparent Substances; and *Catoptrics*, or what relates to the seeing of Bodies by reflected Light. To these we may add a third, which properly comes under neither of the former Distinctions, and that is, *the Doctrine of Colours*, which explains every Thing that relates to the Causes of the Diversity of Colours observable in natural Bodies.

† 'Tis more probable, that they differ either in Magnitude, or Density, than in Velocity or Substance. For, if the Difference of Colours arise from the different Velocity of the Rays of Light, then the Colours of Objects would appear changed to an Eye placed under Water, or within any *Medium* differing from the Air in Density: For when a Ray of Light passes out of a *Medium* into another of different Density, it undergoes an Alteration in its Velocity, as will be explained hereafter. And to suppose them to differ in Substance, is contrary to that Uniformity of Things, which is observable in the Universe; as well as repugnant to that *Homogeneity* in the

prime-

That the Particles, which constitute Light, are exceedingly small, appears from hence, *viz.* that if a Hole be made through a Piece of Paper with a Needle, all the Rays of Light which proceed at the same Time, from all the Objects on one Side of it, are capable of passing through it at once without the least Confusion; for any one of those Objects may as clearly be seen through it as if no Rays passed through it from any of the rest. Further, if a Candle is lighted, and there be no Obstacle in the Way to obstruct the Progress of its Rays, it will fill all the Space within two Miles of it every Way with luminous Particles, before it has lost the least sensible Part of its Substance thereby.

That these Particles proceed from every Point of the Surface of a visible Body, and in

primogeneal Parts of Matter, which from the Experiments hitherto made, is thought to exist every where. Whereas, if we suppose them to differ either in Magnitude or Density, nothing is more easy than to see how those of the same Kind should, however refracted, produce the same Colours; and also how those which produce different Colours, should suffer different Degrees of Refraction in passing through the same *Medium*. As to the first, it is self-evident, because Refraction cannot alter their Magnitudes or Densities; as to the second, it is probable that the more intense and stronger Colours, the Rays of which suffer the least Refraction, are produced by the larger, or more dense Particles of Light: For, that such Particles should be less refracted than others, is quite consonant to the Laws of Attraction of Cohesion, which, as it acts in Proportion to the Surfaces of Bodies only, must necessarily affect the larger, or the more dense Particles, less than it does the rest; because such have larger *Momenta* or Forces in Proportion to their Surfaces, than others have.

all Directions, is clear from hence, *viz*, because where-ever a Spectator is placed with Regard to the Body, every Point of that Part of the Surface which is turned towards him, is visible to him. That they proceed from the Body in right Lines, we are assured, because just so many and no more will be intercepted in their Passage to any Place, by an interposed Object, as that Object ought to intercept, supposing them to come in such Lines.

The Velocity, with which they proceed from the Surface of the visible Body is no less surprising, than their Minuteness: The Method whereby Philosophers estimate their Swiftness, is by Observations made on the Eclipses of *Jupiter's* Satellites, which Eclipses to us appear about seven Minutes sooner than they ought to do by Calculation, when the Earth is placed between the Sun and him, that is, when we are nearest him, and as much later, when the Sun is between him and us, at which Time we are farthest from him; from whence it is concluded, that they require about seven Minutes to pass over a Space equal to the Distance between the Sun and us, which is about eighty one Millions of Miles *.

A

* This affords us another Proof of the surprizing Fineness of the Particles of Light; for the above-mentioned Velocity of the Rays is considerably more than a Million of Times greater than that of a Canon Ball. Were they not therefore inconceivably small, the Eye would be rather wounded, than delighted with them; and the

A Stream of these Particles issuing from the Surface of a visible Body in one and the same Direction, is called a *Ray of Light*.

As Rays proceed from a visible Body in all Directions, they necessarily become thinner and thinner, continually spreading themselves, as they pass along, into a larger Space, and that in Proportion to the Squares of their Distances from the Body *; that is, at the Distance of two Spaces, they are four Times thinner, than they are at one; at the Distance of three Spaces, nine Times thinner, and so on: the Reason of which is, because they spread themselves in a twofold Manner, *viz.* upwards and downwards, as well as Side-Ways.

the tender Flowers of Plants would be so far from being cherished by them, that they would be torn in Pieces, and not able to stand before them.

* This Proposition is demonstrated mathematically thus; Let us conceive two concentric Surfaces ABD, and EFG (*Fig.* 1.) and in these, two similar Portions ELFI, and AHBK; let the Rays CE and CF, with the rest proceeding from the Center C, fall upon the Portion ELFI and cover it; 'tis evident from Inspection of the Figure, that the same Rays at the Distance CH will cover the Portion AHBK only; now these Rays being the same in Number at each Place, will be thinner in the former, than they are in the latter, in Proportion as that is larger than this; but these Spaces being similar Portions of the Surfaces of Spheres, bear the same Proportion to each other, that the Surfaces themselves do, that is, they are to each other as the Squares of their *Radii* CL, CH; the Rays therefore are more diffused, or thinner in Proportion to the Squares of the same *Radii*, or of their Distances from the luminous Point C. *Q. E. D.*

The Particles of Light are subject to the Laws of Attraction of Cohesion like other small Bodies, for if a Ray of Light be made to pass by the Edge of a Knife, it will be diverted from its natural Course, and be inflected towards the Edge of the Knife. The like Inflection happens to a Ray when it enters obliquely into a denser or rarer Substance than that in which it was before, in which Case it is said to be *refracted*; the Laws of which Refraction are the Subject of the following Chapter *.

* The *Cartesian* Notion of Light, was not, that it is propagated from luminous Bodies by the Emission of small Particles, but that it was communicated to the Organ of Sight by their Pressure upon the *Materia subtilis*, with which they supposed the Universe to be full. But according to this *Hypothesis*, it could never be dark; because when a Fluid sustains any Pressure, if that Fluid fills all the Space it takes up, absolutely, without leaving any Pores, which is the Case of the supposed *Materia subtilis*; then that Pressure must necessarily be communicated *equally* and *instantaneously* to every Part: And therefore, whether the Sun were above or below the Horizon, the Pressure communicated, and consequently the Light, would be the same. And farther, as the Pressure would be instantaneous, so would the Light, which is contrary to what is collected, as we observed above, from the Eclipses of *Jupiter*'s Satellites.

CHAP.

CHAP. II.

Of the Cause of Refraction, *and the* Law *by which it is performed.*

WHatever Subſtance a Ray of Light paſſes through, or if it paſs through a Space void of all Subſtance, it is ſaid by Philoſophers to paſs through a *Medium*; and therefore if it paſſes out of any Subſtance, as Air or Glaſs, into a *Vacuum*, it is ſaid to paſs out of one *Medium* into another.

All Bodies being endued with an attractive Force, which is extended to ſome Diſtance beyond their Surfaces; when a Ray of Light paſſes out of a rarer into a denſer *Medium* (if this latter has a greater attractive Force than the former, as is commonly the Caſe *, and what we ſhall hereafter always ſuppoſe, unleſs it be mentioned to the contrary) the Ray juſt before its Entrance, will begin to be attracted towards the denſer *Medium*, and this Attraction will continue to act upon it, till ſome Time after it has entered the *Medium*, as we ſhall ſhew by and by; and therefore if a Ray approaches a denſer *Medium* in a Direction perpendicular to its Surface, its Velocity will

* In oily and inflammable Bodies it happens otherwiſe; for they are obſerved to attract more ſtrongly than others of greater Denſity.

be

be continually accelerated during its Passage through the Space in which that Attraction exerts itself; and therefore, after it has passed that Space, it will move on, till it arrives at the opposite Side of the *Medium*, with a greater Degree of Velocity than it had before it entered. So that in this Case its Velocity only will be altered. Whereas, if a Ray enters a denser *Medium* obliquely, it will not only have its Velocity augmented thereby, but its Direction will become less oblique to the Surface. Just as when a Stone is thrown downwards obliquely from a Precipice, it falls to the Surface of the Ground in a Direction nearer to a perpendicular one, than that with which it was thrown from the Hand. From hence we say a Ray of Light in passing out of a rarer into a denser *Medium*, is refracted *towards the Perpendicular*; that is, supposing a Line drawn perpendicularly to the Surface of the *Medium*, through the Point where the Ray enters, and extended both Ways, the Ray in passing through the Surface is refracted or bent towards the perpendicular Line; or, which is the same Thing, the Line which it describes by its Motion after it has passed through the Surface, makes a less Angle with the Perpendicular, than the Line it described before. All which may be illustrated in the following Manner.

Let

Let us suppose first, that the Ray passes out of a *Vacuum* into the denser *Medium* ABCD, (*Fig.* 2.) and that the attractive Force of each Particle in the *Medium* is extended from its respective Center to a Distance equal to that which is between the Lines AB and EF, or AB and GH; and let KL be the Path described by a Ray of Light in its Progress towards the denser *Medium.* This Ray, when it arrives at L will enter the attractive Forces of those Particles which lie in AB the Surface of the denser *Medium*, and will therefore cease to proceed any longer in the right Line KLM, but will be diverted from its Course by being attracted towards the Line AB, and will begin to describe the Curve LN, passing through the Surface AB in some new Direction as OQ, thereby making a less Angle with a Line as PR drawn perpendicularly through the Point N, than it would have done, had it proceeded in its first Direction KLM.

Farther, whereas we have supposed the attractive Force of each Particle to be extended through a Space equal to the Distance between AB and EF, it is evident the Ray after it has entered the Surface, will still be attracted downwards, till it has arrived at the Line EF; for till that Time, there will not be so many Particles above it which will attract it upwards, as below, that will to attract it downwards. So that after it has entered

tered the Surface at N, in the Direction OQ; it will not proceed in that Direction, but will continue to describe a Curve, as N S, after which it will proceed strait on towards the opposite Side of the *Medium*, being attracted equally every Way; and therefore will at last proceed in the Direction XST still nearer the Perpendicular PR than before.

Now if we suppose the Space ABYZ not to be a *Vacuum*, but a rarer *Medium* than the other, the Case will still be the same; but the Ray will not be so much refracted from its rectilineal Course, because the Attraction of the Particles of the upper *Medium* being in a contrary Direction to that of the Attraction of those in the lower one, the Attraction of the denser *Medium* will in some Measure be destroyed by that of the rarer.

On the contrary, when a Ray passes out of a denser into a rarer *Medium*, if its Direction be perpendicular to the Surface of the *Medium*, it will only lose somewhat of its Velocity, in passing through the Space of Attraction of that *Medium* (that is, the Space wherein it is attracted more one Way than it is another). If its Direction be oblique, it will continually recede from the Perpendicular during its Passage, and by that Means have its Obliquity encreased, just as a Stone thrown up obliquely from the Surface of the Earth increases its Obliquity all the time it rises. Thus, supposing the Ray TS passing out of the denser

denser *Medium* ABCD into the rarer ABYZ; when it arrives at S it will begin to be attracted downwards, and so will describe the Curve SNL, and then proceed in the right Line LK, making a larger Angle with the Perpendicular PR, than the Line TSX in which it proceeded during its Passage through the other *Medium*.

The Space through which the Attraction of Cohesion of the Particles of Matter is extended is so very small, that in considering the Progress of a Ray of Light out of one *Medium* into another, the Curvature it describes in passing through the Space of Attraction is generally neglected; and its Path is supposed to be bent, or in the usual Terms, the Ray is supposed to be refracted only in the Point where it enters the denser *Medium*.

Now the Line, which a Ray describes before it enters a denser or a rarer *Medium* is called the *Incident Ray*; that which it describes after it has entered, is the *Refracted Ray*.

The Angle comprehended between the Incident Ray and the Perpendicular, is the *Angle of Incidence*; and that between the refracted Ray and the Perpendicular, is the *Angle of Refraction*.

There is a certain and immutable Law or Rule, by which Refraction is always performed; and that is this; Whatever Inclination a Ray of Light has to the Surface of any *Medi-*

um before it enters it, the Degree of Refraction will always be such, that the Proportion between the Sine of the Angle of its Incidence, and that of the Angle of its Refraction, will always be the same in that *Medium* *.

To

* *Lemma.* If from a Point as M (*Fig.* 4.) taken any where without the Circle PNQ, a Line as MP be drawn passing through L the Center of the Circle, and terminated in the Circumference at P, the Product of MQ multiplied by MP is equal to the Difference between the Squares of ML and PL.

Dem. Call MQ, a; and the Radius of the Circle LQ or LP, b; then will the Diameter QP be expressible by $2b$, and the whole Line MP, by $a+2b$; then multiplying MQ by MP, that is, a by $a+2b$, we have for the Product of this, $aa+2ab$. Now the Square of the Line ML, which is expressible by $a+b$, is $aa+2ab+bb$, and the Square of PL is bb; but the Difference between these Squares, *viz.* $aa+2ab+bb$ and bb is evidently $aa+2ab$; and therefore the Product of MQ multiplied by MP is equal to the Difference between the Squares of ML and PL. *Q. E. D.*

Dem. When a Ray of Light passes through the Space of Attraction of any *Medium*, it is evident that its Motion will be subject to the like Laws with that of *Projectiles*, provided we suppose it to be acted upon with an equal Degree of Force during its whole Passage through that Space, as is the Case of *Projectiles* to whatever Height they are thrown from the Earth. We will therefore put a Case as nearly parallel as may be to that which was demonstrated of *Projectiles* in the seventh Chapter of the first Part; and suppose first, that the Force of Attraction of the denser *Medium* is at all Distances the same as far as it reaches, and that the Ray proceeds out of a denser into a rarer *Medium*; in which Case it will be attracted back towards the denser *Medium*, during its Passage through the Space of Attraction, in like Manner as a *Projectile* thrown upwards is while it rises from the Earth. Let then AB CD (*Fig.* 4.) represent the denser *Medium*, and ABEF the Space of Attraction; and let GH be a Ray about to enter the Force of Attraction at H, and let GH be produced to M. Now 'tis evident, that in this Supposition, the Ray when at H, is in the same Circumstances with a *Projectile* about to be thrown upwards from H towards M; it will therefore describe a Portion of a *Parabola*

as

To illustrate this, Let us suppose ABCD (*Fig.* 3.) to represent a rarer, and ABEF a denser *Medium*; let GH be a Ray of Light passing

as HI; to which the Line HM will be a Tangent at H; and the Line IK, in which it would proceed after it has passed the Space of Attraction, a Tangent to it at I, for after having left the attractive Force at I, it goes strait on in its last Direction. Let the Perpendicular IR be drawn meeting GH produced in M, and let KI be produced to L. On the Center L with the Radius LI, describe the Circle PNQ, let fall the Perpendicular LO upon MR, and join the Points L and N. Now it is demonstrated in the Case of Projectiles, that the *Parameter* of the Point H is equal to $\frac{HMq}{MI}$, and therefore the *Parameter* multiplied by MI is equal to HMq. And it is there farther demonstrated, that the said *Parameter* is equal to a fourth Part of the Height which a Body must fall from, to acquire the Velocity the Projectile has at H; this Parameter therefore is a Quantity not at all depending on the Direction of the Projectile, but its Velocity only; and consequently in the present Supposition it is a given Quantity, the Ray GH being supposed to have the same Velocity, whatever is its Inclination to the Surface AB. Now the Tangent KI being produced to L, will by the Property of the *Parabola*, bisect the other Tangent HM, wherefore the Line LO being parallel to HR, MR will also be bisected in O; and adding the equal Lines OI and ON to each Part, MN will be equal to IR; but the Line IR is also a Line independent of the Inclination of the Ray GH, its Length being determined by the Breadth of the Space of Attraction ABEF only, and therefore MN is a given Quantity. Now, whereas MI, when multiplied by the Parameter of the Point H, which before was shewn to be a given Line, is equal to the Square of HM, therefore the same Line MI when multiplied by any other given Line, (*viz.* MN) if it is not equal to, will nevertheless bear a given Proportion to the Square of HM: But since MI multiplied by MN bears a given Proportion (*viz.* a Proportion that does not depend on the Inclination of the Ray GH) to the Square of MH, its equal, *viz.* the Product of MQ multiplied by MP (37. El. 3.) or what is equal to this, the Difference between the Squares of ML and PL (by the foregoing *Lemma*) or, which is the same Thing, of ML and LI, (because PL and LI are *Radii* of the same Circle)

passing through the first and entering the second at H, and let HI be the refracted Ray; then supposing the Perpendicular PR drawn through the Point H, on the Center H, and with any

Circle) does so too. Now the Square of ML bears also a given Proportion to the Square of MH (ML being equal to half MH) consequently there is a given Proportion between the Square of ML and the Difference of the Squares of ML and LI; and therefore there is a certain Proportion between the Lines themselves, viz. between ML and LI. But in every Triangle the Sides are proportionable to the Sines of their opposite Angles, therefore in the Triangle MLI, the Sine of the Angle LMI has a given Proportion to the Sine of the Angle LIM, or of its Complement to two right ones MIK (for they have the same Sine): But LMI being an Angle made by the incident Ray AH produced, with the Perpendicular RM, is the Angle of Incidence, and MIK being made by the refracted Ray IK, and the same Perpendicular, is the Angle of Refraction, therefore in this Case there is a constant *Ratio* between the Sine of the Angle of Incidence, and that of the Angle of Refraction. *Q. E. D.*

We have in the foregoing Demonstration supposed that the Force of Attraction is every where uniform, but if it be otherwise, provided it be the same every where at the same Distances from the Surface AB, the Proportion between the forementioned Sines will still be a given one. For, let us imagine the Space of Attraction divided into parallel Planes, and the Attraction to be the same through the whole Breadth of each Plane though different in different Planes, the Sine of the Angle of Incidence out of each will, by what has been demonstrated above, be to the Sine of the Angle of Refraction into the next in a given *Ratio*; and therefore, since the Sine of the Angle of Refraction out of one will be the Sine of the Angle of Incidence into the next, it is evident that the Sine of the Angle of Incidence into the first will be in a given *Ratio* to the Sine of the Angle of Refraction out of the last. Now let us suppose the Thickness of these Planes diminished *in infinitum*, and their Number proportionably increased, the Law of Refraction will still continue the same; and therefore whether the Attraction be uniform or not, there will be a constant *Ratio* between the Sine of the Angle of Incidence and of Refraction. *Q. E. D.*

Radius

Radius describe the Circle APBR, and from G and I where the incident and refracted Rays cut the Circle, let fall the Lines GK and IL perpendicularly upon the Line PR, the former of these will be the Sine of the Angle of Incidence, the latter of Refraction. Now if in this Case, the Ray GH is so refracted at H, that GK is double or treble, *&c.* of IL, then whatever other Inclination the Ray GH might have had, the Sine of its Angle of Incidence would have been double, or treble, *&c.* to that of its Angle of Refraction. For Instance, had the Ray passed in the Line MH before Refraction, it would have passed in some Line, as HN afterwards, so situated that MO should have been double or treble, *&c.* of NQ.

When a Ray passes out of a *Vacuum* into Air, the Sine of the Angle of Incidence is found to be to that of Refraction, as 100036 to 100000.

When it passes out of Air into Water, as about 4 to 3.

When out of Air into Glass, as about 17 to 11.

When out of Air into a Diamond, as about 5 to 2.

CHAP. III.

Of the Refraction of Light *in passing thro' plain and spherical* Surfaces.

AS Rays of Light are capable of having their Progreſs altered by Refraction or Reflection, it is poſſible they may have various Inclinations and Directions different from thoſe which they naturally acquire by proceeding from the Surfaces of viſible Bodies.

When they recede from each other as they paſs along, they are ſaid to *diverge*; and the Point they proceed from, is called the *Radiant Point*.

When they proceed towards any Point approaching nearer together in their Progreſs, they are then ſaid to *converge*; and the Point towards which they tend, is called the *Focus*.

This *Focus* may be either *real* or *imaginary*; it is ſaid to be real, when the Rays actually proceed to it; but if they are intercepted in their Progreſs, or turned another Way before they reach it, it is called their *imaginary Focus*.

Sometimes it happens, that Rays are ſo refracted or reflected, that they proceed afterwards, as from ſome Point, which is not their true Radiant, then alſo that Point is called their *imaginary Focus*.

When

When they proceed in parallel Lines, they are then called *parallel Rays*; and both their Focus and radiant Point is supposed to be at an infinite Distance.

When Rays pass out of one *Medium* into another, they suffer various Alterations in their Motion. All which are expressed in the eighteen following Propositions.

I. When *parallel* Rays fall obliquely on a plain Surface of a *Medium* of different Density, they are *parallel* also after Refraction. For having all the same Inclination to the Surface, they suffer an equal Degree of Refraction.

II. When diverging Rays pass out of a rarer into a denser *Medium* through a plain Surface, they are made thereby to diverge less.

For being all refracted towards their respective Perpendiculars, (which Perpendiculars are parallel to one another) they are brought nearer to a Paralelism themselves, that is, they are made to diverge less than before.

See this and the following Cases expressed more determinately, and demonstrated in the Note below *.

III. When

* I. When Rays pass out of one *Medium* into another of different Density through a plain Surface, if they diverge, the focal Distance will be to that of the radiant Point; if they converge, it will be to that of the imaginary Focus of the incident Rays, as

 the

III. When they proceed out of a denser into a rarer *Medium*, the contrary happens; for then being refracted from their respective Perpendiculars, they are made to diverge more.

IV. So

the Sine of the Angle of Incidence is to that of the Angle of Refraction.

This Proposition admits of four Cases.

Case 1. Of diverging Rays passing out of a rarer into a denser *Medium*.

Dem. Let X (*Fig.* 10.) represent a rarer, and Z a denser *Medium*, separated from each other by the plain Surface AB; suppose CE and CD two diverging Rays proceeding from the Point C, the one perpendicular to the Surface, the other oblique; through E draw the Perpendicular PK. The Ray CD being perpendicular to the Surface will proceed on in the right Line CQ, but the other falling obliquely on it at E, and there entring a denser *Medium*, will suffer a Refraction towards the Perpendicular EK. Let then EG be the refracted Ray, and produce it back till it intersects DC produced also, in F; this will be the focal Point. On the Center E and with the Radius EF, describe the Circle A F B Q, and produce EC to H; draw HI the Sine of the Angle of Incidence, and GK that of Refraction; equal to this is FP or CM which let be drawn. Now if we suppose the Points D and E contiguous, or nearly so, then will the Line HE be almost coincident with FD, and therefore FD will be to CD as HE to CE; but HE is to CE as HI to CM, because the Triangles HIE and CME are similar, that is, the focal Distance of the Ray CE is to the Distance of the Radiant Point, as the Sine of the Angle of Incidence is to that of the Angle of Refraction. *Q. E. D.*

Obs. 1. *Whereas the* Ratio *of* IE *to* ME, *or which is the same Thing, that of* nD *to* CD *bears the exact Proportion of* HI *to* CM, *and because this, (being the* Ratio *of the Sine of the Angle of Incidence to that of the Angle of Refraction) is always the same, the Line* In *is in all Inclinations of the Ray* CE, *at the same Distance from* CM; *consequently had* CE *been coincident with* CD, *the Point* H *had fallen upon* n: *and because the Circle passes through both* H *and* F, F *would also have fallen upon* n; *upon which*

IV. So when converging Rays pass out of a rarer into a denser *Medium*, through a plain Surface, they are made thereby to converge less.

For

which Account the Focus of the Ray CE *would have been there. But the Ray* CE *being oblique to the Surface* DB, *the Point* H *is at some Distance from* n; *and therefore the Point* F *is necessarily so too, and the more so by how much the greater that Distance is: from whence it is clear, that no two Rays flowing from the radiant Point* C *and falling with different Obliquities on the Surface* BD, *will after Refraction there, proceed as from the same Point; therefore strictly speaking, there is no one Point in the Line* D *produced, that can more properly be called the Focus of Rays flowing from* C, *than another: for those which enter the refracting Surface near* D, *will after Refraction proceed, as has been observed, from the Parts about* n; *those which enter near* E, *will flow as from the Parts about* F; *those which enter about* T, *as from some Points in the Line* DF *produced,* &c. *And it is farther to be observed, that when the Angle* DCE *becomes large, the Line* nF *increases apace; wherefore those Rays which fall near* T, *proceed after Refraction, as from a more* diffused *Space, than those which fall at the same Distance from each other near the Point* D. *Upon which Account it is usual with* Optical *Writers to suppose the Distance between the Points where the Rays enter the plain Surface of a refracting* Medium, *to be inconsiderable with Regard to the Distance of the* radiant *Point, if they diverge; or to that of their imaginary Focus, if they converge: and unless there be some particular Reason to the contrary, they consider them, as entring the refracting* Medium *in a Direction as nearly perpendicular to its Surface as may be.*

Case 2. Of diverging Rays proceeding out of a denser into a rarer *Medium*.

Dem. Let X be the denser, Z the rarer *Medium*, FD and FE two diverging Rays proceeding from the Point F; and supposing the Perpendicular PK drawn as before, FP will be the Sine of the Angle of Incidence of the oblique Ray FE, which in this Case being refracted from the Perpendicular, will pass on in some Line as ER, which being produced back to the Circumference of the Circle will cut the Ray FD somewhere, suppose in C, this therefore

For being all refracted towards their respective Perpendiculars, they themselves are brought nearer to a Paralelism, and so converge less.

V. On

fore will be the imaginary Focus of the refracted Ray ER; draw RS the Sine of the Angle of Refraction, to which HI will be equal: but here also FP or its equal CM, is to HI, as EC to EH, or (if the Point D and E be considered as contiguous) as DC to DF; that is, the Sine of the Angle of Incidence is to the Sine of the Angle of Refraction, as the focal Distance to that of the radiant Point. *Q. E. D.*

Case 3. Of converging Rays passing out of a denser *Medium* into a rarer.

Dem. Let Z be the denser, X the rarer *Medium*, and GE the incident Ray, this will be refracted from the Perpendicular into a Line as EH; then all Things remaining as before, GK, or its equal FP, or CM will be the Sine of the Angle of Incidence, and HI that of Refraction: but these Lines, as before, are to each other, as DC to DF, that is, the focal Distance is to the Distance of the imaginary Focus, as the Sine of the Angle of Incidence to that of the Angle of Refraction. *Q. E. D.*

Case 4. Of converging Rays passing out of a rarer into a denser *Medium.*

Dem. Let Z be the rarer, X the denser *Medium*, and RE the incident Ray; this will be refracted towards the Perpendicular into a Line, as EF; C will be the imaginary Focus, and F the real one, HI which is equal to RS, the Sine of the Angle of Incidence, and FP that of the Angle of Refraction: but these are to each other, as DF to DC; and therefore the focal Distance is to that of the imaginary Focus, as the Sine of the Angle of Incidence is to that of the Angle of Refraction. *Q. E. D.*

II. When parallel Rays fall upon a spherical Surface of different Density, the focal Distance will be to the Distance of the Center of Convexity, as the Sine of the Angle of Incidence is to the Difference between that Sine and the Sine of the Angle of Refraction.

This Proposition admits of four Cases.

Case 1.

V. On the contrary, when they proceed out of a denser into a rarer *Medium*, they are refracted the contrary Way, and so made to converge more.

All

Case 1. Of parallel Rays passing out of a rarer into a denser *Medium* through a convex Surface.

Dem. Let AB (*Fig.* 11.) represent a convex Surface, C its Center of Convexity; HA and DB two parallel Rays, passing out of the rarer *Medium* X into the denser Z, the one perpendicular to the refracting Surface, the other oblique: draw CB, this being a Radius, will be perpendicular to the Surface at the Point B; and the oblique Ray DB being in this Case refracted towards the Perpendicular, will proceed in some Line, as BF, meeting the other Ray in F, which will therefore be the focal Point: produce CB to N, then will DBN, or its equal BCA be the Angle of Incidence, and FBC that of Refraction. Now, *whereas any Angle has the same Sine with its Complement to two right ones*, the Angle FCB *being the Complement of* ACB, which is equal to the Angle of Incidence, may here be taken for that Angle; and therefore, *as the Sides of a Triangle have the same Relation to each other, that the Sines of their opposite Angles have*, FB *being opposite to this Angle*, and FC *being opposite to the Angle of Refraction*, they may here be considered as the Sines of the Angles of Incidence and of Refraction; and for the same Reason CB may be considered as the Sine of the Angle CFB, which Angle *being together with the Angle* FBC, *equal to the external one* ACB (32. *El.* 1.) is itself equal to the Difference between those two last Angles; and therefore the Line FB is to CB as the Sine of the Angle of Incidence is to the Sine of an Angle which is equal to the Difference between the Angle of Incidence and of Refraction. Now, because in very small Angles as these are, *for we suppose in this Case also the Distance* AB *to vanish, the Reason of which will be shewn by and by*, their Sines bear nearly the same Proportion to each other that they themselves do, the Distance FB will be to CB as the Sine of the Angle of Incidence is to the Difference between that Sine and the Sine of the Angle of Refraction; but *because* BA *vanishes*, FB and FA are equal, and therefore FA is to CA in that Proportion. *Q. E. D.*

Obs. 2.

All which may be illustrated in the following Manner. 1. Let AB, CD, (*Fig.* 5.) be two parallel Rays falling on the plain Surface EF

Obs. 2. *It appears from the foregoing Demonstration, that the focal Distance of the oblique Ray* DB, *is such, that the Line* BF *shall be to the Line* CB *or* CA *as the Sine of the Angle of Incidence to the Sine of an Angle, which Angle is equal to the Difference between the Angle of Incidence and Refraction; therefore so long as the Angles* BCA, &c. *are small, so long the Line* CB *is pretty much of the same Length, because small Angles have nearly the same Relation to each other that their Sines have. But when the Point* B *is removed far from* A, *so that the Ray* DB *enters the Surface suppose about* O, *the Angles* BCA, &c. *becoming large, the Sine of the Angle of Incidence begins to bear a considerably less Proportion to the Sine of an Angle which is equal to the Difference between the Angle of Incidence and Refraction than before, and therefore the Line* BF *begins to bear a much less Proportion to* BC; *wherefore its Length decreases apace: Upon which Account those Rays which enter the Surface about* O, *not only meet nearer the Center of Convexity than those which enter at* A, *but are collected into a more* diffused *Space. From hence it is, that the Point where those only which enter near* A, *are collected, is reckoned the true* Focus; *and the Distance* AB *in all Demonstrations relating to the* Foci *of parallel Rays entring a spherical Surface whether convex or concave, is supposed to* vanish.

Case 2. Of parallel Rays passing out of a denser into a rarer *Medium* through a convex Surface.

Dem. Let X be the denser, Z the rarer *Medium*, AB the Surface by which they are separated, C the Center of Convexity, and HA and DB two parallel Rays, as before. Through B the Point where the oblique Ray DB enters the rarer *Medium* draw the Perpendicular CN; and let the Ray DB, being in this Case refracted from the Perpendicular, proceed in the Direction BM; produce BM back to H; this will be the imaginary Focus, and DBN, or its equal ACB will be the Angle of Incidence, and CBM, or its equal HBN (for they are vertical) that of Refraction; produce DB to L and draw BF such, that the Angle LBF may be equal to DBH: then because NBD and DBH together are equal to NBH the Angle of Refraction, therefore BCA which is equal to the first, and LBF which is equal to the second, are together equal to the Angle

EF of a *Medium* of different Density: now because they both make equal Angles of Incidence with their respective Perpendiculars GH, IK

Angle of Refraction; but LBF is equal to BFA (as being alternate to it) consequently BFA and BCA together are equal to the Angle of Refraction; and therefore since one of them, *viz.* BCA is equal to the Angle of Incidence, the other is the Difference between that Angle, and the Angle of Refraction. Now FB the Sine of the Angle FCB, or which is the same Thing, of its Complement to two right ones BCA, the Angle of Incidence is to CB the Sine of the Angle BFC, as FB to CB, that is as HB to CB; for the Angles DBH and LBF being equal, the Lines BF and BH are so too; but the Distance BA vanishing, HB is to CB, as HA to CA: that is, the Sine of the Angle of Incidence is to the Sine of an Angle which is the Difference between the Angle of Incidence and Refraction, or because the Angles are small, to the Difference between the Sine of the Angle of Incidence and that of Refraction, as the Distance of the Focus from the Surface is to that of the Center from the same. *Q. E. D.*

Case 3. Of parallel Rays passing out of a rarer into a denser *Medium* through a concave Surface.

Dem. Let X be the denser *Medium* having the concave Surface AB, and let LB and FA be the incident Rays. Now whereas, when DB was the incident Ray, and passed out of a rarer into a denser *Medium*, as in Case the first, it was refracted into the Line BF, this Ray LB having the same Inclination to the Perpendicular, will also suffer the same Degree of Refraction, and will therefore pass on afterwards in the Line FB produced, *v. g.* towards P. So that, whereas in that Case the Point F was the real Focus of the incident Ray DB, the same Point will in this be the imaginary Focus of the incident Ray LB: But it was there demonstrated, that the Distance FA is to CA, as the Sine of the Angle of Incidence is to the Difference between that and the Sine of the Angle of Refraction, therefore the focal Distance of the refracted Ray BP is to the Distance of the Center of Convexity in that Proportion. *Q. E. D.*

Case 4. Of parallel Rays passing out of a denser into a rarer *Medium* through a concave Surface.

Dem. Let X be the rarer *Medium*, having the concave Surface AB, and let LB and FA be the incident Rays, as before. Now whereas

IK, before Refraction, they will make equal Angles of Refraction with them afterwards, and so proceed on in the parallel Lines BL DM.

whereas, when DB was the incident Ray passing out of a denser into a rarer *Medium*, it was refracted into BM, as in Case the second, having a Point as H in the Line MB produced, for its imaginary Focus; therefore LB, for the like Reason as was given in the last Case, will in this be refracted into BH, having the same Point H for its real Focus. So that here also the focal Distance will be to that of the Center of Convexity, as the Sine of the Angle of Incidence is to the Difference between that and the Sine of the Angle of Refraction. *Q. E. D.*

III. When diverging or converging Rays enter into a *Medium* of different Density through a spherical Surface, the *Ratio* compounded of that which the focal Distance bears to the Distance of the radiant Point (or of the imaginary Focus of the incident Rays, if they converge); and of that, which the Distance between the same radiant Point (or imaginary Focus) and the Center, bears to the Distance between the Center and the Focus, is equal to the *Ratio*, which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction.

This Proposition admits of sixteen Cases.

Case 1. Of diverging Rays passing out of a rarer into a denser *Medium*, through a convex Surface, with such a Degree of Divergency, that they shall converge after Refraction.

Dem. Let BD (*Fig.* 12.) represent a spherical Surface, C its Center of Convexity, and let there be two diverging Rays AB and AD, proceeding from the radiant Point A, the one perpendicular to the Surface, the other oblique. Through the Center C produce the perpendicular one to F, and draw the Radius CB and produce it to K, and let BF be the refracted Ray; then will F be the focal Point: produce AB to H, and through the Point F draw the Line FG parallel to CB. AB being the incident Ray, and CK perpendicular to the Surface at the Point B, the Angle ABK, or which is equal to it, *because of the parallel Lines* CB *and* FG, FGH, is the Angle of Incidence. Now *whereas the Complement of any Angle to two right ones has the same Sine with the Angle itself*, the Sine of the Angle FGB, *that being the Complement of* FGH *to two right ones, may be considered as the Sine*

of

DM. 2. Let the diverging Rays AB, AE, AF, (*Fig. 6.*) pass out of a rarer into a denser *Medium*, through the plain Surface GH, and

of the Angle of Incidence; which Sine the Line FB, *as the Sides of a Triangle have the same Relation to each other, that the Sines of their opposite Angles have*, may be taken for. Again, the Angle FBC is the Angle of Refraction, or its equal, *because alternate to it*, BFG, to which BG *being an opposite Side*, may be looked upon as the Sine. But FB is to BG in a *Ratio* compounded of FB to BA, and of BA to BG, *for the* Ratio *that any two Quantities bear to each other, is compounded of the* Ratio, *which the first bears to any other, and of the* Ratio *which that other bears to the second.* Now FB is to BA, *supposing* BD *to vanish*, as FD to DA; and BA is to BG, *because of the parallel Lines* CB *and* FG, as AC to CF. That is, the *Ratio* compounded of FD, *the focal Distance*, to DA, *the Distance of the radiant Point*, and of AC, *the Distance between the radiant Point and the Center*, to CF, *the Distance between the Center and the Focus*, is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

Obs. 3. *Whereas the focal Distance of the oblique Ray* AB *is such, that the compound* Ratio *of* FB *to* BA *and of* AC *to* CF *shall be the same, whatever be the Distance between* B *and* D; *it is evident, that since* AC *is always of the same Length, the more the Line* AB *lengthens, the more* FB *must lengthen too, or else* FC *must shorten; but it appears by Inspection of the Figure, that if* BF *lengthens,* CF *will do so too, and in a greater Proportion with respect to its own Length than* BF *will, therefore the Lengthning of* BF *will conduce nothing towards preserving the Equality of the Proportion: but as* AB *lengthens,* BF *and* CF *must both shorten, which is the only possible Way wherein the Proportion may be continued the same. And it is also apparent, that the farther* B *moves from* D, *towards* O, *the faster* AB *lengthens, and therefore the farther the Rays enter from* D, *the nearer to the refracting Surface is the Place where they meet, but the Space they are collected in, is the more* diffused: *And therefore in this Case, as well as those taken Notice of in the two foregoing Observations, different Rays, though flowing from the same Point, shall constitute different* Focus's; *and none are so* effectual *as those which enter at or very near the Point* D. *And since the same is ob-*

 servable

and let the Ray AB be perpendicular to that Surface; the rest being refracted towards their respective Perpendiculars EK, FM, will proceed

servable of converging as well as of diverging Rays, none except those which enter very near that Point, are usually taken into Consideration; upon which Account it is, that the Distance DB, *in determining the focal Distances of diverging or converging Rays entring a convex or concave Surface, is supposed to* vanish.

Those who would see a Method of determining the precise Point, which the Ray AB, whether it be parallel, converging, or diverging to the Ray AF, converges to or diverges from after Refraction at B or any other given Point in the Surface DO, may find it in the Appendix to *Molineux*'s Optics, which for the Sake of those who have not that Book, I shall subjoin at the End of this Note.

Case 2. Of converging Rays passing out of a rarer into a denser *Medium* through a concave Surface with such a Degree of Convergency, that they shall diverge after Refraction.

Dem. Let the incident Rays be HB and FD passing out of a rarer into a denser *Medium* through the concave Surface BD, and tending towards the Point A, from whence the diverging Rays flowed in the other Case; then the oblique Ray HB having its Angle of Incidence HBC equal to ABK the Angle of Incidence in the former Case, will be refracted into the Line BL such, that its refracted Angle KBL will be equal to FBC the Angle of Refraction in the former Case; that is, it will proceed after Refraction in the Line FB produced, having the same focal Distance FD with the diverging Rays AB, AD, in the other Case. But, by what has been already demonstrated, the *Ratio* compounded of FD, *the focal Distance,* to DA, *in this Case, the Distance of the imaginary Focus of the incident Rays,* and of AC, *the Distance between the same imaginary Focus and the Center to* CF, *the Distance between the Center and the Focus,* is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

Case 3. Of diverging Rays passing out of a rarer into a denser *Medium* through a convex Surface with such a Degree of Divergency as to continue diverging.

Dem. Let AB, AD (*Fig.* 13.) be the diverging Rays, and let their Divergency be so great, that the refracted Ray BL shall also diverge from the other; produce LB back to F which will be the focal Point; draw the Radius CB and produce it to K; produce

BA

ceed in the Directions EN and FO; diverging in a less Degree from the Ray AP, than they did before Refraction. 3. Had they proceeded

BA likewise towards G, and draw FG parallel to BC. Then will ABK be the Angle of Incidence, whose Sine BF may be taken for, as being opposite to the Angle BGF, which is the Complement of the other to two right ones. And LBC is the Angle of Refraction, or its equal KBF, or which is equal to this, BFG, as being alternate; therefore BG the opposite Side to this may be taken for the Sine of the Angle of Refraction. But BF is to BG, for the like Reason as was given in Case the first, in a *Ratio* compounded of BF to BA, and of BA to BG. Now BF is to BA (DB vanishing) as DF to DA, and because of the parallel Lines FG and BC, the Triangles CBA and AGF are similar, therefore BA is to AG as CA to AF, consequently BA is to BA together with AG, that is, to BG, as CA is to CA together with AF, that is, to CF. Therefore the *Ratio* compounded of DF the focal Distance to DA the Distance of the radiant Point, and of CA the Distance between the radiant Point and the Center, to CF the Distance between the Center and the Focus, is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

Case 4. Of converging Rays passing out of a rarer into a denser *Medium* through a concave Surface in such Manner that they shall continue converging.

Dem. Let HB and CD be the incident Rays passing out of the rarer into the denser *Medium* through the concave Surface BD, and tending towards A the same Point from whence the diverging Rays flowed in the last Case. Then because the Ray HB has the same Inclination to the Perpendicular CK that AB had before, it will suffer the same Degree of Refraction, and pass on in the Line LB produced, having its Focus F at the same Distance from the refracting Surface as that of the diverging Ray AB in the other Case. Therefore, &c. *Q. E. D.*

Case 5. Of diverging Rays passing out of a denser into a rarer *Medium* through a convex Surface.

Dem. Let AB, AD (*Fig.* 14.) be the incident Rays passing out of a denser into a rarer *Medium* through the convex Surface BD whose Center of Convexity is C; and let BL be the refracted Ray, which produce back to F, and draw FG parallel to CB. Here ABK is the Angle of Incidence, to which its alternate one

ceeded out of a denser into a rarer *Medium*, they would have been refracted from their Perpendiculars EK, FM, and therefore have diverged

verged

FGB being equal, FB the opposite Side may be considered as the Sine of it. The Angle of Refraction is LBC or FBK, of which BFG being the Complement to two right ones, BG the opposite Side may be looked upon as its Sine. But BF is to BG, in the compound *Ratio* of BF to BA and of BA to BG for the Reason given above. Now (BD vanishing) BF is to BA as DF to DA, and BA is to BG as CA to CF. That is, the *Ratio* compounded of the focal Distance to the Distance of the radiant Point, &c. *Q. E. D.*

Case 6. Of converging Rays passing out of a denser into a rarer *Medium* through a concave Surface.

Dem. Let HB, CD be the incident Rays tending towards the Point A which was the Radiant in the last Case. Then, for the Reasons already given, the oblique Ray will suffer such a Degree of Refraction, as to have its Focus F at the same Distance from the Surface, as the diverging Rays AB, AD had in that Case. Therefore, &c. *Q. E. D.*

When the *Mediums* through which Rays pass, and the refracting Surfaces are such, that Rays flowing from A (*Fig.* 12.) are collected in F, then Rays flowing from F through the same *Mediums* the contrary Way, will be collected in A. For when Rays pass out of one *Medium* into another, the Sine of the Angle of Incidence bears the same Proportion to the Sine of the Angle of Refraction, as the Sine of the Angle of Refraction does to the Sine of the Angle of Incidence, when they pass the contrary Way. This is applicable to each of the six following Cases compared respectively with the six foregoing; therefore they may be considered as the Converse of them; or they may be demonstrated independently of them, as follows.

Case. 7. Of diverging Rays passing out of a denser into a rarer *Medium* through a concave Surface, so as to converge afterwards.

Dem. Let AB, AD (*Fig.* 15.) be two diverging Rays passing through the concave Surface BD into a rarer *Medium*. Let C be the Center of Convexity, and BF the refracted Ray. Draw CB and produce it to K, and draw FG parallel to it meeting AB produced in G. Then will ABC be the Angle of Incidence, of which FB being opposite to its alternate and equal Angle FGB, may be considered as the Sine. The Angle of Refraction is FBK, of which

GB

verged more than before. 4. Let the converging Rays AB, CD, EF (*Fig.* 7.) pass out of a rarer into a denser *Medium*, through the

GB being opposite to its Complement to two right ones GFB, may be taken for the Sine. Now FB is to BG, in a *Ratio* compounded of FB to BA, and of BA to BG. But (BD vanishing) FB is to BA as FD to DA, and because of the parallel Lines CB and FG, BA is to BG as CA to CF. Therefore the focal Distance, &c. *Q. E. D.*

Case 8. Of converging Rays passing out of a denser into a rarer *Medium* through a convex Surface, so as to diverge afterwards.

Dem. Let GB and FD be the incident Rays tending towards A, and produce FB to L. Then as AB in the last Case was refracted into BF, GB will in this be refracted into BL, for the Reasons already given, having F for its focal Point. Therefore, &c. *Q. E. D.*

Case 9. Of diverging Rays passing out of a denser into a rarer *Medium* through a concave Surface, in such Manner as to continue diverging.

Dem. Let AB, AD (*Fig*. 16.) be two Rays passing out of a denser into a rarer *Medium*, through the concave Surface DB whose Center of Convexity is C. Draw CB, produce it to K, and let BL be the refracted Ray, produce BL back to F, and draw FG parallel to CB meeting BA produced in G. Then will ABC be the Angle of Incidence, of which FB being opposite to its alternate and equal Angle FGB, may be considered as the Sine. The refracted Angle is LBK, or its equal CBF, of which BG being opposite to its Complement to two right ones BFG, is the Sine. Now BF is to BG in the compound *Ratio* of BF to BA and of BA to BG: but BF is to BA as DF to DA; and because of the parallel Lines CB and FG, the Triangles BCA, AGF are similar, therefore BA is to AG as CA to AF, and consequently BA is to BG as CA to CF. Therefore, &c. *Q. E. D.*

Case 10. Of converging Rays passing out of a denser into a rarer *Medium* through a convex Surface, in such Manner as to continue converging.

Dem. Let HB, MD be the incident Rays tending towards the Point A. Then will the oblique Ray HB for the Reasons already given be refracted into BF. Therefore, &c. *Q. E. D.*

Case 11. Of diverging Rays passing out of a rarer into a denser *Medium* through a concave Surface.

Dem.

the plain Surface G H, and let the Ray A B be perpendicular to that Surface, then the other Rays being refracted towards their respective

Dem. Let AB, AD (*Fig.* 17.) be the incident Rays passing out of a rarer into a denser *Medium*, through the concave Surface BD whose Center of Convexity is C, and supposing the Line CB drawn and produced to K, the refracted Ray BL drawn and produced back to F, and also FG drawn parallel to CB, ABC will be the Angle of Incidence, of which FB being opposite to its Complement to two right ones BGF, is the Sine. The Angle of Refraction will be LBK or its equal FBC, of which BG being opposite to its equal and alternate one BFG, is the Sine. Now FB is to BG in the compound *Ratio* of FB to BA and of BA to BG. But (BD vanishing) FB is to BA as FD to DA, and because of the parallel Lines FG and CB, BA is to BG as CA to CF. Therefore, &c. *Q. E. D.*

Case 12. Of converging Rays passing out of a rarer into a denser *Medium* through a convex Surface.

Dem. Let HB, MD be the incident Rays tending towards A the radiant Point in the last Case; then, as was explained above, BF will be the refracted Ray. Therefore, &c. *Q. E. D.*

Case 13. Of Rays passing out of a denser into a rarer *Medium* from a Point between the Center of Convexity and the Surface.

Dem. Let AB, AD (*Fig.* 18.) be two Rays passing out of a denser into a rarer *Medium* from the Point A, which let be posited between C the Center of Convexity and the refracting Surface BD; through B draw CK, and let BL be the refracted Ray; produce BL back to F and draw FG parallel to BC. Then will ABC be the Angle of Incidence, of which BF being opposite to its Complement to two right ones BGF, is the Sine. LBK will be the Angle of Refraction, or its equal FBC, of which BG being opposite to its alternate and equal one BFG, is the Sine. But, as before, BF is to BG in a compound *Ratio* of BF to BA and of BA to BG; and (BD vanishing) BF is to BA as DF to DA, and because the Lines CB and FG are parallel, BA is to BG as CA to CF. Therefore, &c. *Q. E. D.*

Case 14. Of Rays passing out of a denser into a rarer *Medium* towards a Point between the Center of Convexity and the Surface.

Dem. Let the incident Rays be MD, HB tending towards A from whence the other proceeded in the last Case. Then as in that

respective Perpendiculars DK, FM, will proceed in the Directions DN, FN converging in a less Degree towards the Ray AN, than they did

that Case the refracted Ray BL being produced back passed through F, in this the refracted Ray itself, for the like Reasons as were given in the foregoing Cases, will pass through that Point. Therefore, &c. *Q. E. D.*

Case 15. Of Rays passing out of a rarer into a denser *Medium* from a Point between the Center of Convexity and the Surface.

Dem. Let AB, AD (*Fig.* 19.) be two diverging Rays passing out of a rarer into a denser *Medium* through the refracting Surface BD, whose Center of Convexity is C, a Point beyond that from whence the Rays flow. Through B draw CK, and let BL be the refracted Ray, produce it back to F, and draw FG parallel to BC meeting BA produced in G. ABC will be the Angle of Incidence, of which BF being opposite to its alternate and equal Angle BGF, is the Sine. The Angle of Refraction is LBK or its equal FBC of which BG being opposite to its Complement to two right ones BFG, is the Sine. But BF is to BG in the compound *Ratio* of BF to BA and of BA to BG; and (BD vanishing) BF is to BA as DF to DA, and because of the parallel Lines CB and GF, the Triangles AFG and ABC are similar, BA therefore is to AG as CA to AF, consequently BA is to BA and AG together, that is, to BG, as CA is to CA and AF together, that is, to CF; and therefore the focal Distance, &c. *Q. E. D.*

Case 16. Of Rays passing out of a rarer into a denser *Medium* towards a Point between the Center of Convexity and the Surface.

Dem. Let HB, MD be the incident Rays having for their imaginary Focus the Point A which was the Radiant in the last Case, and let C the Center of Convexity of the refracting Surface be posited beyond this Point. Then will HB, for the Reasons already given, be refracted into BF, having the Point F for its real Focus which was the imaginary one of the diverging Rays AB, AD in the former Case. Therefore the *Ratio* compounded of that which the focal Distance bears to the Distance of the imaginary Focus of the incident Rays, and of that which the Distance between the same imaginary Focus and the Center, bears to the Distance between the Center and the Focus, is equal to the *Ratio* which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

The

did before. 5. Lastly, had the first *Medium* been the denser, they would have been refracted the other Way, and therefore have converged more.

VI. When

The first Term in the foregoing Proportion (*viz.* that in Proposition the 3d of this Note) being always an unknown Quantity; those who are not well versed in the Use of such Propositions, may think it impossible to investigate the focal Distance of any refracting Surface by it: I shall therefore exemplify it in the following Instance, by which the Manner of doing it in all others will clearly be understood. *v. g.* Let it be required to determine the focal Distance of diverging Rays passing out of Air into Glass through a convex Surface, and let the Distance of the radiant Point be 20, and the *Radius* of Convexity be 5: Now because we must make Use of the focal Distance before we know it, let that be expressed by some Symbol or Character as x: Then, because by the aforesaid Proposition the *Ratio* compounded of that which the focal Distance bears to the Distance of the radiant Point (that is in this Supposition, of x to 20); and of the *Ratio* which the Distance of the same radiant Point from the Center bears to the Distance between the Center and the Focus (in this Case, of 25 to $x-5$) is equal to the *Ratio* which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction (that is, of 17 to 11), we shall have in the Instance before us, the following Proportion; *viz.*
$\left.\begin{matrix} x : 20 \\ 25 : x-5 \end{matrix}\right\} :: 17 : 11$, and compounding them into one, which is done by multiplying the two first Parts together, we have $25x : 20x-100 :: 17 : 11$, and multiplying the extream Terms and middle Terms together, $340x-1700=275x$, which Equation after due Reduction gives $x=\frac{1700}{65}$.

In some Cases which might have been put, the Quantity 65 would have been negative, and then the Quotient arising from 1700 divided by that, would have been so too; that is x the focal Distance would have been negative, in which Case the *Focus* must have been taken on the contrary Side the Surface to that on which it was supposed to fall in stating the Problem; that is, it must have been taken on the same Side with the radiant Point, for in calling the Distance between the Center and the *Focus* $x-5$ it was supposed the *Focus* would fall on the same Side with the Center; or on that which is opposite to the radiant Point, because otherwise

that

VI. When Rays proceed out of a rarer into a denser *Medium*, through a convex Surface, if they are parallel before Refraction, they become converging afterwards.

For

that Distance must have been expressed by $x+5$, as any one may see by Inspection of the 13th or 14th Figure, in which the *Focus* of diverging Rays entring a convex Surface, is supposed to fall on the same Side with the radiant Point.

In like Manner as this Problem was performed, a general Theorem may be raised to solve it in all Cases whatsoever, by using Characters instead of Figures; as every one who is not unacquainted with algebraic Operations very well knows.

See this done, and applied to the Passage of Rays through the Surfaces of Lenses in the second Note to the following Chapter.

A Method of determining the Point which a Ray, entring a spherical Surface at any given Distance from the Vertex of it, converges to or diverges from after Refraction at the same. *From the Appendix to* Molineux's *Dioptrics.*

" *Prop.* To find the *Focus* of any Parcel of Rays *diverging from* " or *converging to* a given Point in the *Axis* of a *spherical Lens* " [Surface] and inclined thereto under the same Angle; the " *Ratio* of the Sines in Refraction being known.

" Let GL (*Fig.* 20.) be the *Lens*, P any Point in its Surface, " V the Pole [Vertex] thereof, C the Center of the Sphere " whereof it is a Segment, O the Object or Point in the Axis to " or from which the Rays do proceed, OP a given Ray; and let " the *Ratio* of Refraction be as r to s; make CR to CO as s to r " for the Immersion of a Ray, or as r to s for the Emersion, (that " is, as the Sines of the Angles in the *Medium* which the Ray enters, to their corresponding Sines in the *Medium* out of which " it comes) and laying CR from C towards O, the Point R shall " be the same for all the Rays of the Point O. Then draw the " Radius PC if Need be continued, and with the Center R and " Distance OP sweep a Touch of an Arch intersecting PC in Q; " the Line QR being drawn shall be parallel to the refracted Ray, " and PF being made parallel thereto, shall intersect the Axis in " the Point F, which is the *Focus* sought. Or make it as CQ : " CP : : CR : CF, and CF shall be the Distance of the *Focus* " from the Center of the Sphere.

E " *Demi*

For in this Case, the Perpendiculars at the Points where the Rays enter the Surface, are all drawn from the Center of Convexity on the

" *Dem.* Pet fall the Perpendiculars PX on the Axis, CY on the " given Ray, and CZ on the refracted Ray. By the Construction " PF and QR are parallel, whence the Triangles QRC and PFC " are similar, and CR to QR, as CF to PF, that is CR to OP as " CF to PF. Now CF : PF : : CZ : PX *ob similia Triang.* " whence CR : OP : : CZ : PX, and CR : CZ : : OP : PX. Again, " CR is to CO as the Sines of Refraction by Construction, that " is, as s to r, or r to s; and as CR to CZ, so (CO=) $\frac{r}{s}$ or " $\frac{s}{r}$ CR to $\frac{r}{s}$ or $\frac{s}{r}$ CZ, and so is PO to PX: But as PO to PX, " so CO to CY. *Ergo* CY=$\frac{r}{s}$ or $\frac{s}{r}$CZ, that is, CY to CZ is as " the Sines of Refraction, but CY is the Sine of the Angle of In- " cidence, and CZ of the refracted Angle. *Ergo constat Proposi-* " *tio.* ———

" Hitherto we have considered only *oblique Rays*; it now re- " mains to add something concerning *Rays parallel* to the *Axis*: " In this Case the Point O must be considered as infinitely distant, " and consequently OP, OC, and CR are all infinite; and OP " and OC are in this Case to be accounted as always equal, (since " they differ but by a Part of the *Radius* of the Sphere GPVL, " which is no Part of either of them) wherefore the *Ratio* of CR " to OP will be always the same, *viz.* as s to r for immerging " Rays, and as r to s for those that emerge. And by this Propo- " sition CF is to PF in the same *Ratio.* It remains therefore to " shew on the Base CP to find all the Triangles CPF, wherein " CF is to PF in the *Ratio* given by the Degree of Refraction. " This Problem has been very fully considered by the celebrated " Dr. *Wallis* in his late Treatise of Algebra, p. 258, to which I " refer; but I must here repeat the Construction thereof. See *Fig.* " 21 and 22.

" Let GPVL be a *Lens*, VC or PC the *Radius* of its Sphere, and " let it be required to find all the Points f, f, such, as Cf may be " to Pf in the given *Ratio* of s to r for immerging Rays, or as " r to s for the emerging. Divide CV in K, and continue CV to " F, that CK may be to VK, and CF to VF in the proposed *Ra-* " *tio.* Then divide KF equally in the Point a, and with that

" Center

the other Side; and therefore, as the Rays are refracted towards those Perpendiculars, they are necessarily refracted towards each other, and thereby made to converge.

Let it be remembered, that in the foregoing Case and in all others where Rays are said to pass through a convex or concave Surface, the Surface is supposed to belong to that Medium, *into which the Rays enter, not to that out of which they pass. Thus, when Rays pass out of a denser* Medium *that has a convex Surface into a* Vacuum, *they are said to pass through a concave Surface, the Surface of the* Vacuum *into which they enter in that Case, being such.*

VII. If they enter diverging, then for the same Reason, they are made to diverge less, to be parallel, or to converge, according to

" Center sweep the Circle FKF; this Circle being drawn gives " readily all the *Foci* of the parallel Rays OP, OP. for having " continued CP till it intersect the Circle in F, PF shall be always " equal to V*f* the Distance of the *Focus* of each respective Parcel " of Rays OP from the Vertex or Pole of the *Lens*.

" To demonstrate this, draw the prickt Line VF, and by what " is delivered by Dr. *Wallis* in the above-cited Place, VF and CF " will be always in the same proposed *Ratio*. Again, V*f* being " made equal to PF, CF and C*f* will be likewise equal, as are " CP, VC; and the Angles PC*f*, VCF being *ad verticem* are also " equal: Wherefore P*f* will be equal to VF, and consequently " C*f* to P*f* in the same *Ratio* as CF to VF, whence, and by what " foregoes, the Points *f*, *f*, are the several respective *Foci* of the " several Parcels of Rays, OP, OP. *Q. E. D.* ——

If any one would see how this is to be applied to all other Instances, he may consult the Place.

 the

the Degree of Divergency they have before they enter.

For if they diverge very much, their being bent towards their respective Perpendiculars in passing through the Surface, may only diminish their Divergency; whereas, if they diverge in a small Degree it may make them parallel, or even to converge. What Degree of Divergency or Convergency before Refraction in this and the following Cases, is necessary to make Rays become parallel, will be shewn at the End of this Chapter.

VIII. If they converge in such Manner as to tend directly towards the Center of Convexity before they enter the Surface, they fall in with their respective Perpendiculars, and so pass on to the Center without suffering any Refraction.

IX. If they converge less than their Perpendiculars, that is, if they tend to a Point beyond the Center of Convexity, they are made by Refraction to converge more; and if they converge more than their Perpendiculars, that is, if they tend towards a Point between the Center and the Surface, then by being refracted towards them, they are made to converge less.

This and the three foregoing Propositions may be illustrated in the following Manner. 1. Let AB, CD (*Fig.* 8.) be two parallel Rays entring a denser *Medium* through the convex Surface DB, whose Center of Convex-

ity is E; and let one of these, *viz.* AB be perpendicular to the Surface. This will pass on through the Center without suffering any Refraction, but the other being oblique to the Surface, will be refracted towards the Perpendicular ED, and will therefore be made to proceed in some Line, as DG, converging towards the other Ray, and meeting it in G, which Point for that Reason is called the *Focus*. 2. Had the Ray CD diverged from the other, suppose in the Line AD, it would, by being refracted towards its Perpendicular ED, have been made either to diverge less, be parallel, or to converge. 3. Let the Line ED be produced to F, and if the Ray had converged, so as to have described the Line FD, it would then have been coincident with its Perpendicular, and have suffered no Refraction at all. 4. If it had proceeded from any Point between C and F, as from H, or which is the same Thing, towards any Point beyond E in the Line BE produced, it would have been made to converge more, by being refracted towards the Perpendicular DE, which converges more than it; and had it proceeded from some Point, as I, on the other Side F, that is, towards any Point between B and E, it would then have converged more than its Perpendicular, and so, being refracted towards it, would have been made to have converged less.

K. When

X. When Rays proceed out of a denser into a rarer *Medium*, through a convex Surface, the contrary happens in each Case.

For being now refracted from their respective Perpendiculars, as they were before towards them, if they are parallel before Refraction, they diverge afterwards; if they diverge, their Divergency is increased; if they converge in the Direction of their Perpendiculars, they suffer no Refraction; if they converge less than their respective Perpendiculars, they are made to converge still less, to be parallel, or to diverge; if they converge more, their Convergency is increased. All which may clearly be seen by the Figure, without any farther Illustration, imagining the Rays AD, CD, *&c.* bent the contrary Way in their Refractions to what they were in the former Cases.

XI. When Rays proceed out of a rarer into a denser *Medium*, through a concave Surface, if they are parallel before Refraction, they are made to diverge.

For in this Case, the Perpendiculars at the Points where the Rays enter the Surface, being drawn from a Point on that Side of the Surface from which the Rays tend, if we conceive them to pass through the Surface, they will be so many diverging Lines on the other Side, and therefore the Rays after they have passed through the same Points, must necessarily be rendered diverging in being refracted towards them.

XII. If

XII. If they diverge before Refraction, then for the ſame Reaſon, they are made to diverge more;

XIII. Unleſs they proceed directly from the Center, in which Caſe they fall in with their Perpendiculars, and ſuffer no Refraction: or from ſome Point between the Center of Convexity and the Surface, for then they diverge more than their reſpective Perpendiculars, and therefore being by Refraction brought towards them, they become leſs diverging.

XIV. If they converge, then being refracted towards their Perpendiculars, they are either made leſs converging, parallel, or diverging, according to the Degree they converged in before Refraction.

To illuſtrate this, and the three foregoing Caſes. 1. Let AB, CD (*Fig.* 9.) be two parallel Rays entring the concave and denſer *Medium* X, the Center of whoſe Convexity is E, and the Perpendicular to the refracting Surface at the Point D, is EF; the Ray AB if we ſuppoſe it perpendicular to the Surface at B will proceed on directly to G; but the oblique one CD being refracted towards the Perpendicular DF, will recede from the other Ray AG in ſome Line as DH. 2. If the Ray CD had proceeded from A diverging in the Direction AD it would have been bent nearer to the Perpendicular, and therefore have diverged more. 3. But if it had diverged from the Center

ter E, it would have fallen in with the Perpendicular EF, and not have been refracted at all: and had it proceeded from I, a Point on the other Side the Center E, it would by being refracted towards the Perpendicular DF have proceeded in ſome Line nearer it than it otherwiſe would have done, and ſo would diverge leſs than before Refraction. 4. If it had converged in the Line LD, it would have been rendered leſs converging, parallel, or diverging, according to the Degree of Convergency, which it had before it entered into the refracting Surface.

XV. If the ſame Rays proceed out of a denſer into a rarer *Medium* through a concave Surface, the contrary happens in each Suppoſition: The parallel are made to converge; thoſe which diverge leſs than their reſpective Perpendiculars, that is, thoſe which proceed from a Point beyond the Center, are made leſs diverging, parallel, or converging, according to the Degree in which they diverge before Refraction; thoſe which diverge more than their reſpective Perpendiculars, that is, thoſe which proceed from a Point between the Center and the refracting Surface, are made to diverge ſtill more. And thoſe which converge, are made to converge more. All which may eaſily be ſeen by conſidering the Situation of the Rays AD CD, *&c.* with Reſpect to the Perpendicular EF; and therefore requires no farther Illuſtration.

XVI. When

Part III. Plate I. *Pag.* 42.

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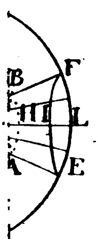


Fig. 2. *P.* 11.

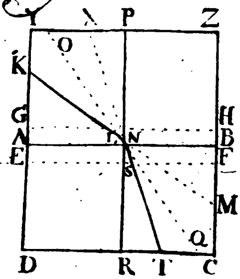
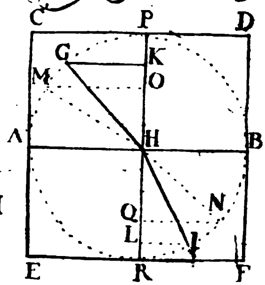


Fig. 3. *P.* 15.



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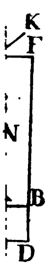


Fig. 5. *P.* 24.

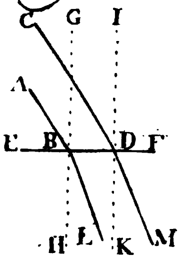


Fig. 6. *P.* 27.

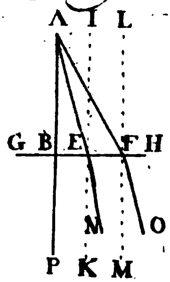
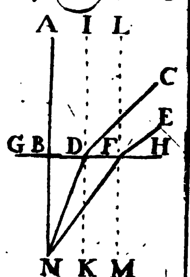


Fig. 7. *P.* 31.



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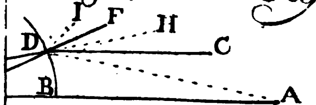
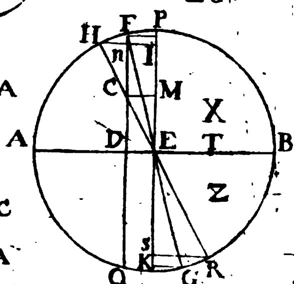


Fig. 10. *P.* 20.



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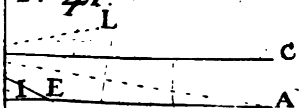

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P 25.

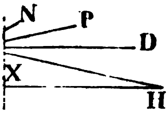
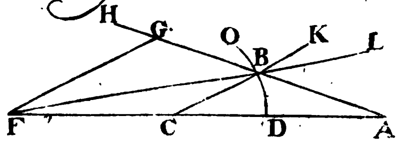


Fig. 12. P. 26.



P. 28.

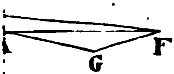
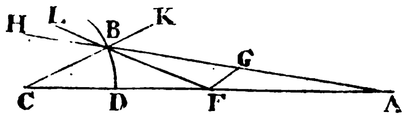


Fig. 14. P. 29.



P 30.

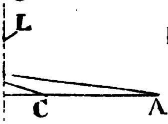
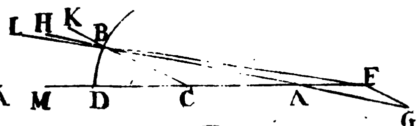
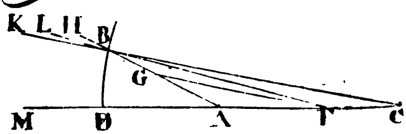


Fig. 16. P. 31.



P 32.

Fig. 18. P. 32.



P 33.

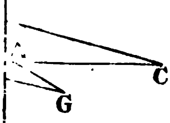
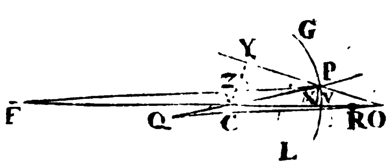


Fig. 20. P. 35.



XVI. When diverging Rays are by Refraction made to converge, the nearer their radiant Point is to the refracting Surface, the farther is their *Focus* from it on the other Side, and *vice versâ*.

For the nearer the radiant Point is to the refracting Surface, the more the Rays which fall upon the same Points of it, diverge before Refraction, upon which Account they converge the less afterwards.

XVII. When the radiant Point is at that Distance from the Surface, at which parallel Rays coming through it from the other Side would by Refraction be collected, then Rays flowing from that Point become parallel on the other Side, and are said to have their *Focus* at an infinite Distance. For the Power of Refraction in the *Medium* is the same, whether the Ray passes one Way or the other. For Instance, if the parallel Rays AB, CD (*Fig.* 8.) in passing through the refracting Surface BD are brought to a *Focus* in G, then Rays flowing from G as a radiant Point will afterwards proceed in the parallel Lines BA and DC. And the Point G, where the parallel Rays AB and CD meet, is called the *Focus of parallel Rays*.

XVIII. When Rays proceed from a Point nearer the refracting Surface than the *Focus* of parallel Rays, they continue to diverge after Refraction, and their *Focus* is then an imagi-

nary one, and situated on the same Side the Surface with the Radiant.

For in this Case, their Divergency being greater than that which they would have, if they had proceeded from the *Focus* of parallel Rays, they cannot be brought to a Parallelism with one another, much less be made to converge, and therefore they continue to diverge, though in a less Degree than before they passed through the refracting Surface; upon which Account, they proceed after Refraction, as if they came from some Point farther distant from the refracting Surface than their Radiant.

CHAP. IV.

Of Lenses, *and the Manner in which Rays are affected in passing through them.*

A *Lens*, is a *Medium* terminated on one Side by a spherical Surface, on the other by a Surface either plain or spherical. And of these there are five Sorts. The first, as A, (*Fig.* 23.) is plain on one Side and convex on the other; the second, B, convex on both Sides; the third, plain on one Side and concave on the other, as C; the fourth, D, concave on both Sides; the fifth, convex on one Side and concave on the other, as E, which is by some called a *Meniscus*.

The

The *Axis* of a *Lens* is a Line paſſing perpendicularly through both its Surfaces: Thus, the Line FG is an *Axis* common to all the five.

Lenſes are diſtinguiſhed into two general Kinds, convex and concave; the firſt and ſecond *Lenſes* are conſidered, as convex; the third and fourth, as concave: the laſt, if its Convexity is greater than its Concavity, is looked upon, as convex; if on the contrary, it is conſidered as concave.

A *Lens* is always ſuppoſed to conſiſt of a *Medium* denſer than the circumambient one, unleſs where the contrary is expreſſed.

When parallel Rays fall upon the Surface of a convex *Lens*, they are refracted towards each other in paſſing through it, and thereby collected to a *Focus* on the other Side.

To explain this, let us trace the Progreſs of a Ray as AB (*Fig.* 24.) through the convex *Lens* CDEH, whoſe Axis is IK. Let L be the Center of the firſt Convexity CDE, and M that of the other CHE; and let the Ray AB be parallel to the Axis; through B draw the Line LN which will be perpendicular to the Surface CDE at that Point. The Ray AB in entring the denſer Subſtance of the *Lens* will be refracted towards the Perpendicular, and therefore proceed after it has entered the Surface at B in ſome Direction inclined towards the Axis as BP. Through M the Center of Convexity of this Surface and the Point P draw

the Line MR, which passing through the Center will be perpendicular to the Surface at P, and the Ray now entering a rarer *Medium* will be refracted from the Perpendicular into some Direction as PF. In like Manner, and for the same Reasons, the parallel Ray ST on the other Side the Axis, and also all the intermediate ones as XZ, *&c.* will meet it in the same Point, unless the Rays AB and ST enter the Surface of the *Lens* at too great a Distance from the Axis IF, the Reason of which has already been fully explained *.

The Point F where the parallel Rays AB, SF, *&c.* are supposed to be collected by passing through the *Lens* CE, is called *the Focus of parallel Rays of that Lens*.

If the Rays converge before they enter the *Lens*, they are then collected at a Point nearer to the *Lens* than the *Focus* of parallel Rays. If they diverge before they enter the *Lens*, they are then collected in a Point beyond F; unless they proceed from a Point on the other Side at the same Distance with the *Focus* of parallel Rays, in which Case they are rendered parallel. If they proceed from a Point nearer than that, they diverge afterwards, but in a less Degree than before they enter the *Lens*.

If the *Lens* is plain on one Side and convex on the other, the Rays are refracted the same Way, but in a less Degree.

* See *Observation* 3, in the foregoing Note.

Had the Rays AB, ST proceeded from a radiant Point on one Side the *Lens*, and been collected in a *Focus* on the other; then if they ſhould be ſuppoſed to proceed from that focal Point as from a Radiant, and paſs through the *Lens* the contrary Way, they would be collected in that Point which was the Radiant in the other Caſe: and the nearer the Radiant Point is to the *Lens*, the farther is the *Focus* from it on the other Side, and *vice verſâ*.

If the Rays AB, CD, EF, *&c.* (*Fig.* 25.) parallel to each other, but oblique to GH the Axis of the *Lens* IK, or if the diverging Rays CB, CF, proceed as from ſome Point C which is not ſituated in the Axis of the *Lens*, they will be collected into ſome Point as L, not directly oppoſite to the Radiant C, but nearly ſo: for the Ray CD which paſſes through M the Middle of the *Lens* and falls upon the Surface of it with ſome Obliquity, will itſelf ſuffer a Refraction at D and N; but then it will be refracted the contrary Way in one Place to what it is in the other, and theſe Refractions will be equal in Degree if the *Lens* has an equal Convexity on each Side, as we may eaſily perceive if we imagine ND to be a Ray paſſing out of the *Lens* both at N and D, for it is evident the Line ND has an equal Inclination to each Surface at both its Extremities. Upon which Account the Difference between the Situation of the Point L and one directly oppo-

ſite

site to C, is so small, that it is generally neglected; and the *Focus* is supposed to be in that Line, which a Ray, that would pass through the middle Point of the *Lens*, were it to suffer no Refraction, would proceed in.

All which is sufficiently clear, from what has been said concerning the Laws of Refraction explained in the foregoing Chapter.

When parallel Rays fall upon a concave *Lens*, they are refracted from each other in passing through it, and thereby made to diverge, proceeding as from an imaginary *Focus* on the first Side the *Lens*.

In order to comprehend this, let AB CD (*Fig.* 26.) represent a concave *Lens*, EF its Axis, GH the Radius of the first Concavity, IK that of the second; produce HG to L, and let MG be a Ray of Light entring the *Lens* at the Point G. This Ray being refracted towards the Perpendicular GL, will pass on to some Point as K in the other Surface more distant from the Axis than G, and being there refracted from the Perpendicular, IK will be diverted farther still from the Axis, and proceed in the Direction KN as from some Point as O on the first Side of the *Lens*. In like Manner other Rays as PQ parallel to the former, will proceed after Refraction at both Surfaces as from the same Point O; which upon that Account will be the imaginary *Focus* of parallel Rays of this *Lens*.

If

If the Rays diverge before they enter the *Lens*, their imaginary *Focus* is then nearer the *Lens* than that of the parallel Rays. If they converge before they enter the *Lens* proceeding towards ſome diſtant Point in the Axis as E, they are then rendered leſs converging: if they converge to a Point at the ſame Diſtance from the *Lens* with the *Focus* of parallel Rays, they then go out parallel; if to a Point at a leſs Diſtance they remain converging, but in a leſs Degree than before they entered the *Lens*.

When the Rays enter the *Lens* diverging, the nearer their radiant Point is to it, the nearer alſo is their imaginary *Focus* after Refraction, and *vice verſâ*.

If the *Lens* is plain on one Side and concave on the other, the Rays ſuffer a like Refraction in each Caſe, but in a leſs Degree.

The Truth of what has been ſaid concerning the Paſſage of Rays through a concave *Lens*, is eaſily to be deduced from the Laws of Refraction delivered in the foregoing Chapter.

But the Method of determining the exact focal Diſtances of *Lenſes* is to be had from the Propoſitions laid down and demonſtrated in the Note in the foregoing Chapter. Thus, the Progreſs of the Rays after their Refraction at the firſt Surface where they enter a *Lens*, is had by one of thoſe which determines the focal Diſtance of Rays entring a denſer *Medium* of ſuch Form: And their Progreſs after their

Refraction at the other Surface where they go out, is had by computing what Progress Rays, moving in the Direction they are found to have after their Entrance at the first Surface, will acquire by being refracted at the other; which is to be effected by one which determines the focal Distance of Rays passing out of a denser *Medium* of like Form with that of the *Lens* *.

When

* Or a general Theorem may be made after the following Manner, to determine the Progress of Rays after Refraction at both Sides of the *Lens*, whatever be the Matter of it, or the Form wherein it is made.

Thus suppose GH (*Fig.* 28.) to be a given *Lens*, and E a Point in its Axis from whence the diverging Rays EL, &c. fall upon the *Lens*, AL the *Radius* of the first Convexity, and CK that of the second; let LK*f* be the Direction of the diverging Ray EL after its Refraction at the first Surface, and KF its Direction after Refraction at both. Then will *f* be the *Focus* of the Rays after their first Refraction, and F the Point they will meet in after both. Let BD be the Thickness of the *Lens*, and let the Proportion which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction be expressed by the *Ratio* of I to R. Call EB, d; BD, t; AB, r; CD, s; Bf, x; DF, y: Now, to find f their *Focus* after Refraction at L where they enter the first Surface of the *Lens*, comes under the third Proposition in the forementioned Note: According to which the *Ratio* compounded of x, *the focal Distance sought*, to d, *the Distance of the radiant Point*; and of $d+r$, *the Distance between the same Point and the Center*, to $x-r$, *the Distance between the Center and the* Focus, is as I to R; compounding these two *Ratios* therefore (that is multiplying them together) we have $dx+rx : dx-dr :: \text{I to R}$; which Proportion being converted into an Equation, and duly reduced, gives $x = \frac{Idr}{Id-Rd-Rr}$.

Thus having found the Distance Bf, and consequently the Point f, to which the Rays converge from L, we must proceed to find F, that to which they will converge after having passed through K, where

When a Ray passes through a *Medium* terminated by two plain and parallel Surfaces, it is

where they suffer a second Refraction: This comes under the same Proposition, but if we would use the same Letters as before, to express the Proportion which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, they must be put one for the other; because when Rays pass out of a denser into a rarer *Medium*, the Sine of the Angle of Incidence bears the same Proportion to the Sine of the Angle of Refraction, that the Sine of the Angle of Refraction does to the Sine of the Angle of Incidence, when they pass out of of a rarer into a denser. This being observed, by the aforesaid Proposition we shall have the *Ratio* compounded of y, *the focal Distance*, to $\frac{Idr}{Id-Rd-Rr}-t$, *the imaginary Focus of the incident Rays*; and of $\frac{Idr}{Id-Rd-Rr}-t+s$, *the Distance between the imaginary Focus and the Center*, to $y+s$, *the Distance between the Center and the Focus*, as R to I. Which Equation, if we reduce the mixed Quantities $\frac{Idr}{Id-Rd-Rr}-t$, and $\frac{Idr}{Id-Rd-Rr}-t+s$ into improper Fractions, will stand thus,

$$\left.\begin{array}{c} y : \dfrac{Idr-Idt+Rdt+Rrt}{Id-Rd-Rr} \\ \text{and} \\ \dfrac{Idr-Idt+Rdt+Rrt+Ids-Rds-Rrs}{Id-Rd-Rr} : y+s \end{array}\right\} :: I : R.$$

And compounding these Ratios we have

$$\frac{Idry-Idty+Rdty+Rrty+Idsy-Rdsy-Rrsy}{Id-Rd-Rr} : \frac{Idry-Idty+Rdty+Rrty+Idrs-Idts+Rdts+Rrts}{Id-Rd-Rr} :: R : I.$$

And multiplying Extreams together and Means together, we have $IIdry-IIdty+IRdty+IRrty+IIdsy-IRdsy-IRrsy = IRdry-IRdty+RRdty+RRrty+IRdrs-IRdts+RRdts+RRrts$; which Equation being reduced, gives

$$y = \frac{IRdrs-IRdts+RRdts+RRrts}{IIdr-IIdt+2IRdt+IRrt+IIds-IRds-IRrs-IRdr-RRdt-RRrt}.$$

is refracted in going out of one, as much as it was in entring the other; and therefore proceeds afterwards not in the same Direction, but in one that is parallel to that which it had before. Thus if the Ray AB (*Fig.* 27.) enters the denser *Medium* CDEF terminated by the parallel Surfaces CD and EF, it is refracted at B towards the Perpendicular BI, proceeding to a Point as G, where it is as much refracted from the Perpendicular GK in going out, and proceeds in the Direction GH not the same, but parallel to the former ABL.

This Theorem may be applied to all Cases whatever, even to plain Surfaces *mutatis mutandis*; *v. g.* the *Radius* of a concave Surface being negative (as lying the contrary Way) with Respect to that of a Convex, and the *Radius* of a plain Surface being an infinite Line; if we would apply this Theorem to a concave Surface, we must change all the Sines of those Members wherein the Symbol expressing the *Radius* of that Surface occurs; and if to a plain Surface, all the Members which involve the *Radius* must be considered as infinite Quantities, that is, all, except them, must be struck out of the Equation as nothing. So likewise, if we would have it extend to other Rays besides diverging ones, the Point where converging Rays would meet, lying on the contrary Side to that from whence the diverging ones were supposed to flow, its Distance must be made negative; and the Distance where parallel Rays meet being infinite, it is only changing the Sines of all those Members in which *d* is found, if the Rays are supposed converging, or making those Members infinite in Case the Rays are supposed parallel; which is done by striking out all the rest, as bearing no Proportion to them.

See the Method of reducing this Equation to fewer Terms, where it is also illustrated with divers Instances, in Dr. *Browne*'s Appendix to *Gregory*'s Optics; or in Dr. *Halley*'s Method of finding the principal Focus of Optic Glasses universally; Philosoph. Transact. No. 205.

CHAP.

CHAP. V.

Of the Eye.

THE Form of the Eye is such as is represented in Figure 29, and would be a perfect Sphere, were not the fore Part AA somewhat more protuberant than the rest.

The Description of it, so far as is necessary to explain the Nature of Vision, is as follows.

It is inclosed in three distinct Coats or Teguments; the outermost of which, *viz.* *aa*, is called *Tunica Sclerotica*; the next *cc*, *Choroides*, or *Uvea*; the third and innermost *dd*, derives its Name from that of its Discoverer, and is called *Tunica Ruyschiana*.

These Coats are contiguous to each other every where, except on the fore Part of the Eye.

That Portion of the *Sclerotica* which lies between A and A, is more protuberant than the rest, is transparent, and has the Name of *Tunica Cornea*.

That Portion of the *Choroides* which is situated between *b* and *b*, is called the *Iris*; and is that which by its Colour denominates an Eye black, grey, *&c.* In the Middle of this there is a round Hole as *pp*, called the *Pupil*.

The *Iris* consists of two Kinds of muscular Fibres; the first lies extended from its Extremity

like the *Radii* of a Circle, and point towards the Middle of the Pupil as towards a Center: the other are circular ones and surround the Pupil having the Middle of it for their common Center. These are connected to the former where they cross: and therefore when these contract, the Pupil is diminished; when the other, it is inlarged.

Within the Cavity of the Eye, and not far behind the Pupil, there is a soft transparent Substance CC, not unlike a double convex *Lens*, one of whose Surfaces as S, is more convex than the other. This is called the *Christalline Humour*; and is suspended within the Eye by certain Ligaments as C*l*, C*l*, called *Ligamenta Ciliaria*, or *Processus Ciliares*: these are convex towards the Pupil, as expressed in the Figure, and concave on the other Side, and are muscular, and therefore capable of Contraction and Dilatation. The convex Side of these Ligaments is lined with a very black *Mucus* or Substance, as is also that Side of the *Iris* which is next it.

The *Tunica Ruyschiana* leaves the *Choroides* at *l*, and passing behind the *Ligamenta Ciliaria* and the Christalline Humour, is contiguous to them, and joins the *Choroides* again at *l*, on the other Side the Pupil.

By means of the forementioned Parts, the Cavity of the Eye is divided into two Portions; the one of which VV, is filled with a Fluid nearly

nearly of the same Density with Water, and is therefore called *Humor Aqueus*; the other TT contains a Fluid whose Consistency is greater than that of the former, and is called *Humor Vitreus*.

At the back Part of each Eye, but not directly opposite to the Pupil, there enters a Nerve as NN, which is called the *Optic Nerve*.

The Fibres of this Nerve, after their Entrance into the Eye at N, spread themselves over the innermost Coat of it as far as the *Ligamenta Ciliaria*, and form a very thin Membrane, called *Tunica Retina*.

The innermost Coat of the Eye is every where covered (except that Part of it which is contiguous to the back Part of the Christalline Humour) with a very black *Mucus*, not unlike that with which the back Part of the *Iris* and fore Part of the *Ligamenta Ciliaria* were observed to be cover'd. This is to hinder any Light from being reflected from those Parts on to the *Retina*; which would render the Images of Objects indistinct, as we shall see when we have explained the Nature of Vision, which is the Subject of the following Chapter.

CHAP.

CHAP. VI.

Of the Nature of Viſion.

SUch is the Subſtance and Form of the Humours of the Eye, when lodged in their proper Receptacles, that Rays of Light in paſſing through them are affected in the like Manner as in paſſing through a convex *Lens*, as we ſhall ſee immediately; and therefore to underſtand the Nature of Viſion, which depends on the Paſſage of Rays flowing from the *ſeveral* Points of a diſtant Object through thoſe Humours, little more is required than to know how the ſame Rays would be affected, were they ſuppoſed to paſs through a convex *Lens*. Which may eaſily be done by applying to this Caſe what has been delivered in the fourth Chapter concerning the Manner in which Rays flowing from a *ſingle* Point, are affected in paſſing through *Lenſes* of that Kind.

We have already ſeen, in the abovementioned Chapter, that Rays flowing from a *ſingle* radiant Point, and afterwards falling on a convex *Lens*, are collected to a *Focus* in ſome Point oppoſite, or nearly ſo, to the Radiant. Let us now ſuppoſe an Object placed before a *Lens*, but farther from it than the focal Diſtance of its parallel Rays, and let it ſend forth

Rays

Rays from each Point of its Surface in every Direction, as from so many radiant Points. Some of the Rays which flow from each Point of that Surface of the Object which is turned towards the *Lens* will necessarily fall upon it, and passing through it will be collected in so many distinct focal Points on the opposite Side, as there are distinct radiant Points in the Surface of the Object from whence they came. Now as the radiant Points are contiguous to each other in the Surface of the Object on one Side of the *Lens*, the focal Points will also be contiguous on the other; and as each focal Point is opposite to its respective Radiant, their Places will have the same Relation to each other, that those of the Radiants have; and consequently these Points taken together will be a true Representation and perfect Image of that Object; for each Point will exhibit the same Colour that its correspondent Point in the Object is of. But because each Point in the Representation is opposite to its respective one in the Object, the Image will be inverted. The Truth of this may easily be experienced, if we hold a clean white Paper facing the *Lens* in the Place where the focal Points are, and take Care to prevent all other Light from falling upon the Paper, except that which passes through the *Lens* *.

To

* On this depends the Structure of the *Obscura Camera*, which is a Contrivance to exhibit the Representation of such Objects as may

To illustrate this; Let PQR (*Fig.* 30) represent an Object placed before the *Lens* AB, and sending forth Rays from each Point in its Surface; and let *q* be the focal Distance of Rays proceeding from Q and passing through the said *Lens*. Then will all the Rays that proceed from the Point Q between the Lines QA and QB be collected in *q*; in like Manner all that flow from P between the Lines PA and PB, will meet in the opposite Point *p*; and so many as proceed from R and pass thorugh the *Lens* will be collected in *r*; and all the Rays that flow from the remaining Points between P and R and fall upon the *Lens*, will be collected in as many Points between *p* and *r*; and *pqr*, if the Rays are received there upon a white Surface, will exhibit an Image of the Object PR, but inverted; because the Rays PL*p* and RL*r*

may be seen from a Window, upon some plain white Surface held before the Window within the Room. In order to do this, a common spectacle Glass or Burning Glass (both which are convex *Lenses*) must be fixed in an Hole in the Windowshut; for then if no Light be suffered to enter into the Room, but what passes through the Hole, and a Sheet of white Paper be held opposite to the Hole at that Distance where the Rays proceeding from the Objects abroad, and passing through the Glass are collected into their respective *Foci*; we shall have the Images of all the Objects which lie before the Hole, represented upon the Paper, inverted; but in a much more lively and exact Manner than can be done by the Pencil; and not only the Objects and their respective Situations, but, what is peculiar to this Sort of Painting, their Motions will also be expressed.

cross

P.36.

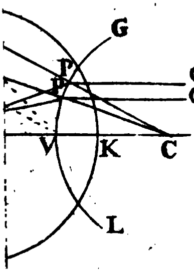
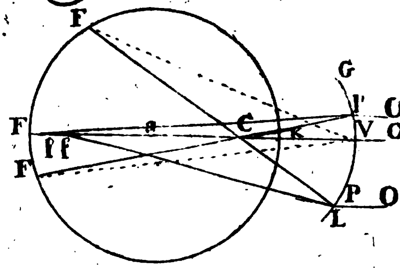


Fig. 22. P.36.



P.44.

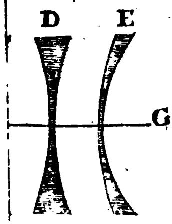
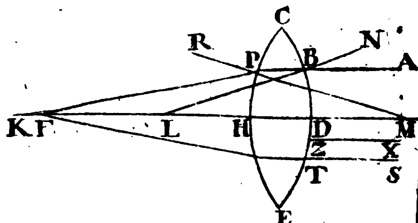


Fig. 24. P.45.



47.

Fig. 26. P.48.

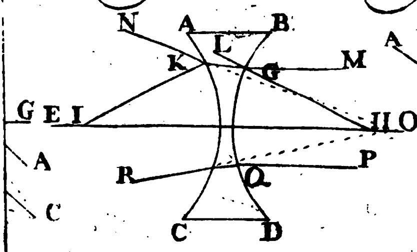


Fig. 27. P.52.

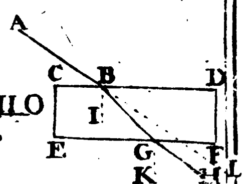
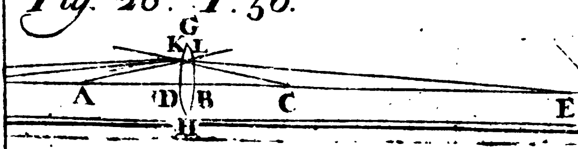


Fig. 28. P.50.



croſs each other at L in paſſing through the *Lens* *.

Thoſe Rays which flow from the ſame Point of an Object, when conſidered together, are called a *Cone* or *Pencil* of Rays. Thus the Rays QA, QL, *&c.* conſtitute a Pencil flowing from the Point Q; ſo the Rays PA, PL, *&c.* a Pencil from the Point P; and the middle Ray of each Pencil, as PL, QL, *&c.* is called the *Axis* of that Pencil, to which it belongs.

Now in like Manner as the ſeveral Pencils of Rays flowing from the diſtinct Points in the Surface of an Object placed before a *Lens*, are collected in ſo many Points at a certain Diſtance on the other Side of the *Lens*, and form an Image there when received upon a white Paper; ſo Pencils proceeding from an Object placed before the Eye, and being refracted in paſſing through the Humours of it, are collected into their reſpective *Foci* upon the *Retina*, where they form a Repreſentation of that Object; and by their Impulſes upon the tender Nerves of the *Retina*, an *Idea* of the Object is excited in the Mind.

* A burning Glaſs, is no other than a Piece of Glaſs ground into the Form of a convex *Lens*; for if the Rays of the Sun are permitted to paſs through ſuch an one, they will burn very ſtrongly in the Place where they are collected into their reſpective *Foci*; upon which Account it is, that the Point where Rays in general are collected, is called their *Focus*, that is, their Place of Burning.

The Progress of the Rays through the Humours of the Eye, are expressed in the 31st Figure: where FG*t* is the Eye, FG the *Tunica Cornea*, *pp* the Pupil, AA the aqueous Humour, HH the christalline, and VV the vitreous. And RS represents an Object placed before it, emitting Pencils of Rays from its several Points R, S, T, *&c.* The Rays which constitute the Pencil GTF, in entring the aqueous Humour, pass out of a rarer into a denser *Medium* through a convex Surface, in which Case diverging Rays are made to diverge less, to become parallel or to converge (Chap. 3. Prop. 7.); in entring the christalline they do the like; and in passing out of that, they enter a rarer through a concave Surface (*viz.* the concave Surface of the vitreous) which also has the same Effect (Chap. 3. Prop. 15.) By which Means they are made to converge, as described in the Figure, and to meet together in a *Focus* at *t* a Point in the *Retina.* In like Manner the Rays flowing from R, and constituting the Pencil GRF, will proceed as described in the Figure, and after Refraction meet in *r*; and the Rays proceeding from S will be collected in *s*, *&c.* by which Means an Image of the Object will be formed in *rts* upon the *Retina*, but because the Pencils cross each other in passing through the Pupil, it will be inverted *.

The

* Of this we have experimental Proof: For if we cut away the back Part of an Eye, and apply a Paper there, we shall see the Image

The leſs the Diſtance between the Object and the Eye is, the more the Rays which come from the Object, are ſaid to diverge, and *è contra*: Not that the Situation of the Eye makes any Alteration in the Progreſs of thoſe Rays, but that, when the Eye is placed nearer the Object, it receives into its Pupil Rays which diverge in a greater Degree than thoſe which it can receive when placed farther off. The following Illuſtration will make this clear: Let AB (*Fig.* 32.) repreſent an Object emitting Rays from each Point of its Surface, and let C*d*, C*r*, *&c.* expreſs thoſe which flow from the Point C: let *m* be a Pupil of an Eye placed at the Diſtance C*m* from it; 'tis plain this Pupil will receive into it the diverging Rays C*r*, C*s*; whereas the Rays C*o*, C*t* will diverge the moſt of any that can enter the ſame Pupil, when placed at the Diſtance C*n*; but theſe diverge leſs than the former, the Angle *o*C*t* being included in the Angle *r*C*s*.

Images of external Objects placed thereon, as accurately as in the *Obſcura Camera*, provided no Light is permitted to fall upon the Paper, except that which paſſes through the Humours of the Eye.

Optic Writers have made it Matter of great Difficulty to determine the Point where the *Axes* of the Pencils which enter the Eye, croſs each other; ſome placing it in the Center of the Eye, others in the Vertex of the chriſtalline Humour, others in that of the *Tunica Cornea*: But as the Rays of each Pencil fill the Pupil, or as the Pupil itſelf is a common Baſe to each Pencil, it is inconceivable how the *Axes* of thoſe Pencils ſhould croſs each other in any other Place than the Center of the Pupil. See Figure 31, or any other where ſeveral Pencils are repreſented, as paſſing through the Pupil of an Eye.

Vision is distinguished into *bright* and *obscure*; *distinct* and *confused*.

It is said to be *bright*, when a sufficient Number of Rays enter the Pupil at the same Time; *obscure*, when too few. It is *distinct*, when each Pencil of Rays is collected to a *Focus* exactly upon the *Retina*; *confused*, when they meet before they come at it, or when they would pass it before they meet; for in either of these last Cases, the Rays flowing from different Points of the Object, will fall upon the same Part of the *Retina*, which must necessarily render the Image confused and indistinct.

Now that Objects may appear with a due *Brightness*, whether more or fewer Rays proceed from them, we have a Power of contracting, or dilating the Pupil by means of the muscular Fibres of the *Iris* (as explained in the foregoing Chapter), in order to take in more or fewer Rays as Occasion requires. But this Power has its Limits. *

And that the Rays may be collected into Points exactly upon the *Retina*, that is, that Objects may appear *distinct*, whether they be nearer or farther off, that is, whether the Rays proceeding from them diverge more or less, we have a Power of contracting or relaxing the *Ligamenta Ciliaria*, and thereby altering the

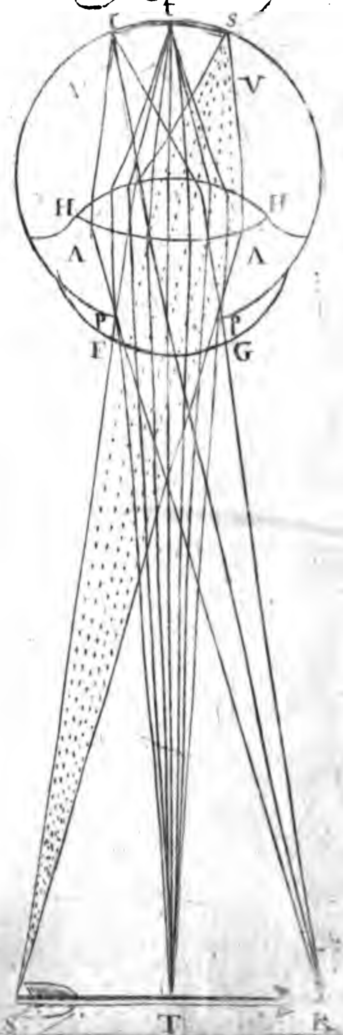
* In some Animals this Power is much greater than in others; particularly in such as are obliged to make Use of their Eyes by Night, as well as by Day, as in Cats, &c.

Form

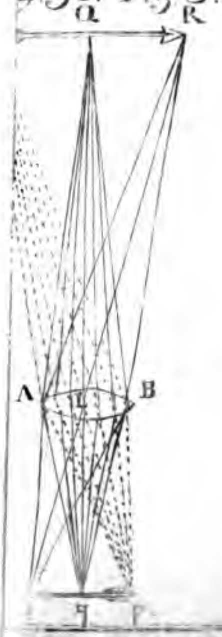
Part III. Plate IV. Pag. 62

ig. 29. P. 53.

Fig. 31. P. 69



g 30. P. 58.



Form of the chriſtalline Humour, and with that the focal Diſtance of the Rays. Thus, when the Object we view is far off, and the Rays fall upon the Pupil with a very ſmall Degree of Divergency, we contract the *Ligamenta Ciliaria*, which being concave towards the vitreous Humour, do thereby compreſs it more than otherwiſe they would do; by this means it is made to preſs harder upon the back Side of the chriſtalline Humour, which is thereby rendered flatter; and ſo the Rays proceed farther before they meet in a *Focus* than otherwiſe they would have done. Add to this, that we dilate the Pupils of our Eyes (unleſs in Caſes where the Light is ſo ſtrong that it offends the Eye) and thereby admit Rays into them, that are more diverging than thoſe which would otherwiſe enter. And when the Rays come from an Object that is very near, and therefore diverge too much to be collected into their reſpective *Foci* upon the *Retina*, by relaxing the *Ligamenta Ciliaria* we give the Chriſtalline a more convex Form, by which means the Rays are made to ſuffer a proportionably greater Degree of Refraction in paſſing through it *.

* Some Philoſophers are of Opinion, that we do this by a Power of altering the Form of the Eye; and others, by removing the Chriſtalline forwards or backwards as Occaſion requires; but neither of theſe Opinions is probable; for the Coats of the Eye are too hard, eſpecially in ſome Animals, for the firſt; and as to moving the Chriſtalline out of its Place, the Cavities of the Eye ſeem to be too well filled with the other Humours to admit of ſuch Removal.

And besides this, by contracting the Pupils of our Eyes, we exclude the more diverging Rays, and admit only such as are more easily refracted into their respective *Foci* *. But Vision is not distinct at all Distances, for our Power of contracting and relaxing the *Ligamenta Ciliaria* is also circumscribed within certain Limits.

The nearer an Object is placed to the Eye, the greater is the Image of it upon the *Retina*. Because the Pencils flowing from the extreme Parts of the Object when near, make a larger Angle with each other in the Pupil where they cross, than the same Pencils do when the Object is placed farther off. Thus AB (*Fig.* 33.) the Image of the Object CD, far exceeds EF that of the same Object GH, placed at a greater Distance from the Eye, as is evident from Inspection of the Figure.

In those Eyes where the *Tunica Cornea* is very protuberant and convex, the Rays of Light suffer a very considerable Refraction at their Entrance into the aqueous Humour, and are therefore collected to a *Focus* before they fall upon the *Retina*, unless the Object be placed very near, so that the Rays which enter the Eye, may have a considerable Degree of Divergency. People that have such Eyes, are

* Accordingly it is observable, that if we make a small Hole with the Point of a Needle through a Piece of Paper, and apply that Hole close to the Eye, making Use of it, as it were, instead of a Pupil, we shall be able to see an Object distinctly through it, though the Object be placed within half an Inch of the Eye.

said

said to be *purblind*. Now the nearer an Object is placed to the Eye, the greater is the Image of it therein; as explained above; these People therefore can see much smaller Objects than others, as seeing much nearer ones with the same Distinctness. And their Sight continues good longer than that of other People, because the *Tunica Cornea* of their Eyes, as they grow old, becomes plainer, for Want of that Redundancy of Humours with which they were filled before.

On the contrary, old Men having the *Cornea* of their Eyes too flat for want of a sufficient Quantity of the aqueous Humour to fill them out, if the Rays diverge too much before they enter the Eye, they cannot be brought to a *Focus* before they reach the *Retina*; on which Account those People cannot see distinctly, unless the Object be situated at a greater Distance from the Eye, than is required for those whose Eyes are of a due Form.

Since the Images of the Objects we look at are inverted in the Eye, it may be thought the Objects themselves ought to appear so; but it must be considered, that there is no *natural* Connection between the *Idea* in our Mind, and the Image upon the *Retina*; we find by Experience, that when such an *Idea* is excited in our Mind, such an Object stands before us in such a Position and of such a Form; when ever therefore the like *Idea* is excited again, we con-

conclude there is a like Cause of it. For it is found by Observation, that People who have been born blind, and have afterwards received their Sight, have had no Information from their Eyes at first, concerning the particular Situation or Form of Bodies; but have been obliged to stay till Experience has taught them what Figures and Situation of Bodies correspond to such and such Sensations in the Mind *.

In like Manner it is from Experience that an Object appears single, though there be an Image of it in each Eye; for after we find, that its Place, according to the Representation of it in each Eye, is the same, we necessarily conceive it to be but one. The Manner how we come to find this, seems to be as follows: There is one Part of the *Retina* upon which when the Image falls, the Object appears brighter and more distinct, than when it falls upon any other, as is evident because we always see one Part of an Object with greater Distinctness than any of the rest. This Point I shall hereafter call *the Point of distinct Vision.* This naturally leads us to turn our Eyes so, that the Object may be situated directly opposite to this Point. And this Action of ours is that which has given Rise to those imaginary Lines, which are supposed to pass di-

* See Mr. *Chesselden's* Observations on a young Gentleman couched by him at the Age of 13 Years. Philosoph. Transact. No. 402.

rectly

rectly through the Eye and to terminate in the Object we view, and are commonly called the *optic Axes*. We therefore turn our Eyes so that the Object may appear in those Lines. Therefore since these Lines concur at the Object, when we indeavour to view it with Distinctness, each Eye affords us an *Idea* of the Object in the same Place, from whence it necessarily appears but one *.

When-

* There are other Methods of accounting for these two last *Phænomena*, some of which, perhaps, the Reader may think more plausible; for the Connection between the Image on the *Retina* and the Idea in the Mind, being purely *metaphysical*, we can never hope to arrive at any Certainty in this Matter.

Some are of Opinion, that we judge those Rays which paint the uppermost Part of the Image in the Eye to proceed from the lowermost Part of the Object, because they strike upon the *Retina*, as coming from that Part; and that we conclude from hence that the Object is erect, though the Image be inverted; as if the Direction wherewith the Rays strike the *Retina*, informed the Mind which Way they came. This Solution serves also to explicate the *Phænomenon* of seeing but one Object with both Eyes; for as the Mind is informed by the Direction with which the Rays strike the *Retina* of the Place from whence they come; therefore when it appears that they enter each Eye as from the same Place, the Object necessarily seems to be but one; because we cant suppose two to exist in the same Place at the same Time. But according to this Doctrine no Object could ever appear double; because wherever it be situated, the Rays cannot affect the *Retina* otherwise than if they came from it alone.

Some have been so absurd as to embrace an Opinion, the first Author of which was *Gassendus*, that we see one and the same Point of an Object only with one Eye at a Time, (*otiante alio*, as they express it) while the other does nothing. Vid. *Gassendi* Epistol. de Magnitud. Solis. Or *Tacquet*. Optic. Lib. I. Prop. 2.

Some imagining that the *optic* Nerves consist of a Bundle of small ones wrapped up in one common Tegument, are of Opinion, that such as lie upon the *Retina* at equal Distances from the Point of distinct Vision, and on the same Side of it in each Eye, are con-

 nected

Whenever the Eyes are so situated with respect to an Object, that the same Part of the *Retina* in each Eye is affected by the Rays that flow from it, which are wont to be affected when two Objects are placed before the Eyes, the Mind receiving no Information from without, but by the Impulses of the Rays upon the *Retina*, judges that there are two Objects. Thus let A, B (*Fig.* 34.) represent two Eyes whose *optic Axes* are directed to the Point C, and let E be an Object on one Side the Point C, and F an Object on the other. Now Objects thus situated must appear separate, otherwise every time we viewed an Object we must imagine all the different Points in its Surface to be but one, which is contrary both to Reason and daily Experience. In this Case, the Point *d* in each Eye will be affected by the Rays which flow from those Objects; but so it will, if a single Object be placed at D, and therefore for the Reason given above, an Object in that Situation shall appear as the two separate ones E and F, that is, double. A-

nected together in one, before they terminate in the Brain; and so whether one or both are affected, only one *Idea* is excited in the Mind. *Gravesand* confirms this Opinion by asserting, that in all Animals which look at the same Object with both Eyes, the optic Nerves concur before they enter the Brain; and that in such as look at one Object with one Eye, and at a different one with the other, they are separate all the Way.

Others, with *Briggius* (see his *Opththalmog.* Chap. 11.) do not contend, that the forementioned corresponding Parts of the *optic* Nerves are connected before they terminate in the Brain; but that they are of an equal Tension, and therefore excite the same Sensation in the Mind.

gain,

gain, let there be an Object placed without the *optic Axes* as at G, Rays flowing from this will affect the ſame Part in each Eye, as if there were two diſtinct Objects, *viz.* one at E and the other at H, this therefore will alſo appear double. Farther, as the Objects D and G are ſituated in this Figure, if both are attended to at the ſame Time (the *optic Axes* being ſtill directed to the ſame Point C) they will appear as three, being ſituated oppoſite to the three Points F, E, and H. And what has been ſaid of the Appearance of the Objects D and G as they are ſituated in this Figure, may be applied to their Appearance as they are placed in the next, where they are repreſented as being beyond the Line HF. So that wherever an Object is placed, provided it be nearer to the Eyes than the Point where the *optic Axes* concur, or farther from them, it appears double.

There is one Part of the *Retina* of each Eye, upon which when the Image falls, the Object cannot be ſeen at all with that Eye; the Proof of this we have from the following Experiment. Fix two Objects upon a Wall, of ſuch Bigneſs that each may hide a ſquare Inch of it, or thereabouts, and at the Diſtance of about a foot or two from each other, and go back from the Wall about three times that Diſtance; then ſhutting the left Eye, look at the left Object with the right one, and while the right Eye is

in that Pofition, the right Object will not be feen. So if in that Station we look at the right Object with only the left Eye, the left Object will difappear. The Reafon of this is fuppofed to be, that the Image then falls upon the blood-veffels of the *optic* Nerve, from which no Senfation is conveyed to the Brain.

The Angle comprehended between the Rays which flow from the extreme Parts of the Object, and crofs in the Pupil, is called the *optic Angle.*

'Tis by means of this Angle that we are able to form fome Judgment of the Magnitude of an Object; becaufe the larger this is, the larger is the Image upon the *Retina*, that is, a greater Portion of it is affected by the Rays which flow from that Object. But this is not fufficient alone, becaufe different Objects at different Diftances from the Eye, may fubtend equal Angles at the Pupil. We ought therefore to know alfo the Diftance of the Object.

This, if the Object be very near, we are able to form a tolerable Judgment of, by the Degree of Divergency, wherein the Rays which flow from the fame Point of the Object enter the Eye; becaufe we find it neceffary to adapt the Eye accordingly, in order to bring them to a *Focus* upon the *Retina*.

But when the Object is at a greater Diftance from us, a confiderable Variation in the Diftance of it makes but a very fmall one in the

Diver-

Divergency of thoſe Rays, and therefore this Rule of Judging ceaſes to be of Uſe. The only Expedient then is, the Angle comprehended between the *optic Axes* at the Object to which they are directed, or which is the ſame Thing, the Poſition of the Eyes with reſpect to each other when they view the Object *. But in very large Diſtances this Poſition varies ſo little, that it is alſo of no Uſe; in which Caſe we make the beſt Judgment we can from the Brightneſs, Diſtinctneſs, and apparent Magnitude of the Object, and likewiſe from its Situation with reſpect to others which are interpoſed †.

When we are unable to judge rightly concerning the Diſtance of an Object, we conceive it greater, the farther we imagine it to be from us, and *vice verſâ*; becauſe it requires a larger Object to exhibit the ſame Image upon the *Retina*, when it is ſituated at a great Diſtance, than when near. Thus we imagine the Sun and Moon to be farther off, when they are in the *Horizon*, than when they are near the *Meridian*, and accordingly think them

* That the Poſition of the *optic Axes* is a Means whereby we judge of Diſtances, is evident from hence, *viz.* that they who have loſt the Sight of one Eye, find it much more difficult to eſtimate the Diſtances of Objects, than they did, when they had the Uſe of both.

† We have a remarkable Inſtance of the Error of our Judgment concerning the Diſtances of very remote Bodies, in that we look upon the Sun, Moon, and Stars to be all at the ſame Diſtance, wheras ſome of them are a thouſand Times farther from us than others.

pro-

proportionably larger in one Situation than in the other, though they are found to exhibit the same Image upon the *Retina* in both Cases *.

We are never able to see very distant Objects with Distinctness; this is not solely owing to the Pupil's not receiving into it a sufficient Number of Rays for that Purpose, or because they are not collected into *Foci* upon the *Retina*, but because the Object being very far off, the Rays which flow from Points of the Object that are contiguous, fall too near each other upon the *Retina* to excite distinct Sensations in the Mind, so that the *Idea* of the Whole is confused.

CHAP. VII.

Of the Appearance of Objects *seen through* Media *of different Forms.*

THat what we shall say upon the Subject of this Chapter may more readily be understood, we shall premise the five following Particulars; which are all comprised in the foregoing Chapter, or follow immediately from what has been there laid down, *viz.*

1. That as each Point of an Object when viewed by the naked Eye, appears in its pro-

* See the Dissertation on the *horizontal* Moon, annexed to this Part.

per

Part III. Plate V. Pag. 72.

Fig. 32. P. 61.

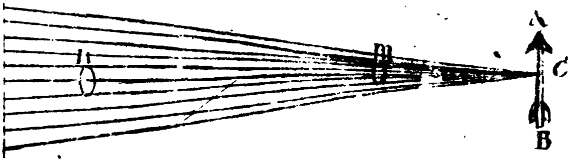
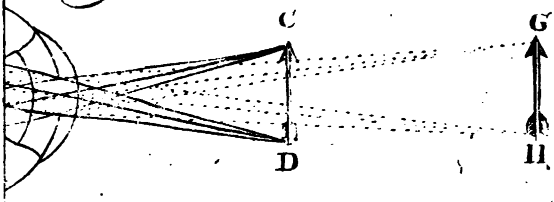


Fig. 33. P. 64.



4. P. 68.

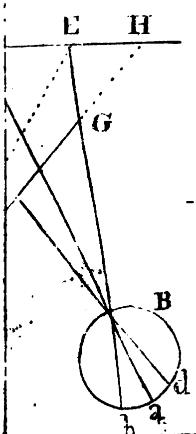
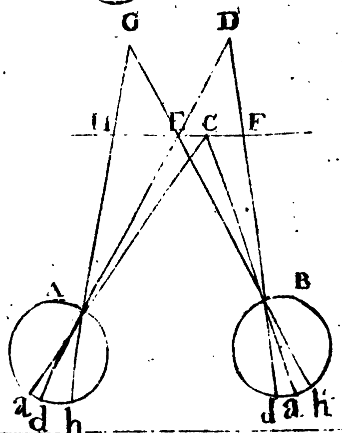


Fig. 35. P. 69.



per Place; and as that Place is always to be found in the Line in which the *Axis* of a Pencil of Rays flowing from it, enters the Eye, we from hence acquire an Habit of judging the Point situated in that Line; and because the Mind is unacquainted with what Refractions the Rays suffer before they enter the Eye, therefore, in Cases where they are diverted from their natural Course by passing through any *Medium*, it judges the Point to be in that Line produced back in which the *Axis* of a Pencil of Rays flowing from it is situated the Instant they enter the Eye, and not in that it was in before Refraction. We shall therefore in what follows, suppose the apparent Place of an Object when seen through a refracting *Medium* to be somewhere in that Line produced back in which the *Axis* of a Pencil of Rays flowing from it proceed after they have passed through the *Medium*.

2. That we are able to judge, though imperfectly, of the Distance of an Object by the Degree of Divergency, wherein the Rays flowing from the same Point of the Object enter the Pupil of the Eye, in Cases where that Divergency is considerable; but because in what follows, it will be necessary to suppose an Object, when seen through a *Medium* whereby its apparent Distance is altered, to appear in some determinate Situation; in those Cases where the Divergency of the Rays at their Entrance

into

into the Eye is considerable, we will suppose the Object to appear where those Lines which they describe in entring, if produced back, would cross each other; though it must not be asserted that this is the precise Distance; because the Brightness, Distinctness, and apparent Magnitude of the Object, on which its apparent Distance in some Measure depends, will also suffer an Alteration by the Refraction of the Rays in passing through that *Medium*.

3. That we estimate the *Magnitude* of an Object by that of the *optic Angle*.

4. That Vision is the *brighter*, the greater the Number of Rays is which enter the Pupil. And,

5. That in some Cases, the *apparent Brightness*, *Distinctness*, and *Magnitude* of an Object are the only Means whereby our Judgment is determined in estimating the Distance of it.

Prop. I. An Object placed within a *Medium* terminated by a plain Surface on that Side which is next the Eye, if the *Medium* be denser than that in which the Eye is (as we shall always suppose it to be, unless where the contrary is expressed) appears *nearer* to the Surface of the *Medium* than it is.

Thus if A be a Point of an Object placed within the *Medium* BCDE (*Fig.* 36), and A*b*, A*c* be two Rays proceeding from thence; these Rays passing out of a denser into a rarer *Medium* will be refracted from their respective Per-

Perpendiculars *bd*, *ce*, and will enter the Eye at H; suppose in the Directions *bf*, *cg*, let then these Lines be produced back till they meet in F; this will be the apparent Place of the Point A; and because the refracted Rays *bf*, *cg* will diverge more than the incident ones A*b*, A*c* (Chap. III. Prop.3.), it will be nearer to the Points *b* and *c*, than the Point A; and as the same is true of each Point in the Object, the Whole will appear to an Eye at H, *nearer* to the Surface BC than it is *.

Prop. II.

* From hence it is, that when one End of a strait Stick is put under Water, and the Stick is held in an oblique Position, it appears bent at the Surface of the Water; *viz.* because each Point that is under Water appears nearer the Surface, and consequently higher than it is.

From hence likewise it is, that an Object at the Bottom of a Vessel may be seen when the Vessel is filled with Water, though it be so placed with Respect to the Eye, that it cannot be seen when the Vessel is empty. To explain this; Let ABCD (*Fig.* 37.) represent a Vessel, and let E be an Object lying at the Bottom of it. This Object, when the Vessel is empty, will not be seen by an Eye at F, because HB the upper Part of the Vessel will obstruct the Ray EH; but when it is filled with Water to the Height GH, the Ray EK being refracted at the Surface of the Water into the Line KF, the Eye at F shall see the Object by Means of that.

In like Manner, an Object situated in the Horizon appears above its true Place, upon Account of the Refraction of the Rays which proceed from it in their Passage through the *Atmosphere* of the Earth. For first, if the Object be situated beyond the Limits of the *Atmosphere*, its Rays in entring it will be refracted towards the Perpendicular, that is, towards a Line drawn from the Point where they enter, to the Center of the Earth which is the Center of the *Atmosphere*, and as they pass on they will be continually refracted the same Way, because they are all along entring a denser Part, the Center of whose Convexity is still the same Point; upon which Account the Line they describe will be a Curve bend-

Prop. II. An Object seen through a *Medium* terminated by plain and parallel Surfaces, appears *nearer*, *brighter*, and *larger*, than with the naked Eye.

For Instance, let AB (*Fig.* 38.) be the Object, CDEF the *Medium*, and GH the Pupil of an Eye, which is here drawn large to prevent Confusion in the Figure. And 1st let RK, RL be two Rays proceeding from the Point R, and entring the denser *Medium* at K and L; these Rays will here by Refraction be made to diverge less (Chap. III. Prop. 2.) and to pro-

ing downwards; and therefore none of the Rays that come from that Object can enter an Eye upon the Surface of the Earth, except what enter the *Atmosphere* higher than they need to do, if they could come in a right Line from the Object; consequently the Object must appear above its proper Place. Secondly, if the Object be placed within the *Atmosphere*, the Case is still the same; for the Rays which flow from it must continually enter a denser *Medium* whose Center is below the Eye, and therefore being refracted towards the Center, that is, downwards as before, those which enter the Eye must necessarily proceed as from some Point above the Object, wherefore the Object will appear above its proper Place.

From hence it is, that the Sun, Moon, and Stars appear above the *Horizon*, when they are just below it, and higher than they ought to do, when they are above it: Likewise distant Hills, Trees, &c. seem to be higher than they are.

Farther, the lower these Objects are in the *Horizon*, the greater is the Obliquity with which the Rays which flow from them, enter the *Atmosphere*, or pass from the rarer into the denser Parts of it, and therefore they appear to be the more elevated by Refraction; upon which Account the lower Parts of them are *apparently* more elevated than the other. This makes them seem shorter than they are; as is evident in the Sun and Moon, which appear of an oval Form when they are in the *Horizon*, their *horizontal* Diameters appearing of the same Length they would do if the Rays suffer'd no Refraction, while their *vertical* ones are shortened thereby.

ceed

ceed afterwards suppose in the Lines K*a* L*b*; at *a* and *b* where they pass out of the denser *Medium*, they will be as much refracted the contrary Way, proceeding in the Lines *ac*, *bd*, parallel to their first Directions (see Chap. IV.); produce these Lines back till they meet in *e*, this will be the apparent Place of the Point R, and 'tis evident from the Figure that it must be nearer the Eye than that Point; and because the same is true of all other Pencils flowing from the Object AB, the Whole will be seen in the Situation *fg*, *nearer* to the Eye than the Line AB. 2d. As the Rays RK, RL would not have entered the Eye, but have passed by it in the Directions K*r*, L*t*, had they not been refracted in passing through the *Medium*, the Object appears *brighter*. 3d. The Rays A*h*, B*i*, will be refracted at *h* and *i* into the less converging Lines *hk*, *il*, and at the other Surface into *k*M, *l*M parallel to A*h* and B*i* produced (see Chap. IV.), so that the Extremities of the Object will appear in the Lines M*k*, M*l* produced, *viz.* in *f* and *g*, and under as large an Angle *f*M*g*, as the Angle A*q*B under which an Eye at *q* would have seen it, had there been no *Medium* interposed to refract the Rays; and therefore it appears *larger* to the Eye at GH, being seen through the interposed *Medium*, than otherwise it would have done. But it is here to be observed, that the nearer the Point *e* appears to the Eye on

Account of the Refraction of the Rays RK, RL, the shorter is the Image *fg*, because it is terminated by the Lines M*f* and M*g*, upon which Account the Object is made to appear less; and therefore the apparent Magnitude of an Object is not much augmented by being seen through a *Medium* of this Form.

Farther it is apparent from the Figure, that the Effect of a *Medium* of this Form depends wholly upon its Thickness; for the Distance between the Lines R*r* and *ec*, and consequently the Distance between the Points *e* and R depends upon the Length of the Line K*a*: Again, the Distance between the Lines AM and *f*M, depends on the Length of the Line *bk*; but both K*a* and *bk* depend on the Distance between the Surfaces CE and DF, and therefore the Effect of this *Medium* depends upon its Thickness.

Prop. III. An Object seen through a convex *Lens*, appears *larger*, *brighter*, and *more distant*, than with the naked Eye.

To illustrate this, let AB (*Fig.* 39.) be the Object, CD the *Lens*, and EF the Eye. 1. From A and B the Extremities of the Object draw the right Lines AY*r*, BX*r* crossing each other in the Pupil of the Eye; the Angle A*r*B comprehended between these Lines, is the Angle under which the Object would be seen with the naked Eye. But by the Interposition of a *Lens* of this Form, whose Property it is to render converging Rays more so (see Chap. IV.) the Rays AY and BX will be made

to cross each other before they reach the Pupil. Therefore the Eye at E, will not perceive the Extremities of the Object by means of these Rays (for they will pass it without entring), but by some others which must fall without the Points Y and X, or between them; but if they fall between them, they will be made to concur sooner than they themselves would have done, and therefore if the Extremities of the Object could not be seen by them, it will much less be seen by these. It remains therefore, that the Rays which will enter the Eye from the Points A and B after Refraction, must fall upon the *Lens* without the Points Y and X; let then the Rays AO and BP be such. These after Refraction entring the Eye at *r*, the Extremities of the Object will be seen in the Lines *r*Q, *r*T produced, and under the *optic Angle* Q*r*T which is larger than A*r*B, and therefore the *apparent Magnitude of the Object will be increased.* 2. Let GHI be a Pencil of Rays flowing from the Point G; as it is the Property of this *Lens* to render diverging Rays less diverging, parallel or converging (see Chap. IV.), it is evident, that some of those Rays which would proceed on to M and N and miss the Eye, were they to suffer no Refraction in passing through the *Lens*, will now enter it; by which means the Object will appear *brighter*. 3. As to the apparent Distance of the Object, that will vary according to the Situation

of

of it with respect to the *Focus* of parallel Rays of the *Lens*. 1. Then, let us suppose the Object placed so much nearer the *Lens* than its *Focus* of parallel Rays, that the refracted Rays KE and LF though rendered less diverging by passing through it, may yet have a considerable Degree of Divergency, so that we may be able to form a Judgment of the Distance of the Object thereby. In this Case, the Object ought to appear where EK, FL produced back concur, which, because they diverge less than the Rays GH, GI, will be beyond G, that is, at a greater Distance from the *Lens* than the Object is. But because both the Brightness and Magnitude of the Object will at the same Time be augmented, Prejudice will not permit us to judge it quite so far off as the Point where those Lines meet, but somewhere between that Point and its proper Place. 2. Let the Object be placed in the *Focus* of parallel Rays, then will the Rays KE and LF become parallel (see Chap. IV.) and though in this Case the Object would appear at an immense Distance, if that Distance were to be judged of by the Direction of the Rays KE and LF, yet upon Account of the Brightness and Magnitude of it, we shall not think it much *farther* from us, than if it were seen by the naked Eye. 3. If the Object be situated beyond the *Focus* of parallel Rays, as in AB (*Fig.* 40), the Rays flowing from thence and falling upon the *Lens*

CD,

CD, will be collected into their respective *Foci* at *a* and *b*, and the intermediate Points *m*, *n*, &c. and will there form an Image of the Object AB; and after crossing each other in the several Points of it, as expressed in the Figure, will pass on diverging as from a real Object. Now if an Eye be situated at *c*, where A*c*, B*c*, Rays proceeding from the extreme Points of the Object, make not a much larger Angle A*c*B, than they would do if there were no *Lens* interposed; and the Rays belonging to the same Pencil do not converge so much as those the Eye would receive, if it were placed nearer to *a* or *b*, the Object upon these Accounts appearing very little larger or brighter than with the naked Eye, is seen nearly in its proper Place; but if the Eye recedes a little Way towards *ab*, the Object then appearing both brighter and larger, seems to approach the *Lens* *: which is an evident Proof of what has

* That the Object should seem to approach the *Lens* in this Case, was a Difficulty that exceedingly puzzled the learned *Barrow*, and which he pronounces insuperable, and not to be accounted for by any Theory we have of Vision. *Molineux* also leaves it to the Solution of others, as that which will be inexplicable, till a more intimate Knowledge of the visive Faculty, as he expresses it, be obtained by Mortals.

They imagined, that seeing an Object appears *farther off*, the *less* the Rays *diverge* which fall upon the Eye; if they should proceed *parallel* to each other, it ought to appear *exceedingly remote*, and if they should *converge*, it should then appear *more distant still*: The Reason of this was, because they looked upon the apparent Place of an Object, as owing to the Direction of the Rays whatever it was, and not at all to its apparent Magnitude or Splendour.

been

been so often asserted, *viz.* that we judge of the Distance of an Object in some Measure by its Brightness and Magnitude *; for the Rays converge the more the farther the Eye recedes from the *Lens*; and therefore if we judged of the Distance of the Object by the Direction of the Rays which flow from it, we ought in this Case to conceive it at a greater Distance, than when the Rays were parallel, or diverged at their Entrance into the Eye.

Prop. IV. If an Object be placed farther from a convex *Lens*, than its *Focus* of parallel Rays, and the Eye be situated farther from it on the other Side, than the Place where the Rays of the several Pencils are collected into their respective *Foci*, the Object appears *inverted*, and *pendulous in the Air*, between the Eye and the *Lens*.

To explain this, let AB (*Fig.* 40.) represent the Object, CD the *Lens*, and let the Rays of the Pencil ACD be collected in *a*, and those of BCD in *b*, forming there an inverted Image of the Object AB, and let the Eye be placed in F: 'Tis apparent from the Figure, that some of the refracted Rays which pass through each Point of the Image, will enter

* Perhaps it may proceed from our judging of the Distance of an Object in some Measure by its Magnitude, that that Deception of Sight commonly observed by Travellers may arise; *viz.* that upon the first appearing of a Building larger than usual, as a *Cathedral* Church, or the like, it generally seems nearer to them, than they afterwards find it to be.

the

the Eye as from a real Object in that Place, and therefore the Object AB will appear there, as the Proposition asserts. But we are so little accustomed to see Objects in this Manner, that it is very difficult to perceive the Image with one Eye; but if both Eyes are situated in such a Manner, that Rays flowing from each Point of the Image may enter both, as at G and H, and we direct our optic Axes to the Image, 'tis easy to be perceived.

If the Eye be situated in *a* or *b*, or very near them on either Side, the Object appears exceedingly confused, *viz.* if at *d*, the Rays which proceed from the same Point of the Object converge so very much, and if at *e*, they diverge so much, that they cannot be collected together upon the *Retina*; but fall upon it as if they were the Axes of so many distinct Pencils coming through every Point of the *Lens*; wherefore little more than one single Point of the Object is seen at a Time, and that appears all over the *Lens*; from whence nothing but Confusion arises.

If the *Lens* be so large that both Eyes may be applied to it, as in *h* and *k*, the Object will appear double; for 'tis evident from the *Figure*, that the Rays which enter the Eye at *h* from either Extremity of the Object A or B, do not proceed as from the same Point with that from whence those which enter the other at *k* seem to flow; the Mind therefore is here deceived,

and looks upon the Object as situated in two different Places, and therefore judges it to be double.

Prop. V. An Object seen through a concave *Lens* appears *nearer*, *smaller*, and less *bright*, than with the naked Eye.

Thus, let AB (*Fig.* 41.) be the Object, CD the Pupil of an Eye, and EF the *Lens*. Now, as it is the Property of a *Lens* of this Form, to render diverging Rays more so, and converging ones less so, the diverging Rays GH, GI, proceeding from the Point G, will be made to diverge more, and so to enter the Eye as from some nearer Point *g*; and the Rays AH, BI, which converge, will be made to converge less, and to enter the Eye as from the Points *a* and *b*; wherefore the Object will appear in the Situation *agb*, *less* and *nearer* than without the *Lens*. Farther, as the Rays which proceed from G, are rendered more diverging, some of them will be made to pass by the Pupil of the Eye which otherwise would have entered it, and therefore each Point of the Object will appear *less bright* *.

Prop. VI.

* From what has been observed about the Properties of convex and concave *Lenses*, we may see the Reason why the former Sort are made Use of by *old People* to help their Sight; and the latter by those who are *purblind*. *Old People*, as was observed before, having the *Tunica Cornea* of their Eyes too flat, require that the Object be placed at a greater Distance from them, than other People whose Eyes are of a just Form, that the Rays which enter the Pupils of their Eyes from the same Point of the Object, may not di-

Prop. VI. An Object seen through a *polygonous* Glass, that is, such as is terminated by several plain Surfaces, is *multiplied* thereby.

For Instance, let A (*Fig.* 42.) be an Object, and BC a polygonous Glass terminated by the plain Surfaces BD, DE, *&c.* and let the Situation of the Eye F be such, that the Rays AB being refracted in passing through the Glass, may enter it in the Direction BF, and the Rays AC in the Direction CF. Then will the Eye by means of the former, see the Object in G, and by the latter in H; and by means of the Rays AI, the Object will appear also in its proper Situation A.

Thus much for the Principles of *Dioptics*, and the Solution of some obvious *Phænomena* which tend to confirm the same: Those which yet remain to be accounted for, shall according to the Method we have hitherto observed, be treated of in the Dissertations of this Part.

verge too much. Now a convex *Lens* makes those Rays diverge less, as they would naturally do if the Object was placed farther off. Again, those who are *purblind*, having the *Tunica Cornea* too protuberant, require such a *Lens* as may render those Rays more diverging, lest they should be collected into their respective *Foci* before they fall upon the *Retina*; and therefore *Lenses* of the concave Sort are of Use to them.

DISSERTATION I.

Of the Horizontal Moon.

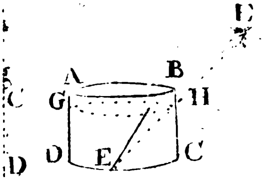
THE *Phænomenon* of the *horizontal* Moon is this: when the Moon is just above the Surface of the Earth, either immediately after she is risen, or just before she sets, she appears four or five Times greater in Diameter, than when she is in her Meridian Altitude: And yet her apparent Diameter, if taken by an Instrument, is found to subtend the same Angle in either Situation *.

The Moon's apparent Diameter being found to subtend the same Angle, whether she be in the Horizon or Meridian, it is evident the Image of her projected upon the *Retina* of an Eye, is of the same Dimensions in either Case; and therefore that she should appear of a different Magnitude in one Situation from what

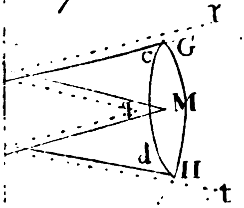
* What is said here of the Moon's Diameter, as taken by an Instrument, must be understood of her *horizontal* Diameter, and not of her *vertical* one, for the Length of this is diminished by Refraction (as explained Chap. VII. Note the first); and therefore if it be taken by an Instrument, it will not be found to subtend the same Angle in the Horizon as in the Meridian: But notwithstanding this, it appears longer to the naked Eye when in the former, than in the latter Situation, as well as the horizontal Diameter.

she

Fig. 37. P. 75.



P. 76.



P. 78.

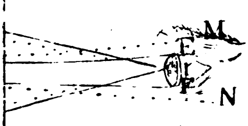


Fig. 42. P. 85.

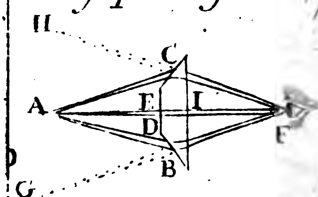
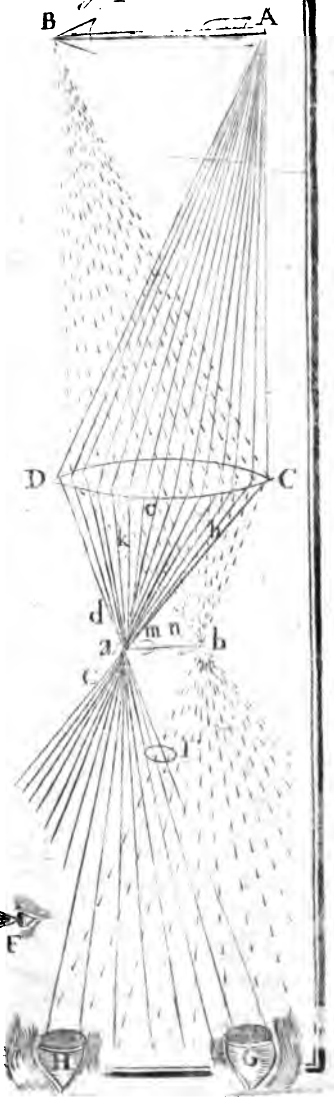


Fig. 40. P. 82.



he does in the other, has always been Matter of great Speculation among the *Connoiſſeurs* both in *Optics* and *Aſtronomy*. *Des Cartes* was of Opinion, that we think the Moon larger when ſhe is in the Horizon, than when ſhe is in the Meridian, becauſe in the former Caſe by comparing her Diſtance with that of interpoſed Objects, we imagine it greater than when ſhe is elevated: And that as we judge her Diſtance greater in that Situation, we of Courſe think her Diameter longer, becauſe it ſubtends the ſame Angle in either Caſe. But more of this by and by, when we come to the Explication Dr. *Wallis* has given of this Matter.

Gaſſendus was of Opinion, that becauſe the Moon appears leſs bright when in the Horizon than in the Meridian, we view her in the former Situation with a larger Pupil, than we do in the latter; and from thence he concludes, that the Image of her upon the *Retina* muſt be larger. But this is contrary to the Laws of *Optics*; for if the refractive Power of the Humours of the Eye collects the Rays of the ſeveral Pencils into their reſpective *Foci* upon the *Retina* (and there is no Reaſon to ſuppoſe the contrary in this Caſe) the Breadth of the Pupil makes no Alteration in the Magnitude of the Image; becauſe the Situation of thoſe *Foci* is determined by the *Axes* of the ſeveral Pencils, which croſſing each other in the Center

ter of the Pupil (as was shewn Chap. VI. pag. 61. in the Note) pass on to the same Points of the *Retina*, whether the Pupil be broad or narrow.

Molineux in the Philosophical Transactions No. 187. tells us of a certain *French Abbé*, that revived the forementioned Supposition of *Gassendus*, and adding two others of his own, endeavour'd to account for this *Phænomenon.* His Suppositions were these, *viz.* "That this "contracting and enlarging the Pupil (sup-"posed by *Gassendus*) causeth a different "Shape in the Eye; an open Pupil making "the Christalline flatter, and the Eye longer, "and the narrower Pupil shortning the Eye, "and making the Christalline Humour more "convex. The first attends our looking at "Objects that are remote, or which we think "so, the latter accompanies the viewing Ob-"jects nigh at Hand. Likewise an open Pu-"pil and flat Christalline attends Objects of "a more sedate Light, whilst Objects of more "forcible Rays require a greater Convexity, "and narrower Pupil. From these Positions, "continues *Molineux*, the *Abbé* endeavoured "to give an Account of our *Phænomenon*, as "follows. When the Moon is nigh the Ho-"rizon, by Comparison with interposed Ob-"jects, we are apt to imagine her much far-"ther from us than when more elevated, and "therefore we order our Eyes as for viewing

"an

"an Object farther from us; that is, we some-
"thing enlarge the Pupil, and thereby make
"the Christalline flatter: moreover the Dus-
"kishness of the Moon in that Posture does not
"so much strain the Sight; and consequently
"the Pupil will be more large, and the Chris-
"talline more flat; hence a larger Image shall
"be projected on the Fund of the Eye, and
"therefore the Moon shall appear larger.
"These two forementioned Accidents, *viz.*
"the Moon's imaginary Distance and Dus-
"kishness gradually vanishing as she rises, a
"different *Species* is hereby introduced in the
"Eye, and consequently she seems gradually
"less and less, till again she approaches nigh
"the Horizon.

As to what is taken for granted in this Solution concerning a Change in the Christalline Humour and Form of the Eye, upon viewing an Object in a *dusky* or faint Light, that seems to be very ill grounded. We know of no such Connection between the Muscles of the *Iris* and those of the *Ligamenta Ciliaria*, as is necessary to produce this Effect. And the Coats of the Eye are not so pliable, as easily to admit of an Alteration in their Form *. Could the Author have made good his other Supposition, *viz. That by Comparison with*

* See what has been observed concerning the Power we have of making an Alteration in the Eye, in order to see *distinctly*. (Chap. VI. pag. 62.)

in-

interposed Objects we are apt to imagine her much farther from us, than when more elevated, he need not have had Recourse to any other; this alone would have been sufficient; but *hic Labor est.* This alone, I say, would have been sufficient; for if by comparing her Distance with that of interposed Objects, we imagine it greater when she is in the Horizon, than when she is in the Meridian; as she subtends an equal Angle in both Cases, we must in Consequence thereof (agreeably to *Des Cartes*'s Notion above-mentioned) imagine her to be bigger in the former than in the latter; because a distant Object cannot subtend the same Angle at the Eye that one which is nearer does, unless it be proportionably larger *.

The famous *Hobbs* endeavour'd at a Solution of this *Phænomenon*, but it is hardly worth mentioning: The Figure he has drawn to explain his Solution by, seems to have been the Occasion of his Errour. He draws a Circle to represent that blue Surface commonly called the Sky, in which the heavenly Bodies seem to be fixed, and concentric to this, a lesser, to represent the Surface of the Earth, but vastly too big in Proportion; so that a Spectator upon the Surface of this Earth, is considerably nearer to the upper Part of the

* See Chap. VI. p. 71.

other

other Circle than to the Sides of it: Wherefore an Object that subtends the same Angle at different Heights, must necessarily hide a greater Portion of that Ark when it is in the Horizon, than when it is in the Meridian; because that Ark is farther behind the Object in the former than in the latter Situation; from whence he concludes that the Moon must appear bigger in that Situation than in the Meridian. Had he drawn his Circles in any tolerable Proportion to that which he designed them to represent, he would easily have seen his Mistake.

A few Years ago Mr. *De Veil* published a Treatise upon the Subject of the horizontal Moon, which he dedicates to the Ladies of *Northampton*. If I remember right, his Solution of it was in the following Manner. 1. When an Object is placed beyond the *Focus* of parallel Rays of a convex *Lens*, the farther the Eye (situated on the other Side the *Lens*) receeds from it towards the *Focus* of the Rays which flow from that Object, the larger that Object appears. 2. Rays of Light flowing from the Moon, and passing thro' the *Atmosphere* of the Earth, are collected into a *Focus* on the other Side of it. 3. When the Moon is in the Horizon, we are nearer to this *Focus* by almost a *Semidiameter* of the Earth, than when she is in the Meridian: And therefore the Moon ought to appear larger when in the former than in the latter Situation.

The ſecond Propoſition in this Solution is true, but not applicable in the preſent Caſe; for unleſs we conſider the Refraction that Rays of Light which flow from the Moon, and paſs through the Atmoſphere of the Earth, ſuffer in their Emerſion, that is, while they paſs through the latter half of it, as well as that which they ſuffer in their Immerſion, or while they paſs through the former half, we ſhall find that they will not be collected into their reſpective *Foci* on the other Side the Earth, as this Gentleman imagines: Which if it can be ſhewn, his Solution falls to the Ground of Courſe; for the Refraction which the Rays ſuffer in their Emerſion is not to be taken into Conſideration, becauſe they reach the Eye of a Spectator upon the Earth as ſoon as they have paſſed through the firſt half of the Atmoſphere, when the Moon is in his Horizon; and before they have paſſed through that half, when ſhe is in his Meridian.

Let us then imagine two Rays flowing from one and the ſame Point of the Surface of the Moon, it being neceſſary in order to conſtitute a *Focus* that ſuch Rays ſhould after Refraction meet in a Point; the meeting of ſuch as flow from different Points in the ſame Surface is not ſufficient; if it were, we might then have *Foci* where we pleaſed, and that as well without refracting or reflecting Surfaces as with them. And let the firſt of thoſe Rays fall perpendicularly upon the Atmoſphere of the Earth, and be

be ſuppoſed to paſs through the Center of it, and let the other after Refraction paſs by the Surface of the Earth. Now the Moon's *Parallax*, that is, the Angle under which the Semidiameter of the Earth is ſeen from the Moon, being about one Degree, it is evident that theſe Rays muſt, before their Incidence upon the Earth's Atmoſphere, diverge the one from the other by ſuch an Angle. But it appears from Sir *Iſaac Newton*'s Table of Refractions publiſhed by Dr. *Halley* in the Philoſophical Tranſactions, No. 368, that when any of the Heavenly Bodies appears in the Horizon, the Rays by which it is ſeen, are refracted but by an Angle of thirty three Minutes and forty five Seconds; and therefore the Ray which we have ſuppoſed after Refraction to paſs by the Surface of the Earth, will be refracted only by ſuch an Angle. Which falling conſiderably ſhort of one Degree, the Angle by which it diverged from the perpendicular one before Refraction; it will be ſo far from being made to converge towards it thereby, that it will ſtill be in a State of Divergency from it. And therefore Rays flowing from the Moon and refracted only in their Immerſion into the Atmoſphere of the Earth, will not be collected into their reſpective *Foci* on the other Side: Which was to be ſhewn.

Dr. *Wallis* in the Philoſophical Tranſactions No. 187, gives us a Solution of the horizontal Moon (or rather an Explication of what *Des Cartes* had given before) which is as follows.

 He

He ascribes this *Phænomenon* to the Deception of the Imagination, and accounts for that Deception in the following Manner. He observes first, that the Imagination doth not estimate the Greatness of an Object seen, by the *optic Angle* only, but by this compared with the supposed Distance. So that if two Things are seen under the same or equal Angles, and if upon any Account whatever we apprehend one of these to be farther from us than the other, that which we apprehend to be farther from us, will to the Imagination appear greater. Secondly, That one great Advantage for estimating the apparent Distance of any Thing, is from the Variety of intermediate Objects between the Eye and the Thing seen; for then the Imagination must allow Room for all these Things.

" Now says he, when the Sun * or Moon is " near the Horizon, the Prospect we have of " Hills and Vallies, Plains and Woods, *&c.* " represent to our Imagination a great Dis" tance capable of receiving all these. Or if " it happens that these interposed Objects are " not actually seen; yet having been accus" tomed to see them, the Memory suggests to " us a View as large as is the visible Horizon.

" But when the Sun or Moon is in an high" er Position, we see Nothing between us and " them (unless perhaps some Clouds) and

* For the Sun appears larger in the *Horizon*, as well as the Moon.

" there-

" therefore Nothing that can present to our
" Imagination so great a Distance as the other
" is. And therefore though both be seen under
" the same Angle, they do not appear (to
" the Imagination) of the same Bigness, be-
" cause not fansied at the same Distance: But
" that near the Horizon is judged bigger (be-
" cause supposed farther off) than the same,
" when at a greater Altitude.

This is the Solution which has hitherto generally been received; but I am told that a more probable one will shortly be given by Dr. *Smith*, Plumian Professor in the University at *Cambridge*, in a Treatise of *Optics* he is now preparing for the Press.

If I might be allowed to mention any Thing of my own, after these great *Genius*'s have given their Opinions upon this Matter, it should be this, *viz.* That I have often thought, that he who would give a rational Account, why the Sun or Moon appears farther from us in the Horizon, than in the Meridian (for that is all that is requisite towards a Solution of the Horizontal Moon, as has been already observed) should first show, why that apparent azure Surface we call the Sky, does not seem to be an entire concave *Hemisphere*, but only a Portion of such an one. For our judging the Heavens to be no more than such a Portion, is undoubtedly the Cause why we judge both the Sun, Moon, and Stars to be farther from us when in the Horizon, than in the Meridian; because

because we have Nothing else we can refer their Places to, but that.

Now possibly the Cause why we think the Heavens of that Form, may after all be only this, *viz.* that, as the Rays which come from the upper Parts of that imaginary Surface the Sky, pass through a less Portion of the *Atmosphere*, than such as come from the horizontal Parts of it, the Sky appears to us more *distinctly* and generally more *bright* in those Parts, than in the latter; and therefore, since we daily observe that those Objects which appear most *distinct*, are, generally such as are *nearest* to us; and also as *bright* Objects, when we have Nothing but bare Imagination to determine us in estimating the Distance of them, appear *nearer* to us than the same Objects when less so *, we think the upper Parts of the Sky *nearer* us than the lower. Wherefore, since we refer all the heavenly Bodies to this Surface, we necessarily imagine them farther from us, and consequently larger, and also more distant from each other †, when near the Horizon, than when they are arrived at their meridian Altitude.

* See what has been said concerning the Brightness of an Object being a Means whereby it appears nearer us, under Prop. 3, of the 7th Chapter.

† The apparent horizontal Distance of two Stars from one another, is observed to be greater when they are in the Horizon, than in the Meridian.